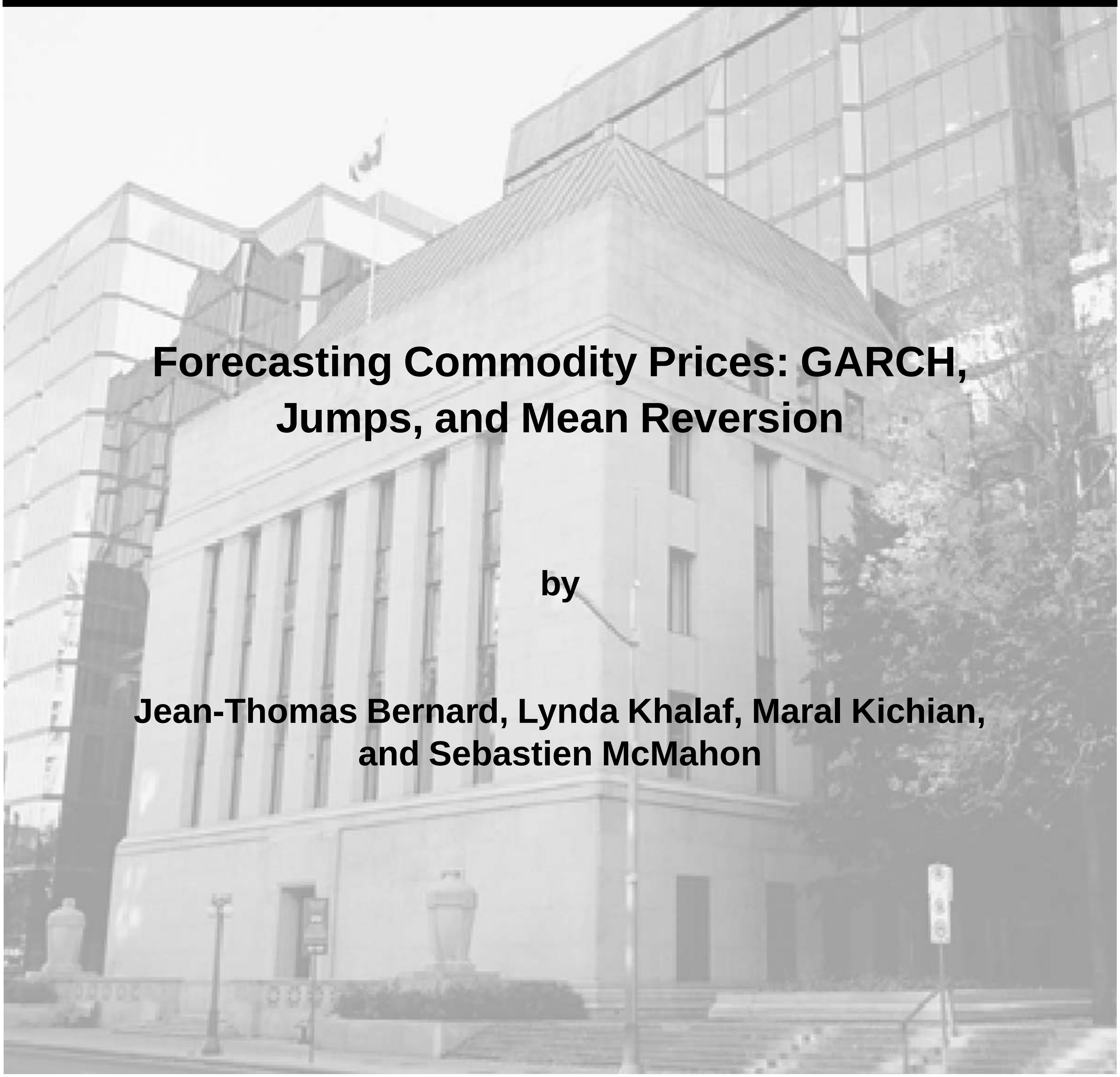
Bank of Canada Banque du Canada

Working Paper 2006-14 / Document de travail 2006-14



Forecasting Commodity Prices: GARCH, Jumps, and Mean Reversion

by

Jean-Thomas Bernard, Lynda Khalaf, Maral Kichian, and Sebastien McMahon

ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2006-14

April 2006

Forecasting Commodity Prices: GARCH, Jumps, and Mean Reversion

by

Jean-Thomas Bernard,[1] Lynda Khalaf,[2] Maral Kichian,[3] and Sebastien McMahon[4]

