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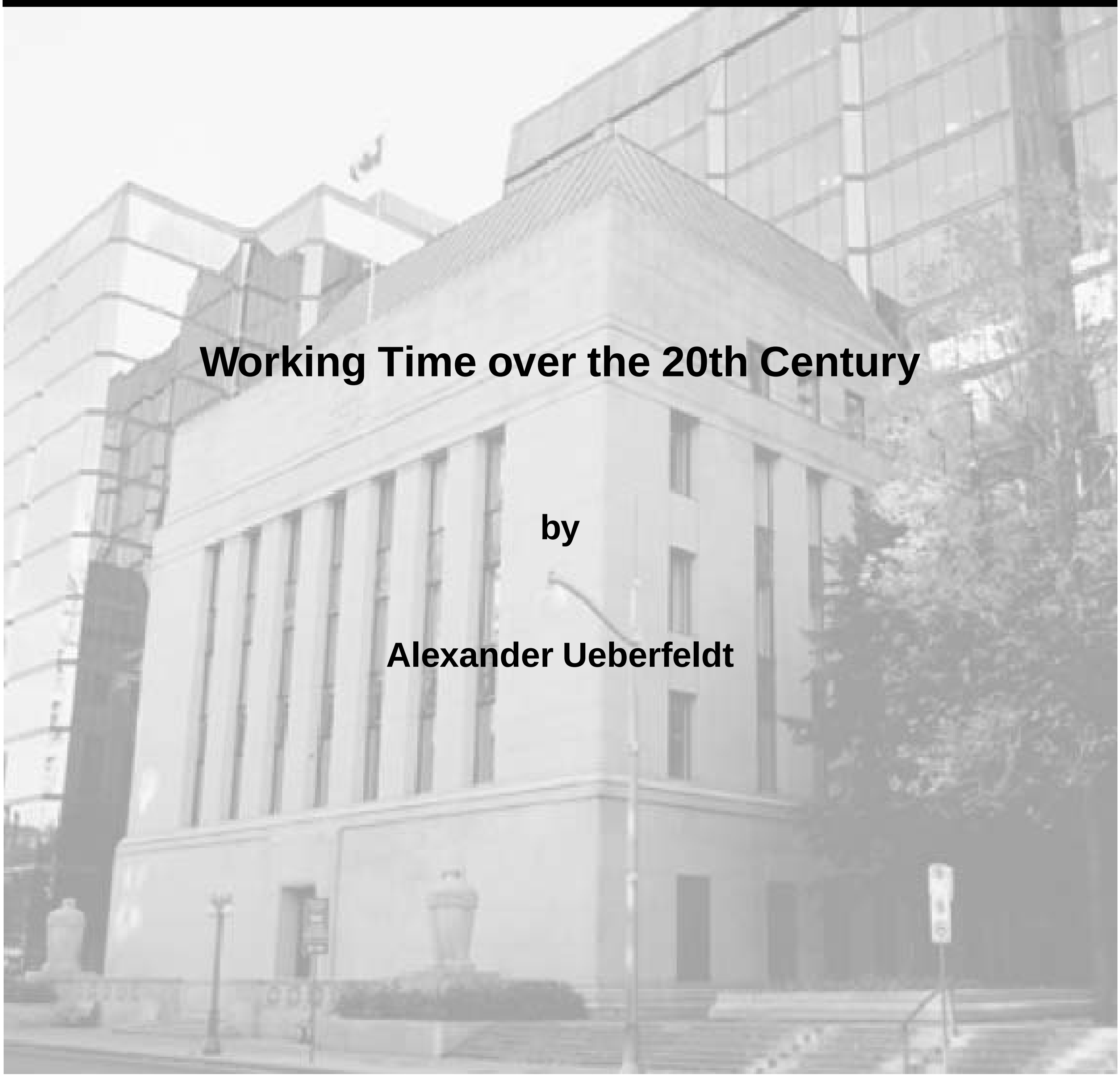

Working Time over the 20th Century

by

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Abstract

From 1870 to 2000, the workweek length of employed persons decreased by 41 per cent in industrialized countries. The employment rate, employment per working age person, displays large movements but no clear secular pattern. This motivated the question: What accounts for the large decrease in the workweek length and developments in the employment rate over the past 130 years? The answer is given in a dynamic general-equilibrium model with supervisory and production workers. Over time, both types of workers become more productive. In a calibrated version of the model, productivity gains of supervisors account for a large fraction of the decline in the workweek length in Japan, the United Kingdom, and the United States. The model, augmented to include taxes, government spending, and technological progress, captures the movement in the employment rates of the three countries.

JEL classification: E13, E24, O11
Bank classification: Economic models; Labour markets; Productivity

Résumé

De 1870 à 2000, la durée hebdomadaire de travail a diminué de 41 % dans les pays industrialisés. Le taux d'emploi, soit le ratio de l'emploi à la population en âge de travailler, a varié considérablement durant cette période sans que se dégage une tendance historique claire. Comment expliquer alors la nette réduction de la durée de la semaine de travail et l'évolution du taux d'emploi pendant ces 130 années? Pour répondre à cette question, il est proposé un modèle dynamique d'équilibre général dans lequel le personnel est affecté soit à la supervision, soit à la production. Ces deux catégories de personnel améliorent leur productivité au fil du temps. Dans une version calibrée du modèle, les gains de productivité du personnel de supervision sont à l'origine d'une large part de la baisse de la durée hebdomadaire de travail observée au Japon, au Royaume-Uni et aux États-Unis. Étoffé par l'ajout des impôts, des dépenses publiques et du progrès technique, le modèle parvient à rendre compte de l'évolution du taux d'emploi dans ces trois pays.

Classification JEL : E13, E24, O11
Classification de la Banque : Modèles économiques; Marchés du travail; Productivité

1. Introduction

During the period 1870 to 2000, there was a large decline in average hours worked in what are today's advanced industrialized countries. Here, average hours worked refers to the total hours worked in the market sector relative to the population age 15 or older. The observation is based on a sample of 15 countries that are now industrialized and for which data are available. The mean decrease in average hours worked was 47 per cent. This decrease can be decomposed into two parts: On the one hand, there was a strong decrease in weekly hours worked per employed person: the workweek length. On the other hand there was no such clear pattern for the number of employed persons per population age 15 or older: the employment rate. There have been large differences in the movement of the employment rate across countries. Some countries experienced a large increase (United States, 34 per cent) while others experienced a large decrease (Spain, 30 per cent). A population weighted average of the workweek and the employment rate over the country sample shows a decline in the workweek by 41 per cent, while the employment rate decreases by eight per cent.

This paper addresses the following questions: What accounts for the decline in average hours worked? What accounts for the changing composition of average hours worked?

A theory that accounts for the movement of both the workweek length and the employment rate can be used to resolve some open issues raised by other researchers. One of them is that "the major puzzle about the U.K. Great Depression is the large fall in the employment rate" (Cole and Ohanian 2002, 41). Another open issue is the decrease in the workweek length in Japan from 1988 to 1993. This decrease is considered by Hayashi and Prescott (2002). They suggest that institutional constraints were responsible for the high workweek length prior to 1988 and that the subsequent decline of the workweek length was due to the removal of these institutional constraints. The theory proposed in my paper can be used to evaluate Hayashi and Prescott's theory as well as to consider Cole and Ohanian's puzzle.

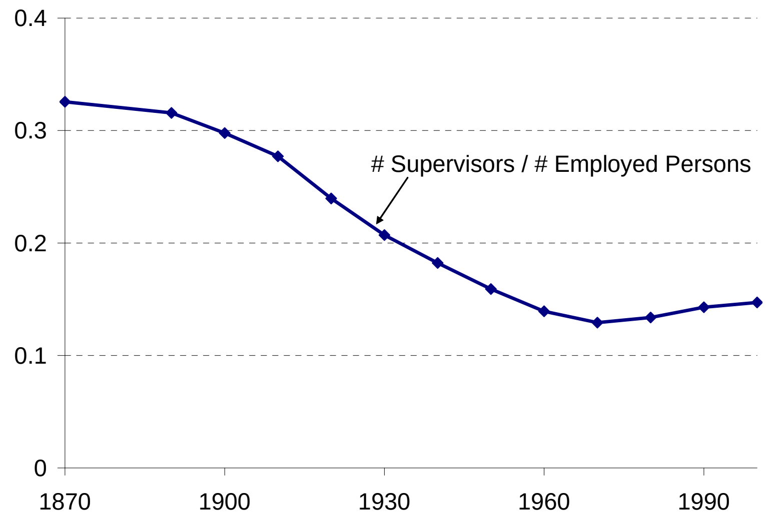
Besides the open issues in the literature this theory also has policy applications. The consequences of both workweek length constraining policies and tax policies with respect to their labour supply implications and their welfare impact can be evaluated. Over the course of the last 130 years, many governments enacted constraints on weekly working time. Examples are the United States' Fair Labor Standards Act of 1938 and the change in taxation of overtime in France in 1998. A recent article points out the significance of taxes for aggregate labour supply (Prescott 2004). Among other things, this article mentions that the 1986 tax reform in the U.S. and the 1998 tax reform in Spain were followed by increases in the aggregate labour supply. When I break the aggregate labour supply into its two margins, I find that all of the increase for these two countries was in the employment rate. Following the tax reform in the U.S., the employment rate increased by 4 per cent from 1986 to 1996, while the workweek stayed constant. A similar picture can be found following the tax reform in Spain. Between 1998 and 2002, the employment rate went up by 12 per cent, while the workweek decreased slightly by 2 per cent. To address the long-run consequences of workweek length constraining policies and tax policies, a theory of aggregate labour supply is needed.

To answer the stated questions, I consider a dynamic general-equilibrium model with two labour supply margins: the workweek length and the employment rate. Previous literature focuses either on one country, a shorter time frame, or just a particular aspect of labour supply. My model differs from the previous literature in the scope of the project and the technology used. I consider multiple countries, a large time frame (1870 to 2000), and both labour supply margins (as mentioned above). The technology is motivated by the fact that employed persons are not a homogeneous group. To capture this fact, I introduce an intermediate production activity. This activity uses an ex ante homogeneous number of persons with a certain workweek length as input. These persons are subdivided into production and supervisory workers. Production workers are all employed persons that are supervised, and supervisory workers are essentially all the other employed persons. The supervisory workers provide a service used as an input to the production activity which produces the final output. Technological progress occurs both for the supervisory and the production activity. The proposed technology is consistent with the secular decline in the number of supervisors relative to the number of all employed persons as displayed in census data (see Figure 1).

Prescott (2003) and Alpanda and Ueberfeldt (2003) consider macroeconomic models with two labour margins. They find that these standard models cannot account for the secular decrease in the workweek length. The proposed theory in this paper can. The main driving force behind the ability of my model to account for the decline in the workweek length is technological progress in the supervisory activity.

For a model calibrated to match certain key characteristics of the U.S. economy, I find that the introduction of technological progress in the supervisory activity accounts for 77 per cent of the secular decline in the U.S. workweek length. Here, secular refers to the period 1870 to 2000. Furthermore, once I incorporate government expenditures, productivity increases for both activities, and taxes on labour income, my model accounts for 65 per cent of the secular increase in the U.S. employment rate. The main driving forces here are technological

Figure 1: Supervisors relative to all employed persons.



progress in the supervision activity and the increase in taxes on labour income.

Using the calibrated model to look at the long-run labour supply of the U.K. and Japan, I find that the model accounts respectively for 45 and 61 per cent of the decline in the workweek length. Furthermore, the model accounts for 87 and 175 per cent of the decrease in the employment rate of the United Kingdom and Japan, respectively. According to the model, the main reason for the decrease in the U.K. employment rate, as compared with the increase in the U.S. employment rate, is the low increase in the U.K.'s productivity (in both sectors). The U.S. experienced twice the increase in productivity over the period 1870 to 2000. Concerning the decrease in the employment rate of Japan, as compared with the increase in the U.S. employment rate, the model suggests that it is mainly due to a larger increase in the effective tax on labour income. Japan experienced a much larger increase in the effective tax on labour income. The model is overpredicting the decrease in the employment rate in the case of Japan, due to the presence of institutional constraints in the Japan of 1928 to 1945. These constraints are not part of the model.

The theory used in this paper is a dynamic version of the model introduced in its basic form by Hornstein and Prescott (1993). The basic model in its dynamic form has been used in various contexts, including the analysis of the Japanese depression of the 1990s (Hayashi and Prescott 2002), the U.S. fiscal policy during World War II (Braun and McGrattan 1993), and the overtime taxation in France (Ozuna and Ríos-Rull 2003).

Some economic historians consider the issue of decreases in the working time per employed person. Notable here are Bienefeld (1972) for the United Kingdom, Schneider (1984) for Germany, and Whaples (1990) for the United States. Each of them looks at a long time period and describes the development of working hours for employed persons. Some general theories for the decline in the workweek length are considered in either a descriptive argumentative way or a partial equilibrium framework.

The paper is organized as follows: Section 2 presents in detail the basic facts outlined above, and section 3 describes the model and theoretical results. Section four presents the calibration of the model's parameters. The same section contrasts the model's implied change in the long-run aggregate labour supply of the U.S., the U.K., and Japan with that in the data. The paper concludes with a summary and suggestions for further research.

2. Facts

During the period 1870 to 2000, working time in today's industrialized countries changed drastically. In this section I will consider this change and derive three main facts. To analyze working time I put together a data set for 15 countries. These countries are: Australia, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, Spain, Sweden, the United Kingdom, and the United States. Using Maddison's[1] measure of real GDP per capita in purchasing power parity terms, I find that the countries considered include 14 of the 20 richest countries in the world in 1870. The only country not in the top 20 group in 1870 is Japan. The main reason for the country selection is data availability with respect to employment and weekly hours worked.

For my study, I make use of the following decomposition of average hours worked:

$$H/N = \underbrace{\frac{H}{E}}_{h} \times \underbrace{\frac{E}{N}}_{e},$$

where H stands for total hours worked, N is the population age 15 or older, and E is the number of working persons.

I will refer to e as the employment rate and to h as the workweek length. Breaking

[1] The dataset is available at <http://www.eco.rug.nl/~Maddison/> under historical statistics. It contains 63 countries for the year 1870.

average hours worked into the workweek and the employment rate is helpful in identifying the main driving force behind the decline in average hours worked.

Below I will present three main facts about average hours worked and its components. As a first step, in Table 1 I show the population weighted averages of the workweek length, the employment rate, and average hours worked for the sample.

Table 1: Average labour supply for the data sample.

	H/N	h	e
1870	41	64	0.63
2000	22	38	0.58
% change 1870 - 2000	-46	-41	-8

These numbers show that there was a strong decline in average hours worked. This decline was mainly driven by the decline in the workweek. They also emphasize the compositional change of average hours worked: a movement from the workweek towards the employment rate. Notice that the displayed decline in the employment rate is misleading. This will become more visible below, when I present three facts.

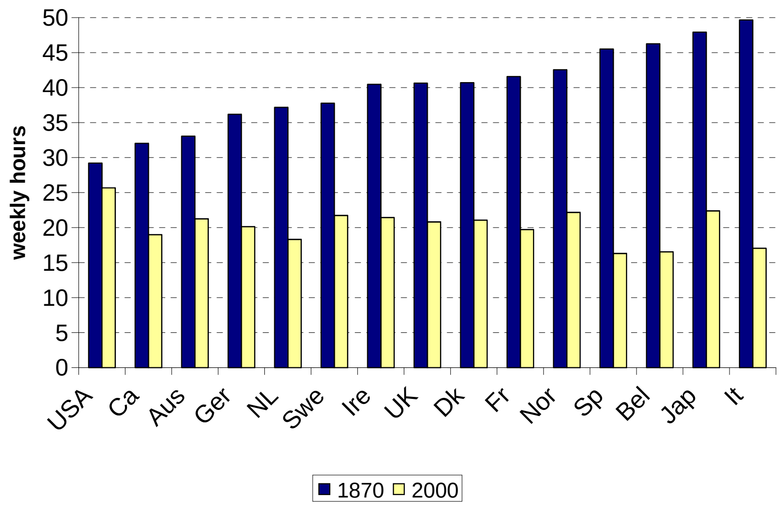
Fact 1: There was a large decline in average hours worked (H/N).

Figure 2 presents average hours worked for the 15 countries in the sample. For each of the countries average hours worked for 1870 and 2000 are displayed. The main thing to note is that for all countries the average hours worked were considerably longer in 1870 than in 2000. Overall, the decrease in average hours worked was between 12 and 66 per cent.

Furthermore, some countries changed their relative position. For example, the U.S. in 1870 had the shortest average working time while Italy had the longest; by 2000 these relative positions had changed. In Italy, average hours worked were among the shortest in the sample, while average hours worked in the U.S. were the longest.

Another aspect of interest is that the countries that initially had a shorter average working time experienced relatively small decreases over time (e.g., the U.S. and Australia), while those with an initially longer average working time had large decreases over time (e.g., Japan and Italy). As a test, I find a high negative correlation between the level of average hours worked in 1870 and the per centage change between 1870 and 2000. This indicates a weak

Figure 2: Average hours worked, 1870 to 2000.



form of convergence with respect to average hours worked.

Fact 2: The decline in average hours H/N is mainly driven by a decline in the workweek (h=H/E).

In Figure 3 the workweek lengths of the 15 countries is displayed. As before, the sample countries are ordered by their position in 1870. The key thing to observe is the large decline in the workweek length, ranging from 32 to 53 per cent.

The decrease in the workweek length is even stronger than the decrease in average hours worked. This leads to the observation that the composition in average hours worked has changed. In particular, the importance of hours worked per employed person has decreased, while the importance of the employment rate has increased.

Fact 3: There is no common pattern in the employment rate.

There are countries that experienced an increase in the employment rate and countries that experienced a decrease (see Figure 4). The per centage change over the whole period ranges from -43 per cent to +35 per cent.

Notice the fairly strong negative relation between the level of the employment rate in 1870 and the decline over the period 1870 to 2000. This observation is most pronounced for the six countries that had the lowest employment rate in 1870 (the U.S., Germany, Canada,

Figure 3: Workweek length, 1870 to 2000.

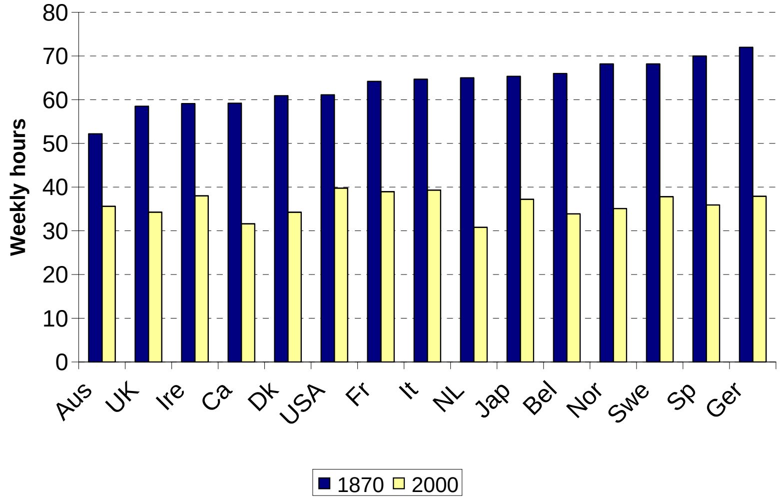
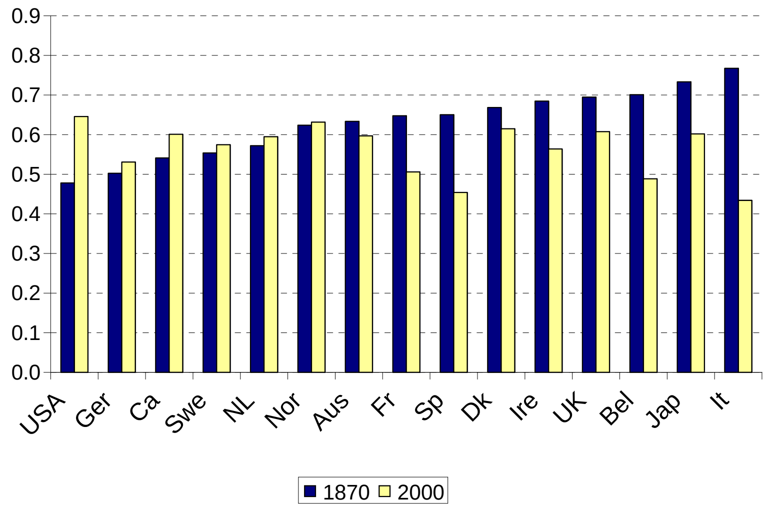


Figure 4: Employment rate, 1870 to 2000.



Sweden, the Netherlands, and Norway). All of these countries experienced an increase, while all the remaining countries experienced a decrease.

In a cross-country comparison, I find that the employment rate across countries for both 1870 and 2000 is about 60 per cent more variable than the workweek length. Here variability is measured as standard deviation relative to the mean value.

This section points out important long-run trends with respect to hours worked. Any good theory of aggregate labour supply must account for the presented main facts.

3. The Model and Its Implication

In this section, I outline the model economy, define an equilibrium, and derive some implications of the model.

3.1 The model economy

The commodities traded at a given point in time are: consumption when employed, c_1, consumption when not employed, c_0, the measure of persons employed, e, the capital stock rented, k, and the investment good, x.

There is an infinitely lived stand-in family with two types of family members: employed and not-employed persons. The family gets to decide what fraction of its members work, e, how much a working person consumes, c_1, and how much a non-working member consumes, c_0. Given a wage relation that depends on hours worked, the family sets a unique weekly working time[2], h, for all working members. Aside from this, the family also determines how much is invested, x, and how much capital is rented out, k:

$$\max_{c,e,h,x,k\geq 0} U(\{c_{1,t}, c_{0,t}, e, h\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t N_0 \lambda^t \left\{ e_t \left[\frac{c_{1,t}^{\alpha} (1-h_t)^{1-\alpha}}{1-\sigma} \right]^{1-\sigma} + (1-e_t) \left[\frac{c_{0,t}^{\alpha}}{1-\sigma} \right]^{1-\sigma} \right\}$$

[2] I focus on weekly hours, since the decline in weekly working time is dominating the decline in working days per year. For Germany the decline in working time is to 80 percent due to a decrease in the weekly working time. The same is true for most countries in the sample.

s.t.

$$\begin{aligned}
(1+\tau_{c,t})\left(e_t c_{1,t}+(1-e_t)c_{0,t}\right)+x_t &\leq (1-\tau_{k,t})\, r_t k_t+(1-\tau_{e,t})\, w_t\left(h_t\right) e_t+\tau_{k,t}\delta k_t+T_t \\
\lambda k_{t+1} &\leq x_t+(1-\delta)\, k_t,
\end{aligned}$$

where I assume that $\alpha, \delta, \beta \in (0,1)$, $\sigma, \lambda \geq 0$.

The family takes the population growth rate, λ, the prices, $\{r_t, w_t\left(h_t\right)\}$, and the government policies $\{\tau_{c,t}, \tau_{e,t}, \tau_{k,t}, T_t\}$ as exogenously given.

The government in our economy taxes consumption, τ_c, labour income, τ_e, and capital income, τ_k. It permits a depreciation allowance, $\tau_k\delta$. The tax revenues are used to finance the government consumption, G_t. Whatever is not needed to fund the government consumption is lump-sum rebated to the households, T_t. The government balances its budget in every period.

There are different technologies, each defined by the workweek length of operation. There is a continuum of workweek lengths $h \in [0,1]$. The technology of type h is given by:

$$\begin{aligned}
\mathcal{Y}(h) = \{ & (Y_h, E_h, K_h) \in R^3_+ | \text{there exist } E_s, E_p \in R_+ \text{ satisfying} \\
& E_h \geq E_s + E_p \\
& E_p \leq A_s h^{\kappa} E_s \\
& Y \leq A_p h^{\phi} K_h^{\theta} E_p^{1-\theta} \}.
\end{aligned}$$

Concerning the basic parameters, I assume: $\theta \in (0,1)$, $A_s, A_p, \phi, \kappa > 0$, and $1 < \phi/(1-\theta) < (5+\sqrt{22})/6$.[3]

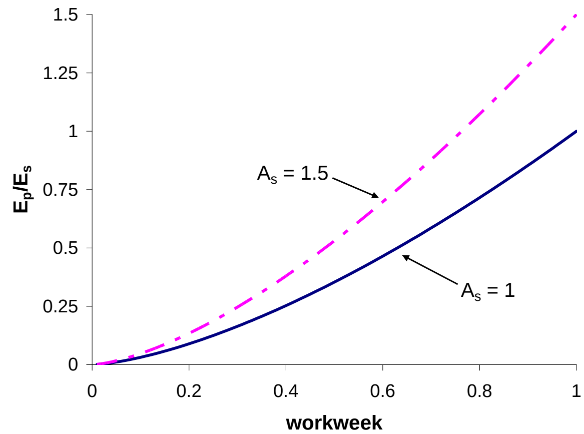
One way of viewing the technology is as a plant with different activities.[4] One activity is providing a supervisory service to the other activity that is producing the final output. I will refer to the first activity as the supervisory activity and to the second activity as the production activity.

As part of the supervisory activity, workers are split into production workers, E_p, and

[3] The assumption of $\phi/\left(1-\theta\right) < (5+\sqrt{22})/6$ is a sufficient condition to guarantee that the wages $w\left(h\right)$ are strictly increasing and convex in h.

[4] An alternative view of the technology is provided in Appendix B.

Figure 5: Impact of an increase in A_s.



supervisors, E_s. For the purpose of this paper I define production workers to be all employed persons who are supervised, and supervisors to be all the remaining employed persons. Splitting employed persons into the two groups is required, since each production worker taking part in the production of the final output of the plant requires supervision. The supervisors spend h hours on supervising production workers. I assume their effective supervisory time is augmented in two ways, by A_s and h^κ. These two components pin down the ratio between production workers and supervisors: $A_s h^\kappa = E_p/E_s$.

In what follows, I motivate how these two assumptions (A_s, h^κ) stand in relation to the ratio between supervisors and production workers.

The functional form $A_s h^\kappa = E_p/E_s$ is based on the assumption that there is a requirement of production workers for supervision. Furthermore, the restriction of $\kappa > 0$ assumes that there is a substitutability between the number of supervisors working h hours and the number of hours each supervisor works.

Let us consider what an increase in A_s does. Here, A_s captures the technology level and the knowledge at a given point in time with respect to the supervisory activity. As A_s increases, the ratio between supervisors and production workers goes down. One way a decrease in the ratio could happen is through a decrease in the number of supervisors relative to the number of production workers. Figure 5 shows the impact of an increase in the productivity of a supervisor, A_s.

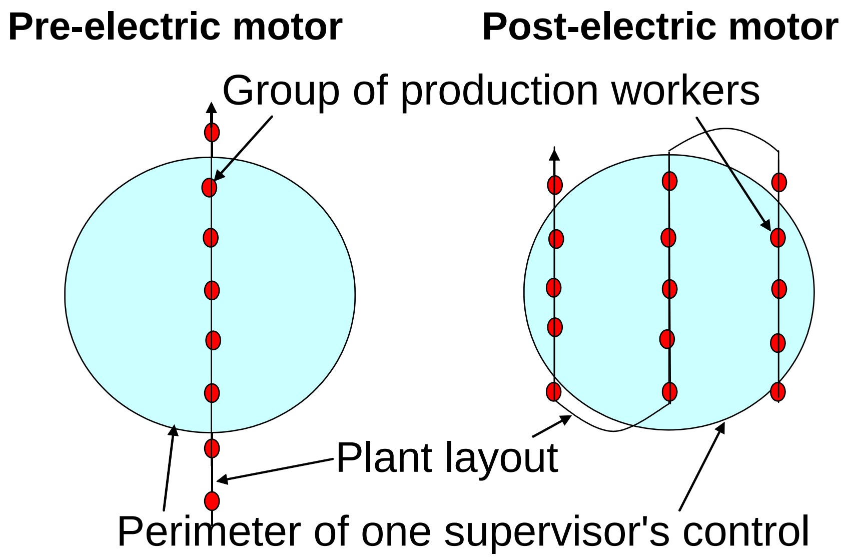
An increase in A_s acts like supervisory-savings technological progress. To illustrate this let's look at a few examples: The introduction of a cash register into restaurants is an example of an increase in A_s. Before the cash register was introduced, each waiter had his own purse and gave change out of that purse. When the waiter's working time was over he returned the purse to the restaurant supervisor. This practice, if uncontrolled, allows for theft. It was virtually impossible to watch all waiters all the time, or it required a lot of supervision. The cash register changed this. It allowed the supervisor of the restaurant to mainly keep an eye on the register and thus reduce the time necessary to monitor the waiters. Notice that even though the cash register made the function of the restaurant supervisor much simpler, it did not reduce the supervisor to a pure cash register person. There is still a need to have somebody on the restaurant floor to make sure that the personnel are treating guests properly. Still, the cash register reduced the number of supervisors. In more recent times, the cash register has become even better as a supervisory savings technology. Today, large grocery stores have centralized all the information from their cash registers to one computer. Thus, to monitor all the production workers linked to that cash register, it suffices to control that one computer.

An example from the manufacturing industry is the use of an assembly line production. The assembly line itself is setting the pace for the production workers at the line. Thus, the supervisor has to control mainly the final output instead of each worker. This leads to a reduction in supervisory requirements. Notice that you still need supervisors along the assembly line to make sure that a high quality of work is maintained.

A final example is the introduction of the electrical motor. It had two main advantages. On the one hand, it made a redesign of the workplace feasible. Before the electrical motor was used you had to design the workplace such that all its machine-driven tools were connected to a few big motors, instead of many small ones. This led to long-stretched-out production places. The electrical motor led to a decrease in the size of the machines. Thus, it became much easier to see all the production workers. If we disregard for a second the visibility problem, in Figure 6, you see the two scenarios. On the left is the situation with an assembly line that is stretched out, and on the right the situation with a compact plant structure based on the electrical motor.

The large circle represents the maximum area of supervision of one supervisor and the small filled circles represent production workers. As you can see, the changes in the plant layout increased the effectiveness of the supervisors. As depicted in the figure, a given supervisor could, at a point in time, control eight instead of six production workers. Hence,

Figure 6: Change in the plant layout.



the introduction of the electrical motor led to an increase in supervisory control.[5]

I take both productivity parameters, that for the supervisory activity, A_s, and that for the production activity, A_p, to grow over time. The series $\{A_{p,t}, A_{s,t}\}$ are externally given and known by everybody in the economy. From now onward I refer to them, respectively, as production total factor productivity (TFP) and supervisory TFP. The problem of the aggregate firm operating the h technology is given by:

$$\max_{(Y_h,K_h,E_h)\in\mathcal{Y}(h)} Y_h - rK_h - w\left(h\right)E_h.$$

Given the technological specifications and the preference structure at a given point in time, only one technology $\mathcal{Y}(h)$ will be operated. Thus I can drop the h subscript.

Proposition 1 ***If the aggregate firm of type h is operating efficiently, then its maximization problem reduces to:***

$$\max_{K,E\geq 0} F\left(K,E;h\right) - rK - w\left(h\right)E$$

[5]For more details concerning the plant layout, see Fisher (1930, Chapter IX).

s.t.

$$F(K,E;h) = A_p \left(\frac{A_s h^{\kappa}}{1 + A_s h^{\kappa}} \right)^{1-\theta} h^{\phi} K^{\theta} E^{1-\theta}.$$

Proof. See Appendix G. ■

Now that all the elements of the model are introduced, I define an equilibrium of the model economy as follows:

Definition 1 ***A competitive equilibrium is an allocation*** $\left(\{c_{1,t}, c_{0,t}, x_t, e_t, k_t\}_{t=0}^{\infty}\right)$, $(\{Y_t, E_t, K_t\}_{t=0}^{\infty})$ ***and prices*** $(\{r_t, w_t(h_t, r_t)\}_{t=0}^{\infty})$ ***such that:***

(i) ***Given the prices and government policies,*** $\left(\{c_{1,t}, c_{0,t}, x_t, e_t, k_t\}_{t=0}^{\infty}\right)$ ***solve the representative family's problem.***

(ii) ***Given the prices and government policies,*** $(\{Y_t(h), E_t(h), K_t(h)\}_{t=0}^{\infty})$ ***solve the firms' problem.***

(iii) ***The resource constraints hold in each period t:***

$$\begin{aligned} e_t c_{1,t} + (1-e_t)\, c_{0,t} + x_t + G_t &= Y_t \\ E_t &= e_t \\ K_t &= k_t. \end{aligned}$$

Assuming interiority, the equilibrium is characterized by the following first-order conditions:

$$\begin{aligned} u_c(c_{1,t}, h_t) &= u_c(c_{0,t}, 0), \\ \frac{u(c_{1,t}, h_t) - u(c_{0,t}, 0) - [u_c(c_{1,t}, h_t)\, c_{1,t} - u_c(c_{0,t}, 0)\, c_{0,t}]}{u_c(c_{0,t}, h_t)} &= -\frac{1-\tau_{e,t}}{1+\tau_{c,t}} \frac{\partial F(k_t, e_t, h_t; t)}{\partial e_t}, \\ -\frac{u_h(c_{1,t}, h_t)}{u_c(c_{1,t}, h_t)} &= \frac{1-\tau_{e,t}}{1+\tau_{c,t}} \frac{\partial F(k_t, e_t, h_t; t)}{\partial h_t} \frac{1}{e_t}, \end{aligned}$$

$$e_t c_{1,t} + (1-e_t)\, c_{0,t} + \lambda k_{t+1} + G_t = (1-\delta) k_t + F(k_t, e_t, h_t; t) + T_t,$$

$$
\begin{aligned}
\frac{u_c\left(c_{0,t},0\right)}{u_c\left(c_{0,t+1},0\right)} &= \beta\frac{1+\tau_{c,t}}{1+\tau_{c,t+1}}\left(1+\left(1-\tau_{k,t+1}\right)\left(\frac{\partial F\left(k_{t+1},e_{t+1},h_{t+1};t+1\right)}{\partial k_{t+1}}-\delta\right)\right), \\
\lim_{t\to\infty}\beta^t u_c\left(c_{0,t},0\right)k_{t+1} &= 0,
\end{aligned}
$$

where $\varepsilon = (1-\alpha)(1-\sigma)/(1-\alpha(1-\sigma))$, $u(c,h) = \left[c^\alpha(1-h)^{1-\alpha}\right]^{1-\sigma}/(1-\sigma)$ and $F(k_t,e_t,h_t;t) = A_{p,t}\left(\frac{A_{s,t}h_t^\kappa}{1+A_{s,t}h_t^\kappa}\right)^{1-\theta} h_t^\phi k_t^\theta e_t^{1-\theta}$.

3.2 Implications of the model

Below, I characterize the equilibrium.

Proposition 2 ***If the equilibrium is interior, then the workweek length is determined by the following equation:***

$$
-\frac{1}{\varepsilon}\left[1-h_t-(1-h_t)^{1-\varepsilon}\right] = \frac{(1-\theta)\left(1/A_{s,t}+h_t^\kappa\right)h_t}{\left(\kappa(1-\theta)+\phi\right)/A_{s,t}+\phi h_t^\kappa}. \tag{1}
$$

Proof. The result follows from the first-order necessary conditions. For more details see Appendix H. ■

Corollary 3 ***Given an interior equilibrium, the following results hold:***

(i) ***The optimal workweek length is independent of***

  (a) ***taxes and***

  (b) ***production TFP,*** $A_{p,t}$***.***

(ii) ***If*** $A_{s,t}\to\infty$ ***as*** $t\to\infty$***, then in the limit, the optimal workweek length is a strictly positive and finite constant.***

(iii) ***If*** $A_{s,t}$ ***is a monotone increasing sequence over time, then the workweek length is a monotone decreasing sequence over time.***

Proof. These results are a direct implication of the last proposition, especially of equation (1). Taking A_s to infinity reduces equation (1) to: $-\frac{1}{\varepsilon}\left[1-h_\infty-(1-h_\infty)^{1-\varepsilon}\right] = \frac{1-\theta}{\phi}h_\infty$. ■

The corollary implies that neither tax changes nor changes in the total factor productivity of the final-goods sector contribute to the decrease in the workweek length. To emphasize the point: changes in A_p are inconsequential for the workweek length. However, this does not imply that changes in the production TFP have no impact on working time. They have a large influence on the employment rate. Furthermore, the corollary states that productivity improvements in the supervision sector have the potential for accounting for the decline in the workweek length.

Note that although taxes do not contribute to the decline in the workweek length, they still have an impact on employment and might be able to account for the large variation in employment across countries and across time.

Remark 1 ***If*** $A_{s,t}$ ***is a monotone increasing sequence and the equilibrium is interior, then*** $E_{s,t}/\left(E_{s,t}+E_{p,t}\right)$ ***is a decreasing sequence.***

This finding conforms with the data presented in the introduction. Further evidence of a decline in supervisory workers relative to all employed persons is provided in Appendix E.1 (Figure E6).

3.3 Context of implications

In this section, I set the just-described findings into a broader perspective. The model presented above is not only an extension of the standard theory of working time (Kydland and Prescott 1991, Hornstein and Prescott 1993), it also has the standard theory as a special case, i.e. for $A_{s,t} \rightarrow \infty$. This means that the model for a large enough supervisory TFP behaves just like the standard model and preserves the basic properties of those models. Furthermore, as in the standard theory, changes in taxes, government spending, or production TFP have no impact on the workweek (see Corollary 1.1). Furthermore, the workweek length is constant in the standard model (see Corollary 1.2) and the employment rate is adjusting. While this seems consistent with macroeconomic data for the United States since the 1960s, it means that the standard model implies a constant workweek and thus cannot account for any long-run movement of the workweek length as documented in section 2 of the paper. In summary: the model I developed has the potential (and, as I show below, actually is able) to account for the long-run change in the workweek length, while preserving desirable properties of the standard model.

4. The Model in Practice

In this section I calibrate the model to match key facts of the U.S. economy; with the calibrated model, I look at the secular labour supply decisions of the U.S., the U.K., and Japan.

4.1 The calibration

The model has eight exogenous processes $(\{N_t, G_t, \tau_{k,t}, \tau_{e,t}, \tau_{c,t}, T_t\})$, $(\{A_{p,t}, A_{s,t}\})$ and seven parameters $(\sigma, \delta, \theta, \kappa, \alpha, \beta, \phi)$. The model parameters are determined, so that the model matches certain moments for a fixed time period.

To determine the model parameters, unless otherwise stated, I use as the reference period 1993 to 1998. This period was chosen since the employment rate, the workweek length, the capital-output, and the consumption-output-ratio displayed a relative constancy over that period. The main alternative periods were 1890 to 1900, 1951 to 1961, and 1954 to 1973. I could not use the earliest period, since the data for that period are of a lesser quality than for the later periods and some data are not available, as in the case of capital consumption. The parameter values for the two later periods are not so different from the ones for the chosen period to lead to significantly different results.

The exogenous processes $(\{N_t, G_t, \tau_{k,t}, \tau_{e,t}, \tau_{c,t}, T_t, A_{p,t}, A_{s,t}\})$ are taken from the data. I assume that the population grows at a constant rate, $N_t = \lambda^t N_0$.

For the population series I use the average growth rate of the population age 15 and older for the period 1870 to 2002 ($\lambda = 1.016$) and the respective population size in 1870.

The government spending is taken from the National Income and Product Accounts. The taxes on income from 1929 till 1975 are from Joines (1981). To extend the series backwards till 1916 and forwards till 2000, for the labour income tax I use a series provided by Marion and Mulligan (2004) and for the capital income tax I use the methodology of Mendoza, Razin, and Tesar (1994) to determine the trend. The consumption tax rate is determined using the indirect business taxes less subsidies from the National Income and Product Accounts together with the Final Private Consumption expenditures. Given my assumption of a period-by-period balanced government budget constraint, the lump-sum transfers are determined residually. That leaves me with the measurement of the series $(\{A_{p,t}, A_{s,t}\})$; these are determined below together with κ, ϕ. Given the technology, all these objects are interlinked.

Next, I determine the parameters $(\delta, \theta, \sigma, \alpha, \beta)$. The annual depreciation rate of the capital stock is taken to be equal to the ratio of capital consumption to the capital stock. It is given by: $\delta = 0.06$. The capital income share is based on the National Income Accounts of the United States and takes the value $\theta = 0.34$.

Using the following first-order conditions for the reference period 1993 to 1998, I determine the parameter values of (α, σ) jointly as the solution to the following equation system:

$$
\begin{aligned}
\alpha &= \left(1 + \frac{1-\tau_{e,t}}{1+\tau_{c,t}} \frac{Y_t}{c_{1,t}} \frac{1-h_t}{e_t} \frac{(1-\theta)}{-\frac{1}{\varepsilon}\left(1-h_t-(1-h_t)^{1-\varepsilon}\right)}\right)^{-1}, \\
\frac{c_{0,t}}{c_{1,t}} &= (1-h_t)^{-\varepsilon}, \\
\text{where } \varepsilon &= \frac{(1-\alpha)(1-\sigma)}{1-\alpha(1-\sigma)}.
\end{aligned}
$$

Following Kydland and Prescott (1991), I use the fact that the consumption expenditures of a non-working person relative to the consumption expenditures of a working person are approximately 3/4. Given this, together with data on the employment rate, the workweek, and some data from the National Income and Product Accounts for the United States, I am able to solve for the leisure weight in the utility function, $\alpha = 0.31$, and the intertemporal elasticity of substitution parameter, $\sigma = 2.10$.

This in turn allows me to utilize the first-order condition

$$
\beta = \frac{\left[c_{0,t+1}/c_{0,t}\right]^{1-\alpha(1-\sigma)}}{(1+\tau_{c,t})/(1+\tau_{c,t+1})\left(1+(1-\tau_{k,t+1})\left(\theta\frac{Y_{t+1}}{K_{t+1}}-\delta\right)\right)}
$$

to determine the families discount factor, $\beta = 0.98$.

This leaves me with the determination of the TFP processes and the parameters (ϕ, κ). These four objects are interlinked in such a way that I have to determine them simultaneously. To reduce the number of parameters, I assume that the supervision productivity is proportional to the production productivity, $A_{s,t} = \mu A_{p,t}$.

Given this assumption, I can solve for all objects as the solution to the following equation system:

$$\begin{aligned} A_{p,t}\left(\frac{\mu A_{p,t}h_t^{\kappa}}{1+\mu A_{p,t}h_t^{\kappa}}\right)^{1-\theta} &= \frac{Y_t^{1-\theta}}{\left(K_t/Y_t\right)^{\theta} E_t^{1-\theta}h_t^{\phi}}, \\ \frac{\kappa\left(1-\theta\right)+\phi\left(1+\mu A_{p,j}h_j^{\kappa}\right)}{1+\mu A_{p,j}h_j^{\kappa}} &= \frac{(1-\theta)h_j}{-\frac{1}{\varepsilon}\left(1-h_j-\left(1-h_j\right)^{1-\varepsilon}\right)}, \\ \mu A_{p,i}h_i^{\kappa} &= \frac{E_{p,i}}{E_{s,i}}. \end{aligned} \tag{2}$$

The first equation is based on the basic production technology, the second equation follows from the first-order conditions, and the last equation follows from the fact that for an efficiently producing firm $E_{p,t} = A_{s,t}h_t^{\kappa}E_{s,t}$. The first equation, considered for $t = 1870, ..., 2002$, pins down the value and the relative movement of the production TFP, $\{A_{p,t}\}$. The next equation, for the period j from 1993 to 1998, is used to determine ϕ, and the final equation, for i from 1993 to 1998 and for 1870, is used to determine, respectively, κ and μ. This last equation ensures that the ratio between the number of supervisors and the number of production workers has the correct value. It turns out that choosing 1870 to determine μ leads to the least favourable value of κ with respect to the model's predicted decline of the workweek length. For a more detailed description of the calibration process, please see Appendix D.

Before I turn to the results, I would like to stress that the value I find for the standard parameter ϕ is consistent with the literature. Equation (2) establishes a relationship between ϕ, κ such that $\phi = f(\kappa)$ is a decreasing function in κ. Both Bils and Cho (1994) and Osuna and Rios-Rull (2003) argue for a value of ϕ between the labour income share, $1-\theta$ and 1, since ϕ captures the fatigue of the production workers. At the one extreme is the case of $\phi = 1-\theta$, which brings us back to the regular Cobb-Douglas production function, with workers tiring as the workweek length increases. At the other extreme is the case of $\phi = 1$, with workers not tiring at all. It is reasonable to assume that the true value of ϕ is somewhere in between the two extremes. As part of my calibration, I determine the value of $\kappa = 1.49$, which results in a ϕ of 0.84. This is close to the benchmark value set by Osuna and Rios-Rull, $\phi = 0.85$, and well within the interval of $1-\theta$ and 1.

Table 2 contains all the parameters. These parameters are taken to be universal, in the sense that they are not country specific. Thus they are not changing across countries.

Table 2: Parameters.

Name	Description	Values
κ	Supervision technology workweek importance parameter	1.49
σ	Intertemporal elasticity of substitution parameter	2.10
δ	Annual depreciation rate of the capital stock	0.06
θ	Capital income share	0.34
α	Consumption-leisure importance parameter	0.31
β	Discount rate	0.98
ϕ	Final-goods-producing sector workweek length productivity parameter	0.84

4.2 Experiments for the U.S. economy

Given the model specifications, I conduct a few experiments to determine how well the model works in accounting for the labour supply developments. I focus on the secular trend, i.e., 1870 to 2002. Since I am dealing with secular movements, I use time series that are smooth in the sense that I control for business cycle fluctuations.

Before turning to the experiments, I describe the changes in those exogenous processes that are most relevant for the labour supply decisions in the model. In Table 3, I report the key economic determinants of the model economy for 1870 and for 2002.

Table 3: Comparison of processes over time for the United States.

	G/Y (in %)	$\frac{\tau_c+\tau_e}{1+\tau_c}$ (in %)	γ_{local} (in %)	A_p	A_s
1870	5.2	4.1	1.6	4.8	3.9
2002	19.4	32.2	1.8	30.1	24.3
per centage change	271	692	11	527	527

The second column presents the government share of GDP. It used to be quite low in 1870 and has increased subsequently by 271 per cent. Today, government expenditures in the U.S. are at about 20 per cent of the total output. In column two, the effective average marginal tax rate on labour income is shown. This tax rate increased considerably over the past 130 years. The next column shows the short-run growth rate of production TFP. By short-run I mean, in the case of 1870, the growth rate of output from 1870 to 1890, and, in the case of 2002, the growth rate of output from 1982 to 2002. This growth rate is determined for 1870 as the average growth rate of $(A_{p,t+1}/A_{p,t})^{1/(1-\theta)}$. This growth rate is to some extent capturing the output growth expectations that decision makers in the economy have for the future. Here the differences are relatively small. The final two columns show the technology levels for the final-goods sector and the supervisory sector. Both increased by about the same amount, i.e. about 527 per cent.

Below I show the implications for labour supply of the following exogenous processes: the productivity measures, the government expenditures, and the distortionary taxes. At first I will look at the effects on labour supply of each of these processes individually and then combined. To get rid of perfect foresight effects, I will solve local systems in the sense that the economic decision makers only know the exogenous processes for 20 periods around the time period they are living in. This is the place where the γ_{local} become important. For all the experiments I take these local growth rates and the tax on capital income as given.

Table 4: Comparison between secular change of labour supply in the data and in experiments for the United States.

Name	h	e
Data	**-35**	**34**
Benchmark	0	-22
Experiment 1	0	-2
Experiment 2	0	-48
Experiment 3	-27	56
Experiment 4	-27	22

As my benchmark model, I consider an economy in which technological progress is just in the final-goods sector, the government is taxing capital income as in the data to lump-sum rebate the tax revenue, and there is lack of supervisory technology.[6] This benchmark model does very poorly in the sense that it neither accounts for the change in the workweek nor for the increase in the employment rate. In experiment 1, I add government expenditures to the benchmark model. This version of the model still fails for both labour measures, but does considerably better with respect to the employment rate relative to the benchmark. In experiment 2, I look at the case where the government taxes the labour income of households and rebates the tax revenue lump sum back. As I have already theoretically shown, taxes

[6] To exclude the supervisory technology from my experiments, I consider an economy with $\mu \to \infty$, which implies that $E_p = E$.

have no impact on the workweek length. Apart from this, they imply a large secular decrease in the employment rate. The next step is to look at the impact of the supervision technology alone. I do this in experiment 3. Improvements in the productivity of supervisors account for a large part of the workweek length decline. The supervision technology alone implies an increase in the employment rate that is much too large compared with the data. In the final experiment, I consider the model with all elements. I am getting very good results. The model with production TFP, government spending, taxes, and technological progress in supervisory technology is able to account for 77 per cent of the decrease in the workweek length and 65 per cent of the increase in the employment rate.

4.3 Labour supply in the U.K. and Japan

Given the previous experiments for the United States, I will now focus on the case of the model with government expenditures, taxes, production, and supervision TFP. The countries considered in this section are the United Kingdom and Japan. In contrast to the United States, these two economies experienced a decline in the employment rate.

The exogenous processes for $(G_i, \tau_{c,i}, \tau_{e,i}, \tau_{k,i})_{i=UK,Japan}$ are taken from the data, respectively, and tax rates are determined using the method described in Mendoza, Razin, and Tesar (1994). For the population process, I am left with finding the long-run average annual growth rate.

Finally, concerning the processes $(A_{p,t}, A_{s,t})$, I follow the same procedure as described for the U.S. case, taking the parameters κ, ϕ as before. The reference year, i, in the equation $\mu A_{p,i} = \frac{E_{p,i}}{E_{s,i} h_i^{\kappa}}$ is 1930 for Japan and 2002 for the U.K.

Table 5: Supervision TFP factor and population growth rate.

	μ	λ (in %)
U.K.	1.24	0.73
Japan	0.81	1.13

In Table 5, I report the factor relating production TFP and supervisory TFP and the population growth rate. Note that for Japan μ is about the same as for the United States, while the value is larger in the United Kingdom than in the United States. In the case of the United States I found $A_{p,US} = 0.81\ A_{s,US}$. Furthermore, the population growth rates are quite different across the countries considered. Compared with the U.S., whose population grew at an average rate of 1.6 per cent annually, both Japan and the U.K. experienced fairly low population growth rates.

In Table 6, I report the key variables determining the labour supply decisions for the different countries.

Table 6: Comparison of processes over time for Japan and the United Kingdom.

		G/Y (in %)	$\frac{\tau_c+\tau_e}{1+\tau_c}$ (in %)	γ_{local} (in %)	A_p	A_s
Japan	1930	12.8	3.6	1.6	5.8	4.7
	2000	10.9	33.8	1.9	30.6	24.8
	% change	-15	832	27	431	431
U.K.	1870	4.6	5.4	0.4	6.8	6.8
	2002	19.5	29.5	2.1	21.3	23.3
	% change	325	448	425	240	240

The most striking difference between the two economies is their long-run growth performance as reflected in the growth of their production and supervision TFP. The U.K. grew on average by 1.4 per cent per year, while Japan grew by 3.7 per cent. As is indicated by the local growth rates, γ_{local}, the U.K. experienced fairly low growth early on and did much better at the end of the 20th century. Regarding the effective tax on labour, Japan experienced a nearly twice as large tax increase when compared with the U.K. The other major difference is in the government spending share of GDP. While Japan actually decreased its government spending starting from a low level, the U.K. increased its government spending to a level comparable with that of the U.S.

For the full model, I determine the per centage changes of the workweek length and the employment rate over the long-run. The results are reported in Table 7.

As we might have expected, given the long-run growth performances of the two economies, the predicted decline in the workweek length is much larger for Japan than for the U.K. Furthermore, the relatively low level of production TFP together with the large increase in taxes depressed the employment rate in Britain. The difference between the previous results for the United States and those for Britain is the sluggish productivity growth. This is also driving the different employment rate experiences. Turning to Japan, the picture changes:

Table 7: Comparison of working time changes for the U.K. and Japan.

% change		h	e
U.K. (1870-2002)	**Data**	**-42**	**-15**
	Model	-19	-13
Japan (1930-2000)	**Data**	**-41**	**-12**
	Model	-25	-21

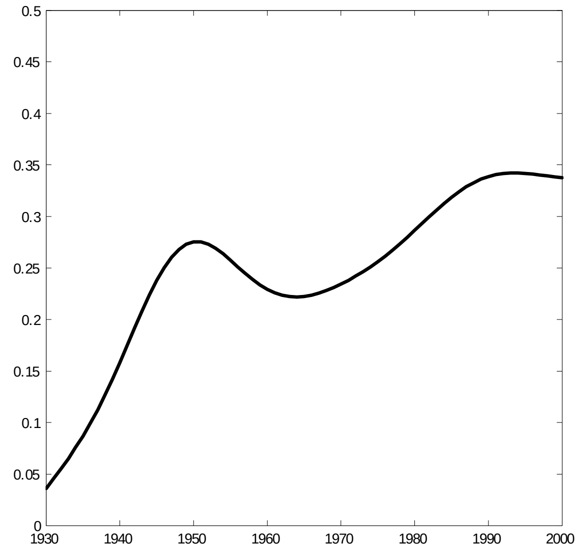
the strong decrease in the employment rate is mainly due to the larger tax increases.

A comparison between the model and the data for the two countries reveals that the model is able to account for a large part of the workweek length change: In the case of Japan the model accounts for 61 per cent of the decline, and in the case of the United Kingdom the model accounts for 45 per cent of the decline. Concerning the employment rate, the model does very well for the U.K., namely it accounts for 87 per cent of the decline in the employment rate. For Japan, the model overpredicts the decrease in the employment rate by 75 per cent.

Concerning this overprediction I argue that the model did not have a chance to accurately capture the Japanese labour supply from 1930 to 1945, because of institutional constraints that are not captured by the model. My conjecture is that Japanese employed persons in the 1930s were strongly encouraged to work longer hours. This in turn led to a decrease in the employment rate. After the war the optimal workweek level became feasible and thus was chosen, leading to a large drop in the workweek length. Theoretically, one would expect in response an increase in the employment rate. The main reason for the absence of a large increase in the employment rate is higher taxes relative to the 1930s level. In Figure 7, I display the effective tax rate on labour: $(\tau_c + \tau_e)/(1 + \tau_c)$.

A couple of observations provide circumstantial evidence for my conjecture. First, the model accounts for 85 per cent of the decline in average hours worked. Second, the discrepancy between the employment rate in the data and the one the model predicts arises after the Second World War. On the one hand, the model implies that the decline in average hours worked after 1945 is driven by a decrease in the employment rate. On the other hand, in the data the decline in average hours worked is driven by a decline in the workweek length.

Figure 7: Japan, effective tax on labour income, 1930 to 2000.



Third, before the 1940s, Japan experienced a strong increase in the workweek length and a decline in the employment rate. Furthermore, the Japanese economy in this pre-World War II period was mainly controlled by large corporations and the government. Finally, after 1945 taxes on labour were at a relatively high level, compared with the pre-1940 situation. All these observations combined support my conjecture. Institutional constraints appear to be the main reason for the discrepancy between the model's prediction and the data.

5. Conclusion

This paper has established important facts about the secular behaviour of the labour supply. The main observation is a considerable decline in the working time per employed person. For most countries, this decline was not offset by an increase in the number of employed persons. In many cases, the number of employed persons relative to the population age 15 and older actually declined. Combining the strong decline in weekly hours worked per employed person with the change in the number of employed persons in the population age 15 and older, I find for all countries a considerable decrease in the average hours worked per person.

I have used an applied dynamic general-equilibrium framework to determine the main driving forces behind the secular movement of the workweek length and the employment rate. I find that a major part of the decline of the workweek length can be accounted for by technological progress in the supervision technology. For a calibrated model, technological progress in the supervisory TFP accounts for 77 per cent of the workweek length decline in the case of the United States. For other considered countries it accounts for 45 to 61 per cent of the decline. Concerning the employment rate, the model is able to capture 75 per cent of the increase in the case of the United States. A similar good prediction is found for the United Kingdom: the model accounts for 87 per cent of the decline in the employment rate. In the case of Japan, the model captures the fact that the employment rate declined, but due to institutional constraints not present in the model, the level of the decline is not accurately predicted. The main driving forces of the change in the employment rate are taxes on labour income and technological progress both in the production TFP and the supervisory TFP.

Concerning future research, I suggest enlarging the number of countries considered. Here, an extension of the model is also required. The model assumes that there are no particular disincentives for working besides the ones generated by the preferences. For some European countries this might not be true. Over the past 30 years several countries (e.g., France, Germany, and Italy) have made use of early retirement policies. These policies encouraged employed persons age 55 and older to retire. Data on the employment rate of the age group 55 to 65 in some European countries suggest that these policies were successful. For example, in France the labour participation of 55-65 year olds decreased from 56 per cent in 1970 to 37 per cent in 2003. Taking the incentives for early retirement into account might help to explain the large cross-country differences with respect to the employment rate.

Furthermore, an analysis of my conjecture concerning institutional constraints on the Japanese workweek length during the period 1928 to 1945 would be a step towards a validation of the proposed theory.

Concerning the remaining discrepancy between the model and the data for the late 20th century in the U.S., a natural extension would be to consider a model with a female labour supply decision. There are two striking facts about the employment rate that deserve special attention, but have not been addressed in this paper: first, for many countries over the past 40 years the employment rate of women increased (especially married women). This is not universal among OECD countries, but it is common enough to deserve a thorough analysis. Second, for many countries there was a large decrease in the employment rate of men over the past 40 years. This seems to be closely related to the decrease in the employment rate in the age group 55-64 and thus linked to the introduction of early retirement incentives by many governments. Again, not all countries in the OECD show this trend.

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Appendix A: Overview

The appendix is organized as follows: In Appendix B an alternative view of the technology is considered. In Appendix C a version of the model is presented in which supervisors are productive and enter the final output production directly. After that, in Appendix D, I describe the calibration procedure in more detail. Then, I consider the model's performance in more detail. To do this I conduct a sensitivity analysis in Appendix E for the parameter κ and look at the model's predictions for the capital-output and the consumption-output ratio compared with a standard real business cycle model. Section E.1 shows the figures of time series data on labour supply. In Appendix F, I present a data appendix naming the sources used. In Appendix G, I consider the supervision technology and analyze the implications of this technology for the wage function and the aggregate production technology. Then in Appendix H, I use the resulting firms and household problems to characterize the equilibrium using first-order necessary conditions for interior equilibria. As an aside I also look at the limiting economy as time goes to infinity. I conclude in Appendix I by examining the equivalence between an equilibrium in an economy with a workweek length indivisibility and the equilibrium for the economy as used in this paper.

Appendix B: Alternative view of the technology

There is an alternative way of viewing the technology described in the main text. Concerning the primary inputs and outputs, this view is observational equivalent to the previous formulation. In the second view, there is an intermediate plant of type h whose problem is given by:

$$\begin{aligned}
&\max\,[qS - w_s(h)E_s] \\
&\text{s.t.} \\
&S \leq A_s h^{\kappa} E_s
\end{aligned}$$

Here, q is the price of the intermediate input and S is the amount of intermediate goods produced. I refer to these intermediate goods as supervisory services. As the sole input the plant hires supervisory workers, E_s.

These supervisory services are used as inputs to the final-good-producing sector[1]:

$$\max_{K_h, S, E_p \geq 0} A_p h^{\phi} K_h^{\theta} \left[\min\{S, E_p\}\right]^{1-\theta} - qS - w_p(h)E_p - rK_h.$$

The first thing to note is that in equilibrium all workers with the same workweek have to earn the same wage. Thus we have: $w_p(h) = w_s(h)$. Notice that under efficient production and positive prices we get the following relationships:

$$\begin{aligned}
qS &= w_s(h)E_s, \\
S &= A_s h^{\kappa} E_s, \\
S &= E_p.
\end{aligned}$$

The first two equations follow from the intermediate sector's problem. The last equation follows from the assumption of an efficient production in the final-goods sector. Combining these conditions with the market clearing condition for labour ($E_s + E_p \leq E_h$) leads to:

[1] All the key theoretical results in this paper remain valid for a more general view of the final-good sector's production function: $A_p h^{\phi} K_h^{\theta} \left[\left(aS^{\rho} + (1-a)\, E_p^{\rho}\right)^{1/\rho}\right]^{1-\theta}$, $a \in (0,1)$, $\rho < 0$. The key requirement is that the supervisory services and the number of production workers are complements.

$$\max_{K_h, E_h \geq 0} A_p \left(\frac{A_s h^\kappa}{1 + A_s h^\kappa} \right)^{1-\theta} h^\phi K_h^\theta E_h^{1-\theta} - rK_h - w(h) E_h.$$

Notice that this is the same reduced problem as described in the previous proposition. The alternative view of the technology can also be written in the form of technology sets indexed by h:

$$\begin{aligned} \mathcal{Y}(h) \quad = \quad & \{(E, K, Y) \in R_+^3 | \quad \text{there exist } S, E_p, E_s \geq 0 \text{ satisfying} \\ & E \geq E_s + E_p \\ & S \leq A_s h^\kappa E_s \\ & Y \leq A_p h^\phi K^\theta \left[\min\{S, E_p\}\right]^{1-\theta}\}. \end{aligned}$$

Appendix C: Alternative technology: the case of productive supervisors

In this appendix, I present a model augmented on the technology side, assuming that supervisors enter production in two ways: directly as a productive input and indirectly through the number of workers that they can supervise.

The technology set for this setting is given by:

$$\begin{aligned}
\mathcal{Y}(h) \;=\; & \{(Y_h, E_h, K_h) \in R_+^3 | \text{there exist } E_s, E_p \in R_+ \text{ satisfying} \\
& E_h \geq E_s + E_p \\
& E_p \leq A_s h^\kappa E_s \\
& Y \leq A_p h^\phi K_h^{1-\theta_s-\theta_p} E_s^{\theta_s} E_p^{\theta_p} \}.
\end{aligned}$$

Concerning the basic parameters, I assume: $\theta_s + \theta_p \in (0,1)$, θ_s, θ_p, $A_s, A_p, \phi, \kappa > 0, \phi - \kappa\, \theta_s > 0$ and $1 < \phi/(1-\theta_s-\theta_p) < (5+\sqrt{22})/6$.

In this case, the optimal workweek length is determined by the following equation:

$$-\frac{1}{\varepsilon}\left[1 - h_t - (1-h_t)^{1-\varepsilon}\right] = \left\{ \begin{array}{cc} \frac{\theta_s+\theta_p}{\phi} h_t & \text{if} \quad \left(\frac{\theta_p}{\theta_s A_{s,t}}\right)^{1/\kappa} < h_t \\ \frac{(\theta_s+\theta_p)(1+A_{s,t}h_t^\kappa)h_t}{(\kappa\theta_p+\phi)+(\phi-\kappa\theta_s)A_{s,t}h_t^\kappa} & \quad\;\; \left(\frac{\theta_p}{\theta_s A_{s,t}}\right)^{1/\kappa} \geq h_t \end{array} \right\}.$$

Furthermore, I find that the following holds for the number of supervisors and production workers:

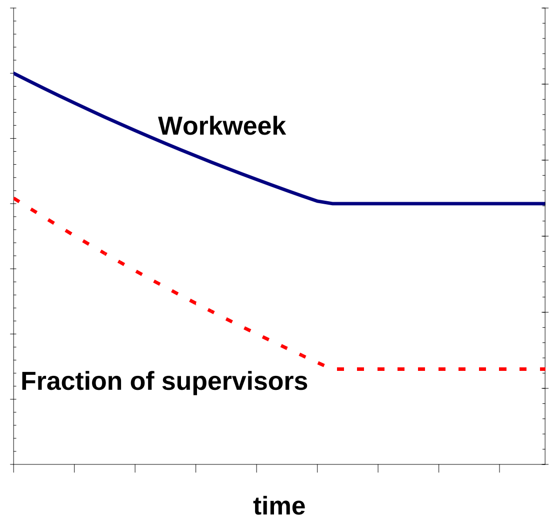
$$(E_s, E_p) = \left\{ \begin{array}{cc} \frac{E}{\theta_s+\theta_p}(\theta_s, \theta_p) & \text{if} \quad \left(\frac{\theta_p}{\theta_s A_{s,t}}\right)^{1/\kappa} < h \\ \frac{E}{1+A_{s,t}h_t^\kappa}(1, A_{s,t}h^\kappa) & \quad\;\; \left(\frac{\theta_p}{\theta_s A_{s,t}}\right)^{1/\kappa} \geq h \end{array} \right\}.$$

Finally, for the reduced-form aggregate production function, I find the following solution:

$$F(K,E,h) = \left\{ \begin{array}{cc} A_p \frac{\theta_s^{\theta_s}\theta_p^{\theta_p}}{(\theta_s+\theta_p)^{\theta_s+\theta_p}} h^\phi K^{1-\theta_s-\theta_p} E^{\theta_s+\theta_p} & \text{if} \quad \left(\frac{\theta_p}{\theta_s A_{s,t}}\right)^{1/\kappa} < h \\ A_p \frac{(A_{s,t}h_t^\kappa)^{\theta_p}}{(1+A_{s,t}h_t^\kappa)^{\theta_s+\theta_p}} h^\phi K^{1-\theta_s-\theta_p} E^{\theta_s+\theta_p} & \quad\;\; \left(\frac{\theta_p}{\theta_s A_{s,t}}\right)^{1/\kappa} \geq h \end{array} \right\}.$$

Figure C1 summarizes the development of an economy that starts with a very low initial level of supervisory TFP and increases fairly rapidly in supervisory TFP.

Figure C1: Supervision and workweek if supervisors are productive.



In terms of capturing the development both of the workweek and the fraction of supervisors among the employed persons, this extended model captures the U.S. experience better than the model in which supervisors are not productive.

Appendix D: Calibration in detail

This appendix describes the calibration algorithm in detail. To make the calibration more transparent, I break the process down into a couple of steps. There are eight exogenous processes

$$\{(N_t, G_t, \tau_{k,t}, \tau_{e,t}, \tau_{c,t}, T_t, A_{p,t}, A_{s,t})\},$$

and seven parameters

$$(\delta, \theta, \sigma, \alpha, \beta, \kappa, \phi).$$

As my reference period I use, unless otherwise stated, the time from 1993 to 1996. This was a period when all the key variables and ratios of the model were fairly constant and the model was relatively close to the limiting economy.

Step 1: Using data of the working age population and from the National Income and Product Accounts (NIPA), I determine the following processes $\{N_t, G_t\}$, where for every period the family size, N_t, is identified as the working age (15 and older) population in a given period and the government consumption, G_t, is taken from the NIPA as the government expenditures. I also take from the data the marginal tax rates on capital income, labour income, and consumption, $\{\tau_{k,t}, \tau_{e,t}, \tau_{c,t}\}$. Given the government spending and the tax rates, the government transfers are determined as the residual using NIPA data and the government budget constraint.

Step 2: I determine the depreciation rate, δ, and the production parameter, θ, using the NIPA. For δ, I make use of the private net fixed assets and the depreciation together with the capital accumulation formula: $k_{t+1} = (1 - \delta) k_t + x_t$. For θ, I utilize the income account side of the NIPA to find the capital income share. Concerning the proprietors income I assume that 70 per cent of it is labour income.

Step 3: To determine the values for α and σ, I make use of the observation by Kydland and Prescott (1991) that the consumption of a not-working person relative to the consumption of a working person takes the value 0.75. This together with the two relationships (derived from the first-order conditions):

$$\begin{aligned}
\alpha &= \left(1+\frac{1-\tau_{e,t}}{1+\tau_{c,t}}\frac{Y_t}{c_{1,t}}\frac{1-h_t}{e_t\ h_t}\frac{(1-\theta)h_t}{-\frac{1}{\varepsilon}\left(1-h_t-(1-h_t)^{1-\varepsilon}\right)}\right)^{-1}, \\
c_{1,t} &= c_{0,t}\left(1-h_t\right)^{\varepsilon}, \\
\varepsilon &= (1-\alpha)\left(1-\sigma\right)/\left(1-\alpha\left(1-\sigma\right)\right),
\end{aligned}$$

allows me to write an algorithm to solve for α and σ at the same time. Basically, I am solving two equations in two unknowns.

Step 4: To solve for β, I make use of the following first-order condition together with NIPA data

$$\beta = \frac{\left[c_{0,t+1}/c_{0,t}\right]^{1-\alpha(1-\sigma)}}{(1+\tau_{c,t})/(1+\tau_{c,t+1})\left(1+(1-\tau_{k,t+1})\left(\theta\frac{Y_{t+1}}{K_{t+1}}-\delta\right)\right)}.$$

Step 5: There are two processes and two parameters left to solve for: $\{A_{p,t}, A_{s,t}\}_{t=1870}^{2002}, (\phi,\kappa)$. The key relationships used to solve for these four objects are:

$$A_{p,t}\left(\frac{A_{s,t}h_t^{\kappa}}{1+A_{s,t}h_t^{\kappa}}\right)^{1-\theta} = \frac{Y_t^{1-\theta}}{\left(K_t/Y_t\right)^{\theta}E_t^{1-\theta}h_t^{\phi}}, \tag{D1}$$

$$A_{s,i} = \frac{E_{p,i}}{E_{s,i}h_i^{\kappa}}, \tag{D2}$$

$$\frac{\kappa\left(1-\theta\right)+\phi\left(1+A_{s,i}h_i^{\kappa}\right)}{1+A_{s,i}h_i^{\kappa}} = \frac{(1-\theta)h_i}{-\frac{1}{\varepsilon}\left(1-h_i-(1-h_i)^{1-\varepsilon}\right)}. \tag{D3}$$

To make this system solvable, I assume that $A_{s,t}=\mu A_{p,t}$. This condition implies that the technological progress in the supervisory technology is linked with the technological progress in the production technology, up to a normalization factor. The example of the electric motor given in the main text suggests that this seems to be a reasonable assumption.

Thus the problem reduces to solving an equation system in 136 unknowns, namely: $\{A_{p,t}\}_{t=1870}^{2002}, (\mu,\phi,\kappa)$. The equations I use are: to pin down the production TFP, I use equation (D1) for all 133 periods. Furthermore, to pin down μ, I use equation (D2) for 1870. In this way, I make sure that the model captures the right ratio of supervisors to production workers for 1870. To capture the right value for ϕ, I solve equation (D3). Finally, to determine κ, I take equation (D2) for the reference period 1993 to 1998.

What I have to work with is the following:

$$\begin{aligned}
\kappa &= f_1\left(\phi, \mu, \{A_{p,t}\}\right); \\
\phi &= f_2\left(\kappa, \mu, \{A_{p,t}\}\right); \\
A_{p,t} &= f_{t+2}\left(\phi, \mu, \kappa\right), \forall t = 1...133; \\
\mu &= f_{t+2}\left(\phi, \kappa, \{A_{p,t}\}\right).
\end{aligned}$$

My solution method is to iterate and use the uniqueness of the solution together with a contraction property of the function to get a solution.

Given κ,

Given ϕ,

I solve

$$\begin{aligned}
A_{p,t} &= f_{t+2}\left(\phi, \mu, \kappa\right), \forall t = 1...133; \\
\mu &= f_{t+2}\left(\phi, \kappa, \{A_{p,t}\}\right).
\end{aligned}$$

for $\left(\mu, \{A_{p,t}\}\right)$.

Then I update $\phi = f_2\left(\kappa, \mu, \{A_{p,t}\}\right)$.

If the new value for ϕ is equal to the old one, I close the 'Given-ϕ'-loop,

else I use the updated $\phi-$value to re-solve the equation system.

Once I have a ϕ that is constant, I update $\kappa = f_1\left(\phi, \mu, \{A_{p,t}\}\right)$.

If the new value for κ is equal to the old one, I stop the program,

else I use the updated $\kappa-$value to re-enter the 'Given-κ'-loop.

Appendix E: Sensitivity analysis of κ

To analyze the sensitivity of the model to the κ value, I conduct a sensitivity analysis. To do this, I perform a simple experiment and then look at the real model of the U.S.

The parameters chosen for the experiment are:

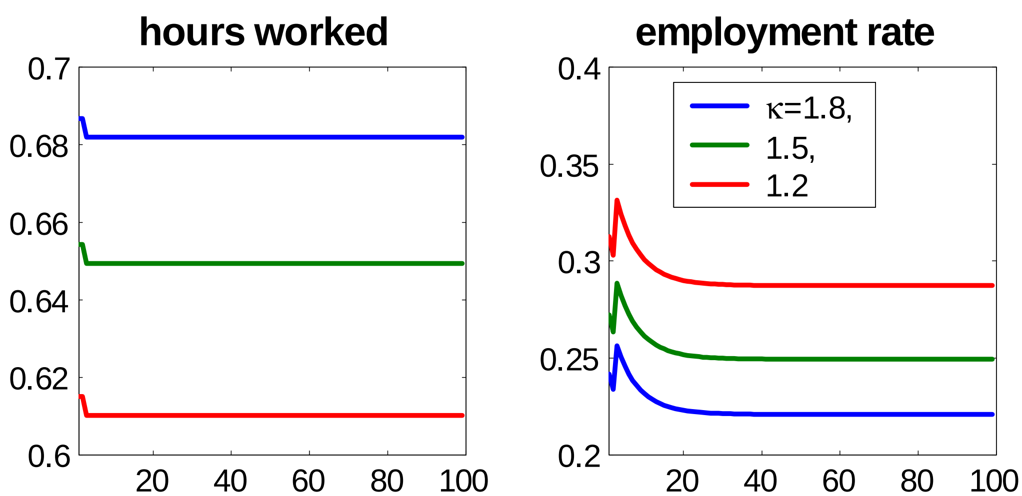
$$\left(\begin{array}{c} \alpha = 0.25, \sigma = 2.00, \lambda = 1.00, \beta = 0.98, \theta = 0.30, \phi = 1.00, \gamma = 1.02, \\ \delta = 0.05, A_p = 1.00, G = 0.00, \tau_{i=c,e,k} = 0.00 \end{array} \right)$$

Furthermore I use $A_{s,t} = \left\{ \begin{array}{ccc} 1 & & = 0, 1, 2 \\ & for \;\; t & \\ 1.1 & & > 2 \end{array} \right\}$.

The experiment looks at the labour variables for $\kappa = 1.2,\ 1.5,\ 1.8$. Recall the relationship between κ and the amount of time a supervisor spends on supervision instead of other responsibilities: other time spent $= \left(\frac{1}{\kappa}\right)^{\kappa/(\kappa-1)} (\kappa - 1)$. From here we find that $\kappa = (1.2, \quad 1.5, \quad 1.8)$, respectively, implies time spent on other things than supervision to be $(6.7, \quad 14.8, \quad 21.3)$ hours per week. While 6.7 hours per week seems to be a low number, 21.3 hours per week seems to be a very high number for non-supervisory time of a supervisor, given a 40 hour workweek.

For the experiment, I find the following:

Figure E1: Sensitivity analysis for $\kappa = 1.2, 1.5, 1.8$.



There is a large level effect both with respect to the workweek length and the employment rate. Furthermore, there is a slope effect. The decrease in the workweek length from the 10 per cent shock to supervision productivity results in a per centage change over 100 periods of h between -0.81 per cent and -0.69 per cent, see Table 8.

Table E1: Sensitivity analysis for isolated technological progress.

κ	$(h_{100} - h_0)/h_0$ (in %)	$(e_{100} - e_0)/e_0$ (in %)
1.2	-0.81	-11.21
1.5	-0.75	-11.57
1.8	-0.69	-11.85

In Table 9, I consider the model economy, calibrated to the U.S., with all key characteristics.

Table E2: Sensitivity analysis for the example of the United States.

κ	$(h_{2002} - h_{1870})/h_{1870}$ (in %)	$(e_{2002} - e_{1870})/e_{1870}$ (in %)
Data	-35.4	34.3
1.2	-28.5	23.8
1.5	-28.2	29.1
1.8	-27.1	31.7

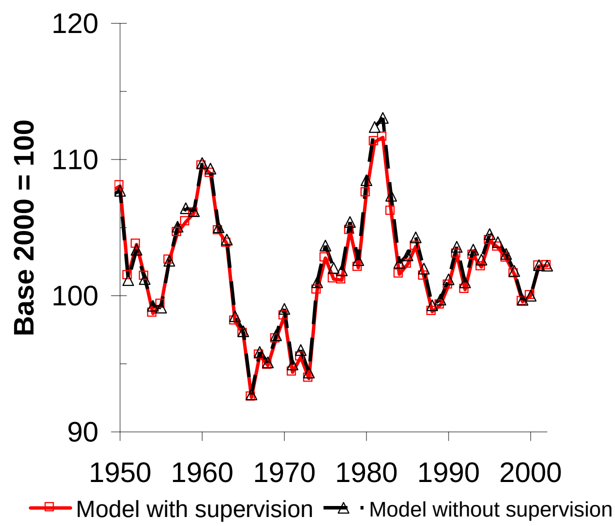
As can be seen, the change in κ has little effect on the workweek length result. On the other hand, the choice of κ has some effect on the employment rate. In particular, a decrease in κ decreases the change in the employment rate. Note that although the employment rate change decreases with κ, the model still accounts for a major part of the decline in the employment rate.

In summary, the model seems to be fairly robust with respect to the particular value of κ.

There are certain features of the real business cycle (RBC) model that are worth preserving. Among those are its ability to capture the movements of the capital-output and the consumption-output ratio in a given economy. Part of this ability relies on the existence of a steady state. The economy presented in this paper does not have a steady state, but converges asymptotically to a steady state. In this part, I wish to show that the model still does very well with respect to the main ratios compared with a standard RBC model. To do this, I present the capital-output ratio and the consumption-output ratio for the model economy with and without a supervisory technology. Note that the latter case is a standard RBC model and thus can serve as a benchmark for the closeness between the two models.

The parameters are taken from the paper. I consider the period from 1950 to 2002.

Figure E2: Comparison of the capital-output ratio for different models, U.S. 1950 to 2002.



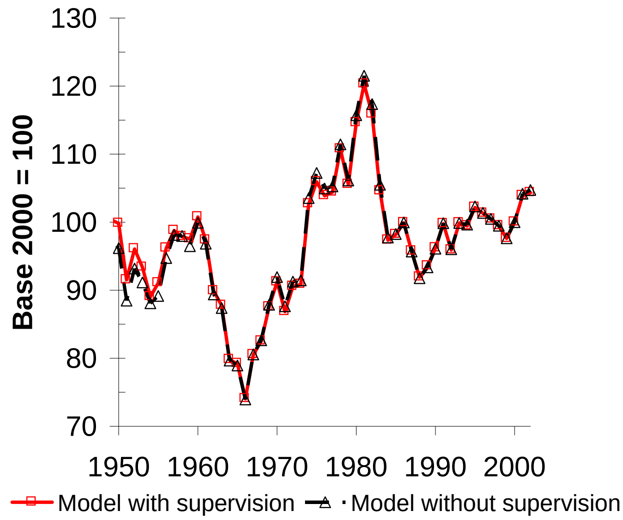
Figures E2 and E3 present, respectively, the capital-output ratio and the consumption-output ratio for the model with supervision (solid line with squares) and the model without supervision (dashed line with triangles). Both figures show that there is very little difference between the two models along the two measures presented. In fact, the time series are 99 per cent correlated. This suggests that the model with supervisors has inherited the basic RBC properties.

E.1 Figures: Time series data

As a supplement to the data appendix, I will show time series of annual hours worked per employed person and the employment rate for some sample countries. Even though annual hours worked are not perfectly correlated with weekly hours worked, for the nine countries considered about 90 per cent of the decline in annual hours is due to a decrease in weekly hours worked. Thus the large secular movements in the two variables are strongly connected.

The first thing to observe from Figure E4 is that the decline in annual hours worked

Figure E3: Comparison of the consumption-output ratio for different models, U.S. 1950 to 2002.



is relatively smooth over the past 130 years. Furthermore, as in the case of the workweek length, I find a strong decline across all countries.

In Figure E5 I show the movement in the employment rate for three selected countries. These countries are representative for key patterns that occurred in the sample. The presented employment rates show no clear pattern over time. Long periods of increase, decrease, and stagnation can occur. A representative for countries that stayed at about the same employment rate level over time is Germany. Another pattern is that of an increase in the employment rate starting from a low level. This is exemplified by the United States. Finally, France is representative of the countries that started with a fairly high employment rate that eventually declined.

Figure E6 shows a plot with different measures of supervisory employment relative to total employment for the United States.

The two measures presented are based on different sources. The sources are explained in Appendix F). The main observation to be taken from this figure is: in the data I find a decline in the number of supervisors relative to the number of employed persons. This observation is visible when considering the data from the census. For the time period where other data sources are available (1964 onwards: Current Employment Statistics and data from

Figure E4: Annual hours worked per employed person, 1870 to 2002.

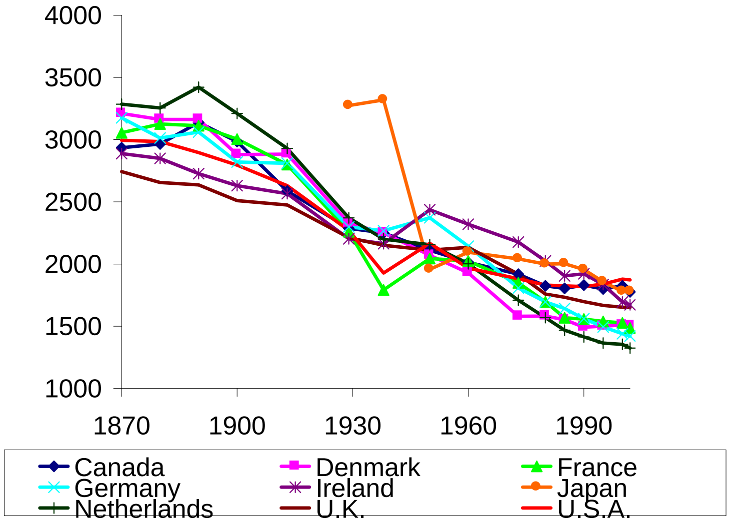


Figure E5: Main patterns for employment rates over time.

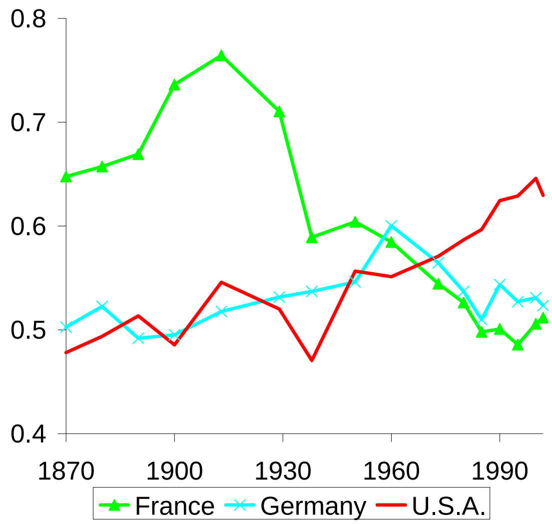
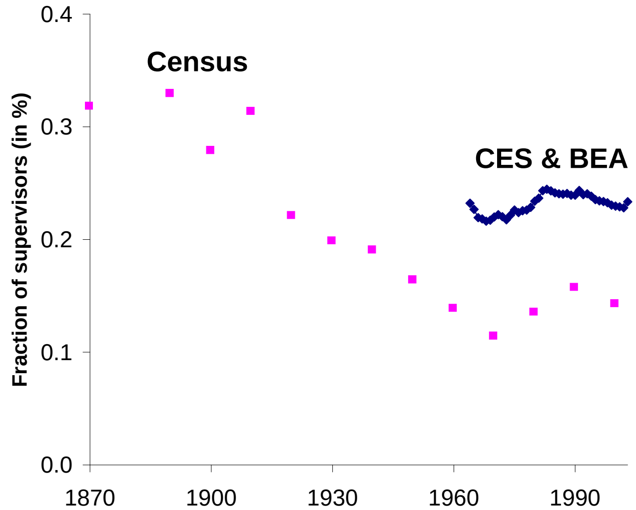


Figure E6: Measures of supervisors relative to measures of employed persons.



the Bureau of Economic Analysis) I find a strong co-movement of the respective measures. Hence, the data are consistent with the model with respect to the decline in the number of supervisors relative to all employed persons.

Appendix F: Data appendix

F.1 Weekly hours worked

The following data sources were used to construct the weekly hours worked for the respective countries:

U.S.A.:

- Robert M. Waples *The Shortening of the American Work Week: An Economic and Historical Analysis*, Dissertation University of Pennsylvania, 1990.
- U.S. Department of Labor, Bureau of Labor Statistics *Employment and Earning*, Publications 1959.1-2004

Japan:

- International Labor Organization, Geneva: Laborsta Internet Database, http://laboursta.ilo.org, downloaded June, 2004.
- Hayashi, Fumio and Prescott, Edward C., "Data Appendix to The 1990s in Japan: A Lost Decade," *Review of Economic Dynamics*, 5(1), 2002, downloadable at:

  http://ideas.repec.org/p/red/append/hayashi02.html.
- Nihon Ginko Tokeukyiku *Hundred-Year Statistics of the Japanese Economy*, Statistics Department Bank of Japan, 1966.
- Thelma Liesner *One Hundred Years of Economic Statistics*, The Economist Publications, Fact on File, New York and Oxford, 1989.

U.K.:

- Hans-Joachim Voth "The Longest Year: New Estimates of Labor Input in England, 1760-1830," *Journal of Economic History*, 61(4), 2001.
- Thelma Liesner *One Hundred Years of Economic Statistics*, The Economist Publications, Fact on File, New York and Oxford, 1989.

- Department of Employment and Productivity, *British Labour Statistics: Historical Abstract 1886-1968*, London 1971.

- Michael Huberman "Working hours of the world unite? New international evidence of working time, 1870-2000, Part 1 and 2," Working Papers at the University of Montreal, March 2003.

Germany:

- Michael Schneider *Streit um Arbeitszeit, Geschichte des Kampfes um Arbeitszeitverkuerzung in Deutschland*, Bund Verlag, Koeln, 1984.

- International Labor Organization, Geneva: Laborsta Internet Database, http://laboursta.ilo.org, downloaded June, 2004.

For all countries, except Japan, data were available for the period 1870 to 2000. In the case of Japan the working time hours are for the period 1923 to 2000.

For some countries only the weekly hours worked for the years 1870 and 2000 were available. Below are the specific references:

1870:

Denmark, France, Ireland, Netherlands, Norway, Sweden, Spain, Italy, and Belgium. Since Norway was a province of Sweden in that year, I used the Swedish working time for Norway.

- Michael Huberman "Working hours of the world unite? New international evidence of working time, 1870-2000, Part 1 and 2," Working Papers at the University of Montreal, March 2003.

2000:

Denmark, France, Ireland, Netherlands, Norway, Sweden, Spain, and Italy.

- International Labor Organization, Geneva: Laborsta Internet Database, http://laboursta.ilo.org, downloaded June, 2004.

  Note: I also used the OECD labour database to get data on: employment, population age 15 and older, and working time per employed person. These data were used for the statements in the introduction.

Belgium: since no weekly data were available I made use of average annual hours worked provided by the OECD-Labor database, which can be found under

http://www.oecd.org/statsportal/0,2639,en_2825_293564_1_1_1_1_1,00.html

F.2 Employment rates

For the construction of the population of age 15 and older and the number of employed persons, various sources were used:

(i) Organization for Economic Co-operation and Development: OECD Statistics Portal, download 05/03/2004,

http://www.oecd.org/statsportal/0,2639,en_2825_293564_1_1_1_1_1,00.html

(ii) B.R. Mitchell, *International Historical Statistics, The Americas 1750-2000*, ed 5, Palgrave Macmillan, 2003

(iii) B.R. Mitchell *International Historical Statistics, Europe 1750-2000*, ed 5, Palgrave Macmillan, 2003

(iv) B.R. Mitchell *International Historical Statistics, Africa, Asia & Oceania 1750-2000*, ed 5, Palgrave Macmillan, 2003

(v) Nihon Ginko Tokeukyoku *Hundred-Year Statistics of the Japanese Economy*, Statistics Department Bank of Japan, 1966

(vi) U.S. Department of Labor, Bureau of Labor Statistics *Employment and Earning*, Publications 1959.1-2004

(vii) Ed. William Lerner, U.S. Department of Commerce and Bureau of the Census *Historical Statistics of the United States, Colonial Times to 1970, Part 1 and 2*, 1975.

(viii) "Groningen Growth and Development Centre and The Conference Board, Total Economy Database, June 2004, http://www.ggdc.net"

(ix) Ed. F.H. Leacy, M.C. Urquhart, and K.A.H. Buckley *Historical Statistics of Canada*, 2nd ed, Minister of Supply and Services Canada, 1983.

(x) Angus Maddison *Dynamic Forces in Capitalist Development, A Long-Run Comparative View*, Oxford: Oxford University Press, 1991.

(xi) Thelma Liesner *One Hundred Years of Economic Statistics*, The Economist Publications, Fact on File, New York and Oxford, 1989.

(xii) Bundesamt fuer Statistik *Statistisches Jahrbuch 2003*, Wiesbaden, pg. 44.

(xiii) Statistics Bureau Japan *Statistical Yearbook of Japan,* 2004, Data download at:

http://www.stat.go.jp/english/data/nenkan.

(xiv) Editor: C.F. Feinstein *National income, expenditure and output of the United Kingdom, 1855-1965*, Cambridge: Cambridge University Press, 1972.

(xv) Bairoch, P., Deldycke, T., Gelders, H., and Limbor, J.-M. *La Population active et sa Structure, The Working Population and its Structure*, Universite Libre de Bruxelles, Insitute de Sociologie, Centre d'Economie Politique, 1968.

In some cases, employment estimates for the year 1870 were not available. In that case, the employment rate for the next year for which data were available was used. I will note such cases and the year used.

Canada

E: 1, 9, and 10.

N: 1 and 2.

Denmark

E: 1 and 10.

N: 1 and 3.

France

E: 1, 3, and 11.

N: 1, 3, and 11.

Germany

E: 1 and 11.

N: 1, 3, 11, and 12.

Ireland

E: 1 and 3.

N: 1 and 3.

Japan

E: 1, 11, 4, 13 and 5.

N: 1, 11, 4, and 5.

Netherlands

E: 1, 3, and 10.

N: 1 and 3.

Norway

E: 1, 3, and 10.

N: 1 and 3.

United Kingdom

E: 1, 11, and 14.

N: 1, 11, and 14.

United States of America

E: 1, 6, and 11.

N: 6 and 7.

Australia (Data for 1901)

E: 4

N: 4

Sweden (Data for 1870)

E: 3

N: 3

Spain (Data for 1877)

E: 3

N: 3

Italy (Data for 1871)

E: 3

N: 3

The working population was adjusted to include the age group of the 14 year old. The employment numbers refer to age groups of 14 and older.

Belgium (Data for 1866)

E: 3

N: 3

The working population was adjusted to include the age group of the 14 year old. The employment numbers refer to age groups of 14 and older.

The employment number was adjusted by 2 per cent to account for a double counting of employed persons in transportation.

To cross-check the correctness of some employment and working age data, I made use of source 15.

F.3 National accounts for Japan, the U.K., and the U.S.A.

For the National Income and Product Accounts as well as the capital stock I used the following sources:

Japan:

1950-2000: Data appendix of Hayashi and Prescott (2003).

1920-1950: 5 and 11.

To get the capital stock prior to 1956 I used the perpetual inventory method, i.e., capital stock yesterday is capital stock today less investment today plus depreciation today.

For Japan there were no data available for the year 1945. I used labour input data, which were available for the year, and estimated the missing numbers.

United Kingdom:

11, 8, 9, 14, and SourceOECD.

United States of America:

For the early period I used Kendrick *Productivity Trends* in the U.S. and BEA.

For the tax revenues of Japan and the United Kingdom I used 11 (pre WWII) and SourceOECD (post WWII).

F.4 Tax measures

Depending on the data availability, taxes are determined in different ways. Wherever available, I made use of existing literature.

For the United States, in the period 1929 to 1975, taxes on capital and labour income from 1929 till 1975 are from Joines (1981). To extend the series backwards till 1916 and forwards till 2000, for the labour income tax I use a series provided by Marion and Mulligan (2004) and for the capital income tax I use the methodology of Mendoza, Razin, and Tesar (1994) to determine the trend.

The consumption tax rate is determined using the Indirect Business Taxes from the National Income and Product Accounts together with the Final Private Consumption expenditures.

Concerning the consumption tax rate for the United Kingdom and Japan, I followed the same approach as described for the United States.

To get the tax rates on labour and capital income, I used tax revenue statistics from the sources already mentioned for the national accounts. These tax revenue data were then combined with data on the national accounts via the method described in Mendoza, Razin, and Tesar (1994) to get the desired tax rates on labour and capital income. I compared the findings with those provided by Mendoza, Razin, and Tesar and found that they were fairly close.

F.5 Supervisory technology

Concerning the supervisory technology, the main sources used for the respective countries:

U.S.A.:

(i) The Current Employment Statistics (CES) publishes an establishment survey. This survey supplies data on employees. Among others it reports hours paid for, the number of paid employees, and the number of production workers in manufacturing and non-supervisory workers in services. From this I extract the employee part of supervisors. Notice that this just gives me an upper bound, since some of the non-production workers aren't supervisors. One thing to note about the CES data is that small establishments and newly started establishments are underrepresented.

I downloaded the data from http://www.bls.gov/ces/ in August 2004.

2. To get a long-run data series on supervisors in the economy, I made use of the census data provided by IPUMS. In particular, I made use of the following variables: AGE, EMPSTAT, LABFORCE, and OCC1950. I restricted my attention to the persons employed for the census years 1900 and 1920-2000, respectively, and to the persons in the labour force for the census years 1850-1890 and 1910. For these years, I consider persons of an age older than fifteen. Within that set, I identified the persons who under OCC1950 fall in the groups of Farmers or Managers, Officials, and Proprietors. These latter groups I associated with the supervisory group in the model.

   With respect to IPUMS see:

   Steven Ruggles, Matthew Sobek, Trent Alexander, Catherine A. Fitch, Ronald Goeken, Patricia Kelly Hall, Miriam King, and Chad Ronnander. Integrated Public Use Microdata Series: Version 3.0 [Machine-readable database]. Minneapolis, MN: Minnesota Population Center [producer and distributor], 2004.

I downloaded the data from http://www.ipums.umn.edu/usa/ in July / August 2004.

3. Concerning the group of self-employed persons in the economy, I use the Bureau of Economic Analysis data for the national economy section 6. I downloaded the data from

   http://www.bea.doc.gov/bea/dn/nipaweb/SelectTable.asp?Selected=N#S7 in August 2004.

For the calibration of μ I used the source 2.

U.K.:

From the International Labor Organization (ILO) I downloaded on the 5 October 2004, the annual data table 2C. The data can be downloaded at http://laboursta.ilo.org. The table 2C reports the number of total employed persons by occupation. Majorgroup 1 consists of corporate and general managers. Here, general managers include proprietors.

The particular numbers found are: Majorgroup 1: 4015.6 thousand persons, and total employment: 28414.5 thousand persons.

Japan:

In the book *Hundred-Year Statistics of the Japanese Economy,* on page 52, I found the number of gainfully occupied persons, as well as the number of employers and workers on own account.

Table F1: Data on supervisors from *Hundred-Year Statistics of the Japanese Economy.*

	Gainfully occupied	Business Proprietors	Workers on own account
1930	29341	6150	3396

Nihon Ginko Tokeukyoku, *Hundred-Year Statistics of the Japanese Economy*, Statistics Department Bank of Japan, 1966.

F.6 Employment rate by age groups

The data for all countries both for the years 1970 and 2002 are taken from the OECD-Labor database, which can be found under:

http://www.oecd.org/statsportal/0,2639,en_2825_293564_1_1_1_1_1,00.html

The data were downloaded on 13 October 2004.

Appendix G: Supervision technology

Consider the type h's firms problem. If I assume efficient production and disregard for a second capital and TFP, the problem reduces to:

$$\max h^{\phi} E_p - wE$$
$$\text{s.t.}$$
$$E_s + E_p = E$$
$$E_p = A_s h^{\kappa} E_s.$$

This problem simplifies to:

$$\max h^{\phi} \left(\frac{A_s h^{\kappa}}{1 + A_s h^{\kappa}} \right)^{1-\theta} E^{1-\theta} - w\ E,$$

which in turn implies

$$\begin{aligned} w &= h^{\phi} \left(\frac{A_s h^{\kappa}}{1 + A_s h^{\kappa}} \right)^{1-\theta} (1-\theta)\, E^{-\theta}; \\ w'(h) &= \left(\frac{A_s h^{\kappa}}{1 + A_s h^{\kappa}} \right)^{2-\theta} \frac{h^{\phi-\kappa-1}}{A_s} \left[\phi + \kappa\,(1-\theta) + A_s \phi h^{\kappa} \right] E^{-\theta} \end{aligned}$$

$$\Rightarrow \frac{w}{w'} = \frac{(1-\theta)\,(1 + A_s \mu h^{\kappa}) h}{(\phi + \kappa\,(1-\theta) + \phi A_s h^{\kappa})}.$$

I define:

$$R(h, A_s) = \frac{(1-\theta)\,(1 + A_s h^{\kappa}) h}{\kappa\,(1-\theta) + \phi + \phi A_s h^{\kappa}}.$$

Then I get:

- $R(1, A_s) = \frac{(1-\theta)(1+A_s)}{\kappa(1-\theta)+(1+A_s)\phi} > 0, R(0, A_s) = 0,$
- $\lim_{A_s \to \infty} R(h) = \frac{1-\theta}{\phi} h,$

- $\frac{\partial R}{\partial A_s} = \frac{(1-\theta)\ \kappa\ h^{\kappa+1}}{(\kappa(1-\theta)+\phi+A_s\phi h^{\kappa})^2} > 0$, which implies $A_{s,1} > A_{s,0} \Rightarrow R(h, A_{s,1}) \geq R(h, A_{s,0})$ with strict inequality for $h > 0$.

- Note that $\frac{\partial R}{\partial h} > 0, \forall h \geq 0$, given $1 < \phi/\left(1-\theta\right) < \left(5+\sqrt{22}\right)/6$.

Now I will start to include capital and production Total Factor Productivity into the considerations.

Then the firm of type h's problem reduces to:

$$\max_{K_t, E_t \geq 0} A_{p,t} \left(\frac{A_{s,t} h^{\kappa}}{1 + A_{s,t} h^{\kappa}} \right)^{1-\theta} h_t^{\phi} K_t^{\theta} E_t^{1-\theta} - r_t K_t - w_t\left(h_t\right) E_t.$$

Remark 2 *If $A_{s,t}$ is an increasing sequence over time going to infinity, then the production technology behaves more and more like one characterized by the production function:*

$$F(h, K, E; t) = A_{p,t} h_t^{\phi} K_t^{\theta} E_t^{1-\theta}.$$

Remark 3 *Under the parameter constraints $1 < \phi/(1-\theta) < (5+\sqrt{22})/6$, the implied wage function*

$$w\left(h; r\right) = \frac{\left(1-\theta\right) A_s h^{\kappa + \phi/(1-\theta)}}{1 + A_s h^{\kappa}} \left(\frac{\theta}{r}\right)^{\theta/(1-\theta)}$$

is strictly increasing and strictly convex for $h > 0$.

Appendix H: Characterization of the equilibrium

From Appendix G I get the following reduced problem:

$$\max_{E_t, K_t \geq 0} A_{p,t} h_t^{\phi} \left(\frac{A_{s,t} h_t^{\kappa}}{1 + A_{s,t} h_t^{\kappa}} \right)^{1-\theta} K_t^{\theta} E_t^{1-\theta} - w_t \left(h_t\right) E_t - r_t K_t,$$

which in turn implies:

$$\begin{aligned}
r_t &= \theta A_{p,t} h_t^{\phi} \left(\frac{A_{s,t} h_t^{\kappa}}{1 + A_{s,t} h_t^{\kappa}} \right)^{1-\theta} \left(\frac{E_t}{K_t} \right)^{1-\theta}, \\
w_t &= (1-\theta) A_{p,t} h_t^{\phi} \left(\frac{A_{s,t} h_t^{\kappa}}{1 + A_{s,t} h_t^{\kappa}} \right)^{1-\theta} \left(\frac{K_t}{E_t} \right)^{\theta}, \\
w_t \left(h_t; r_t\right) &= \left(1-\theta\right) A_{p,t}^{1/(1-\theta)} \left(\frac{\theta}{r_t} \right)^{\theta/(1-\theta)} \frac{A_{s,t} h_t^{\kappa+\phi/(1-\theta)}}{1 + A_{s,t} h_t^{\kappa}}, \\
\frac{\partial w \left(h_t; r_t\right)}{\partial h_t} &= \left(1-\theta\right) A_{p,t}^{1/(1-\theta)} \left(\frac{\theta}{r_t} \right)^{\theta/(1-\theta)} \left[\frac{h_t^{\kappa+\phi/(1-\theta)-1} A_{s,t} \left(\kappa \left(1-\theta\right) + \phi + A_{s,t} h_t^{\kappa}\right)}{\left(1-\theta\right) \left(1 + A_{s,t} h_t^{\kappa}\right)^2} \right] \\
&= A_{p,t} h_t^{\phi-1-\kappa} A_{s,t}{}^{1-\theta} \left(\kappa \left(1-\theta\right) + \phi + A_{s,t} \phi h_t^{\kappa}\right) \left(\frac{h_t^{\kappa}}{1 + A_{s,t} h_t^{\kappa}} \right)^{2-\theta} \left(\frac{K_t}{E_t} \right)^{\theta}.
\end{aligned}$$

The stand-in-household's problem:

$$\max \sum_{t=0}^{\infty} \left(\beta\lambda\right)^t \left[e_t u\left(c_{1,t}, h_t\right) + \left(1-e_t\right) u\left(c_{0,t}, 0\right)\right]$$

s.t.

$$\begin{aligned}
\left(1-\tau_{c,t}\right) \left(e_t c_{1,t} + \left(1-e_t\right) c_{0,t}\right) + x_t &\leq \left(1-\tau_{e,t}\right) w_t \left(h_t\right) e_t + \left(1-\tau_{k,t}\right) r_t k_t + T_t + \tau_{k,t} \delta k_t \\
x_t &\geq \lambda k_{t+1} - \left(1-\delta\right) k_t,
\end{aligned}$$

implies for interior solutions the following:

$$(1-\tau_{c,t})\left(e_t c_{1,t}+(1-e_t)\,c_{0,t}\right)+\lambda k_{t+1}=(1-\tau_{e,t})\,w_t\,(h_t)\,e_t+(1+(1-\tau_{k,t})\,(r_t-\delta))k_t+T_t,$$

$$\frac{u\left(c_{1,t},h_t\right)-u\left(c_{0,t},0\right)-\left[u_c\left(c_{1,t},h_t\right)c_{1,t}-u_c\left(c_{0,t},0\right)c_{0,t}\right]}{-u_c\left(c_{1,t},h_t\right)}=\frac{1-\tau_{e,t}}{1+\tau_{c,t}}w_t\left(h_t\right),$$

$$\begin{aligned}
u_c\left(c_{1,t},h_t\right) &= u_c\left(c_{0,t},0\right),\\
\frac{u_c\left(c_{0,t},0\right)}{u_c\left(c_{0,t+1},0\right)} &= \beta\frac{1+\tau_{c,t}}{1+\tau_{c,t+1}}\left(1+(1-\tau_{k,t+1})\left(r_{t+1}-\delta\right)\right),\\
-\frac{u_h\left(c_{1,t},h_t\right)}{u_c\left(c_{1,t},h_t\right)} &= \frac{1-\tau_{e,t}}{1+\tau_{c,t}}w_t'\left(h_t\right),\\
\lim_{t\to\infty}\beta^t u_c\left(c_{0,t},0\right)k_{t+1} &= 0,
\end{aligned}$$

where $u\left(c,h\right)=\frac{\left[c^{\alpha}(1-h)^{1-\alpha}\right]^{1-\sigma}-1}{(1-\sigma)}$.

H.1 *Variable transformation*

In order to solve the model on the computer I transform the system into an asymptotically-stationary form. For this I assume that $A_{p,t}=\gamma^{t(1-\theta)}\hat{A}_{p,t}$, with $\hat{A}_t$ being bounded. Then I redefine the variables using the growth trend of the total factor productivity: γ.

In particular:

$$\hat{c}_{i,t}=c_{i,t}/\gamma^t, i=0,1; \hat{k}_t=k_t/\gamma^t; \hat{e}_t=e_t; \hat{h}_t=h_t; \hat{g}_t=G_t/\gamma^t.$$

Given this redefinition, the first-order necessary conditions become:

$$\left(\gamma\frac{\hat{c}_{0,t+1}}{\hat{c}_{0,t}}\right)^{1-\alpha(1-\sigma)} = \tag{H1}$$

$$\beta\frac{1+\tau_{c,t}}{1+\tau_{c,t+1}}\left(1+(1-\tau_{k,t+1})\left(\theta\hat{A}_{p,t+1}\hat{h}_{t+1}^{\phi}\left(\frac{A_{s,t+1}\hat{h}_{t+1}^{\kappa}}{1+A_{s,t+1}\hat{h}_{t+1}^{\kappa}}\right)^{1-\theta}\left(\frac{\hat{e}_{t+1}}{\hat{k}_{t+1}}\right)^{1-\theta}-\delta\right)\right),$$

$$\frac{(1-\alpha)\,\hat{c}_{1,t}}{\alpha\left(1-\hat{h}_t\right)} = \tag{H2}$$

$$\frac{1-\tau_{e,t}}{1+\tau_{c,t}}\hat{A}_{p,t}\hat{h}_t^{\phi-1-\kappa}\left(\frac{\kappa\,(1-\theta)+\phi}{A_{s,t}}+\phi\hat{h}_t^{\kappa}\right)\left(\frac{\hat{h}_t^{\kappa}}{1/A_{s,t}+\hat{h}_t^{\kappa}}\right)^{2-\theta}\left(\frac{\hat{k}_t}{\hat{e}_t}\right)^{\theta},$$

$$\hat{c}_{1,t} = \hat{c}_{0,t}\left(1-\tilde{h}_t\right)^{\varepsilon}, \tag{H3}$$

$$\hat{c}_{1,t}-\hat{c}_{0,t} = \frac{-\alpha\,(1-\sigma)}{1-\alpha\,(1-\sigma)}\frac{1-\tau_{et}}{1+\tau_{ct}}(1-\theta)\hat{A}_{p,t}\hat{h}_t^{\phi}\left(\frac{A_{s,t}\hat{h}_t^{\kappa}}{1+A_{s,t}\hat{h}_t^{\kappa}}\right)^{1-\theta}\left(\frac{\hat{k}_t}{\hat{e}_t}\right)^{\theta}, \tag{H4}$$

$$\tilde{e}_t\hat{c}_{1,t}+(1-\tilde{e}_t)\,\hat{c}_{0,t}+\lambda\gamma\hat{k}_{t+1}+\hat{g}_t = \hat{A}_{p,t}\hat{h}_t^{\phi}\left(\frac{A_{s,t}\hat{h}_t^{\kappa}}{1+A_{s,t}\hat{h}_t^{\kappa}}\right)^{1-\theta}\hat{k}_t^{\theta}\hat{e}_t^{1-\theta}+(1-\delta)\hat{k}_t, \tag{H5}$$

$$\lim_{t\to\infty}\beta^t u_c\left(\hat{c}_{0,t},0\right)\hat{k}_{t+1} = 0.$$

Using equations (H3) and (H2), I get:

$$\begin{pmatrix}\hat{c}_{1,t}\\ \hat{c}_{0,t}\end{pmatrix} = \begin{pmatrix}\frac{1-\tau_{e,t}}{1+\tau_{c,t}}\frac{(1-h_t)^{\varepsilon}}{(1-h_t)^{\varepsilon}-1}\frac{(\sigma-1)\alpha}{1-\alpha(1-\sigma)}w_t\left(h_t;r_t\right)\\ \frac{1-\tau_{e,t}}{1+\tau_{c,t}}\frac{1}{(1-h_t)^{\varepsilon}-1}\frac{(\sigma-1)\alpha}{1-\alpha(1-\sigma)}w_t\left(h_t;r_t\right)\end{pmatrix}.$$

This in turn can be used together with equations (H4) and (H3), to get the following key result:

If we have an interior equilibrium, then there exists an optimal workweek length that is determined by the equation

$$-\frac{1}{\varepsilon}\left[1-\hat{h}_t-\left(1-\hat{h}_t\right)^{1-\varepsilon}\right]=\frac{(1-\theta)\left(1+A_{s,t}\hat{h}_t^{\kappa}\right)\hat{h}_t}{\kappa(1-\theta)+\phi+\phi A_{s,t}\hat{h}_t^{\kappa}}, \tag{H6}$$

where $\varepsilon=\frac{(1-\sigma)(1-\alpha)}{1-\alpha(1-\sigma)}$.

Notice that in the last case the working time is purely determined by the deep parameters of the model. In particular, the workweek is independent of taxes and total factor productivity of the final-goods-producing sector.

H.2 Limiting economy

If t is large enough and $A_{s,t}\to\infty$ as $t\to\infty$, then the first-order conditions are approximately equal to:

$$\left(\gamma\frac{\hat{c}_{0,t+1}}{\hat{c}_{0,t}}\right)^{1-\alpha(1-\sigma)}=\beta\frac{1+\tau_{c,t}}{1+\tau_{c,t+1}}\left(1+(1-\tau_{k,t+1})\left(\theta\hat{A}_{p,t}\hat{h}_t^{\phi}\left(\frac{\hat{e}_t}{\hat{k}_t}\right)^{1-\theta}-\delta\right)\right),$$

$$\frac{(1-\alpha)\,\hat{c}_{1,t}}{\alpha\left(1-\hat{h}_t\right)}=\frac{1-\tau_{e,t}}{1+\tau_{c,t}}\hat{A}_{p,t}\hat{h}_t^{\phi-1}\phi\left(\frac{\hat{k}_t}{\hat{e}_t}\right)^{\theta},$$

$$\hat{c}_{1,t}=\hat{c}_{0,t}\left(1-\tilde{h}_t\right)^{\varepsilon},$$

$$\hat{c}_{1,t}-\hat{c}_{0,t}=\frac{-\alpha(1-\sigma)}{1-\alpha(1-\sigma)}\frac{1-\tau_{e,t}}{1+\tau_{c,t}}(1-\theta)\hat{A}_{p,t}\hat{h}_t^{\phi}\left(\frac{\hat{k}_t}{\hat{e}_t}\right)^{\theta},$$

$$\tilde{e}_t\hat{c}_{1,t}+(1-\tilde{e}_t)\,\hat{c}_{0,t}+\lambda\gamma\hat{k}_{t+1}+\hat{g}_t=\hat{A}_{p,t}\hat{h}_t^{\phi}\hat{k}_t^{\theta}\hat{e}_t^{1-\theta}+(1-\delta)\hat{k}_t,$$

$$\lim_{t\to\infty}\beta^t u_c\left(\hat{c}_{0,t},0\right)\hat{k}_{t+1}=0.$$

Now this limit economy will be in a steady state if $\tau_{c,t}=\tau_c,\tau_{k,t}=\tau_k,\tau_{e,t}=\tau_e,\hat{g}_t=\hat{g}$ and $\hat{A}_{p,t}=\hat{A}_p$. This steady state is described by the following equations:

$$
\begin{aligned}
\gamma^{1-\alpha(1-\sigma)} &= \beta\left(1+(1-\tau_k)\left(\theta\hat{A}\hat{h}^{\phi}\left(\frac{\hat{e}}{\hat{k}}\right)^{1-\theta}-\delta\right)\right), \\
\frac{(1-\alpha)}{\alpha} &= \frac{1-\tau_{et}}{1+\tau_{ct}}\hat{A}_p\hat{h}^{\phi-1}\phi\left(\frac{\hat{k}}{\hat{e}}\right)^{\theta}\frac{\left(1-\hat{h}\right)}{\hat{c}_1\hat{h}}, \\
\hat{c}_1 &= \hat{c}_0\left(1-\tilde{h}\right)^{\varepsilon}, \\
\hat{c}_1-\hat{c}_0 &= \frac{-\alpha\left(1-\sigma\right)}{1-\alpha\left(1-\sigma\right)}\frac{1-\tau_e}{1+\tau_c}(1-\theta)\hat{A}_p\hat{h}^{\phi}\left(\frac{\hat{k}}{\hat{e}}\right)^{\theta}, \\
\tilde{e}\hat{c}_1+\left(1-\tilde{e}\right)\hat{c}_0+\lambda\gamma\hat{k}+\hat{g} &= \hat{A}_p\hat{h}^{\phi}\hat{k}^{\theta}\hat{e}^{1-\theta}+(1-\delta)\hat{k}.
\end{aligned}
$$

This equation system is equivalent to:

$$
\begin{aligned}
\hat{c}_1 &= \hat{c}_0\left(1-\hat{h}\right)^{\phi}, \\
\gamma^{1-\alpha(1-\sigma)} &= \beta\left(1+(1-\tau_k)\left(\theta\hat{A}_p\hat{h}^{\phi}\left(\frac{\hat{e}}{\hat{k}}\right)^{1-\theta}-\delta\right)\right), \\
\frac{\left(1-\alpha\right)\hat{c}_1}{\alpha\left(1-\hat{h}\right)} &= \frac{1-\tau_e}{1+\tau_c}\phi\hat{A}_p\hat{h}^{\phi-1}\left(\frac{\hat{k}}{\hat{e}}\right)^{\theta}, \\
\frac{(1-\theta)}{\phi}\hat{h} &= -1/\varepsilon\left(\left(1-\hat{h}\right)-\left(1-\hat{h}\right)^{1-\varepsilon}\right), \\
\hat{e}\hat{c}_1+\left(1-\hat{e}\right)\hat{c}_0+(\gamma\lambda+\delta-1)\hat{k} &= \hat{A}_p\hat{h}^{\phi}\hat{k}^{\theta}\hat{e}^{1-\theta}.
\end{aligned}
$$

This equation system can be solved in the following fashion:

$$
\begin{aligned}
\hat{h} &\leftarrow \frac{1-\theta}{\phi}h=-\frac{1}{\varepsilon}\left(1-h-(1-h)^{1-\varepsilon}\right), \\
\hat{c}_1 &\leftarrow \hat{c}_1=\frac{1-\tau_e}{1+\tau_c}\frac{\alpha\phi\hat{A}_pQ^{-\theta}}{1-\alpha}\hat{h}^{(1-\theta-\phi)/(1-\theta)}\left(1-\hat{h}\right), \\
\hat{c}_0 &\leftarrow \hat{c}_0=\left(1-\hat{h}\right)^{-\varepsilon}\hat{c}_1, \\
\hat{k} &\leftarrow \hat{k}=\hat{c}_0/\left(\hat{A}_pQ^{1-\theta}-Q\hat{h}^{-\phi/(1-\theta)}\left(1-\hat{c}_1+\hat{c}_0\right)+1-\delta-\gamma\lambda\right), \\
\hat{e} &\leftarrow \hat{e}=Q\hat{h}^{-\phi/(1-\theta)}\hat{k},
\end{aligned}
$$

where $Q = \left(\left[\frac{\left(\gamma^{1-\alpha(1-\sigma)}/\beta-1\right)}{1-\tau_k}+\delta\right]/(\theta\hat{A}_p)\right)^{1/(1-\theta)}$ and $\varepsilon = (1-\alpha)(1-\sigma)/(1-\alpha(1-\sigma))$.

Appendix I: Equivalence HP-model and wage-function model

There is a continuum of feasible workweeks, $H = [0,1]$.

In this appendix I assume that $\lambda = 1$. This does not affect the main argument.

Definition 2 *Let a Hornstein-Prescott equilibrium be defined as an allocation* (μ, k) *and a price system* (r, w) *such that the following conditions are satisfied:*

Consumer problem (CP1)

$$\max_{\mu,k\geq 0} U(\mu) = \sum_{t=0}^{\infty} \beta^t \int u(c_{ht}, h_t)\mu_t \left(dh \times dc\right)$$

s.t.

$$\begin{aligned} \int \left[c_{ht} - w_{ht}\right] \mu_t \left(dh \times dc\right) + k_{t+1} &\leq \left(1 - \delta + r_t\right) k_t, \\ \int \mu \left(dh \times dc\right) &\leq 1. \end{aligned}$$

Firms problem in period t (FP1(t))

$$\max_{(\nu,k)\in\mathcal{Y}_t} \int \left[Y_{ht} - w_{ht}\right] \nu_t \left(dh \times dc\right) - r_t K_t.$$

Resource constraints (RC1)

$$\begin{aligned} k_t &= K_t, \\ \mu &= \nu. \end{aligned}$$

Definition 3 *Let a wage-function equilibrium be defined such that the allocation* (c_1, c_0, e, h, k) *and the price system* (r) *satisfy the following conditions:*

Consumer problem (CP2)

$$\max_{c_h,c_0,e,k,h\geq 0} U(c,e,h) = \sum_{t=0}^{\infty} \beta^t [e_t u(c_{h,t},h_t) + (1-e_t) u\left(c_{0,t},0\right)]$$

s.t.

$$\begin{aligned} \int e_t\left[c_{h,t} - w_t\left(h\right)\right] + \left(1-e_t\right)c_{0,t} + k_{t+1} &\leq \left(1-\delta+r_t\right)k_t, \\ e_t, h_t &\leq 1. \end{aligned}$$

Firms of type h's problem in period t (FP2(t))

$$\max_{(E,K)\geq 0} F\left(h_t, K_t, E_t\right) - w_t E_t - r_t K_t,$$

Resource constraints (RC2)

$$\begin{aligned} e_t c_{h,t} + \left(1-e_t\right)c_{0,t} + k_{t+1} &\leq F\left(h_t, K_t, E_t\right) + \left(1-\delta\right)k_t, \\ k_t &= K_t, \\ e_t &= E_t. \end{aligned}$$

Proposition 4 *If there is a continuum of feasible workweek lengths,* $H = [0,1]$*,* $u\left(c,h\right) = \left(c^{\alpha}\left(1-h\right)^{1-\alpha}\right)^{1-\sigma}/\left(1-\sigma\right)$ *and the plant of type h technology is given by:*

$$f(k,e;h,t) = \left\{ \begin{array}{ccc} 0 & for & 0 \leq e < \bar{e} \\ A_{p,t}h^{\phi}k^{\theta} & & \bar{e} \leq e \end{array} \right\}, \theta \in \left(0,1\right),$$

then the optimal allocation of the HP- and wage-function equilibrium is such that:

$$\begin{aligned} \mu\left(dh \times dc_1\right) &= e, \\ \mu\left(dh \times dc_1\right) + \mu\left(d0 \times dc_0\right) &= 1, \\ k &= k, \\ r &= r, \\ w_h &= w(h). \end{aligned}$$

Proof. See Prescott (2003) or Alpanda and Ueberfeldt (2003). ■

Lemma 5 *Let there be a set of activities each characterized by the workweek length of operation h, such that:*

$$f(k,e;h,t) = \left\{ \begin{array}{ccc} 0 & & 0 \leq e < \bar{e} \\ & for & \\ A_{p,t}h^{\phi}k^{\theta} & & \bar{e} \leq e \end{array} \right\}, \theta \in (0,1).$$

Then the aggregate plant h production function is given by:

$$F(K_h, E_h; t) = \tilde{A}_{p,t}h^{\phi}K_h^{\theta}E_h^{1-\theta}.$$

Proof. First the aggregate production problem:

$$\max_{e \geq 0} \sum_i A_{p,t}h^{\phi}k_i^{\theta}e_i$$

s.t.

$$\begin{aligned} \sum_i e_i k_i &= K_h, \\ \sum_i e_i &= E_h. \end{aligned}$$

I define: $f(e) = \left\{ \begin{array}{ccc} 0 & & 0 \leq e < \bar{e} \\ & for & \\ 1 & & \bar{e} \leq e \end{array} \right\}.$

Notice that the function $f(e)e^{\theta-1}$ has a maximum at $\bar{e}$. I also know, from linear programming, that this problem has at most two points with positive mass $(e_i, k_i)_{i=1,2}$.

Let me first show that all plants in operation will employ $\bar{e}$ workers.

Notice that I can do better than any (e_i, k_i) by redistributing everything to $e_i/\bar{e}$ plants with inputs $\left(\bar{e}, k_i/\left(e_i/\bar{e}\right)\right)_i$:

$$
\begin{aligned}
A_{p,t}h^{\phi}f\left(e_i\right)k_i^{\theta} &\leq A_{p,t}h^{\phi}f\left(e_i\right)e_i^{\theta-1}\left(\frac{k_i}{e_i}\right)^{\theta}e_i, \\
&\leq A_{p,t}h^{\phi}f\left(\bar{e}\right)\bar{e}^{\theta-1}\left(\frac{k_i}{e_i}\right)^{\theta}, \\
&= \frac{e_i}{\bar{e}}A_{p,t}h^{\phi}f\left(\bar{e}\right)\left(\frac{k_i}{e_i/\bar{e}}\right)^{\theta}.
\end{aligned}
$$

The next step is to establish that the capital stock will be the same, too.

Since the labour inputs are equated across plants I get the following:

$$
\begin{aligned}
\sum_i k_i^{\theta} &= K_h^{\theta}(E_h/\bar{e})^{\theta}\sum\left(\frac{(E_h/\bar{e})k_i}{K_h}\right)^{\theta}, \\
&\leq K_h^{\theta}(E_h/\bar{e})^{-\theta}\left(\sum\frac{k_i}{K_h/(E_h/\bar{e})}\right)^{\theta}, \\
&\leq K_h^{\theta}(E_h/\bar{e})^{-\theta}.
\end{aligned}
$$

Notice that by the previous result: $\sum e_i k_i = \sum \frac{E_h}{\bar{e}}k_i$ and, furthermore, using the basic constraint on the capital stock I have:

$\sum \frac{E_h}{\bar{e}}k_i/K_h = 1$. Thus the second inequality is just an application of the concavity of k^{θ}.

This completes the proof. ■

Lemma 6 *For the given setting we have that for HP-equilibria:*

1. $\mu\left(dh \times dc_h\right) + \mu\left(d0 \times dc_0\right) = 1$,

2. *and if* $\mu\left(dh \times dc_h\right) = e$,

$$\text{then either } e < 1, h \in (0,1) \text{ or } e = 1, h \leq 1.$$

Proof. See Prescott (2003) or Alpanda and Ueberfeldt (2003). ■

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