[1]Holder of the Chair on the Economics of Electrical Energy
Département d'économique and GREEN
Université Laval, Quebec, Canada G1K 7P4
jtber@ecn.ulaval.ca

[2]Holder of the Canada Research Chair in Environment
Département d'économique and GREEN
Université Laval, Quebec, Canada G1K 7P4
and CIREQ, Université de Montréal, Quebec, Canada H3C 3J7
lynda.khalaf@ecn.ulaval.ca

[3]Research Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
mkichian@bankofcanada.ca

[4]Ministry of Finance
Quebec City, Quebec, Canada G1R 5L3
and GREEN, Université Laval, Quebec, Canada G1K 7P4
sebastien.mcmahon@finances.gouv.qc.ca

Contents

Acknowledgements

This work was supported in part by the Institut de Finance mathématique de Montreal (IFM2), the Canadian Network of Centres of Excellence (program on Mathematics of Information Technology and Complex Systems [MITACS]), the Social Sciences and Humanities Research Council of Canada, the Fonds FQRSC (Government of Quebec), the Chair on the Economics of Electrical Energy, and the Canada Research Chair in Environment. We would like to thank Paul Corrigan, René Dufresne, Yves Richelle, Sylvie Giquel, and Robert Weiner.

Abstract

Fluctuations in the prices of various natural resource products are of concern in both policy and business circles; hence, it is important to develop accurate price forecasts. Structural models provide valuable insights into the causes of price movements, but they are not necessarily the best suited for forecasting given the multiplicity of known and unknown factors that affect supply and demand conditions in these markets. Parsimonious representations of price processes often prove more useful for forecasting purposes. Central questions in such stochastic models often revolve around the time-varying trend, the stochastic convenience yield and volatility, and mean reversion. The authors seek to assess and compare alternative approaches to modelling these effects, focusing on forecast performance. Three econometric specifications are considered that cover the most up-to-date models in the recent literature on commodity prices: (i) random-walk models with autoregressive conditional heteroscedasticity (ARCH) or generalized ARCH (GARCH) effects, and with normal or student-*t* innovations, (ii) Poisson-based jump-diffusion models with ARCH or GARCH effects, and with normal or student-*t* innovations, and (iii) mean-reverting models that allow for uncertainty in equilibrium price.

The authors' empirical application uses aluminium price series at daily, weekly, and monthly frequencies. The authors use one-step-ahead out-of-sample forecasts, where parameter estimates are repeatedly updated at every step of the procedure. In addition, in models with jumps, where analytical formulae are not readily available for obtaining conditional expected forecast errors, the authors devise a simple simulation-based procedure to approximate these errors. Their results are as follows. The mean-reverting model with stochastic convenience yield outperforms, to a large extent, all other competing models for all forecast horizons, with high-frequency (daily and weekly) data; within the non-mean-reverting GARCH class of processes analyzed for these frequencies, models with jumps or asymmetries fare best, yet the latter remain dominated by the mean-reverting models. With monthly data, the mean-reverting model still fares well in comparison with the random-walk GARCH class; nevertheless, depending on the forecast horizon and evaluation criteria, non-mean-reverting models with GARCH-in-mean effects dominate to some extent, suggesting that expected risk has a non-negligible effect on price behaviour.

JEL classification: C52, C53, E37
Bank classification: Econometric and statistical methods

Résumé

Les fluctuations des prix des matières premières préoccupent tant les responsables des politiques publiques que les entreprises; il importe donc de disposer de prévisions de bonne qualité à leur sujet. Les modèles structurels fournissent de précieuses indications sur les causes de l'évolution des prix, mais ils ne se prêtent pas nécessairement à la prévision compte tenu de la multiplicité des facteurs, connus ou non, qui agissent sur les conditions de l'offre et de la demande sur les marchés des produits de base. Les représentations parcimonieuses de la dynamique des prix s'avèrent souvent mieux adaptées à la prévision. Dans les spécifications stochastiques de ce genre, les principaux effets à modéliser concernent généralement la tendance (variable dans le temps), le rendement d'opportunité et la volatilité stochastiques ainsi que la stationnarité. Les auteurs évaluent et comparent différentes modélisations de ces effets sous l'angle de la qualité des prévisions. Les trois spécifications qu'ils considèrent englobent les plus récents modèles utilisés dans la littérature sur les prix des produits de base : i) les modèles de marche aléatoire intégrant des effets ARCH ou GARCH et dans lesquels les chocs sont distribués selon la loi normale ou la loi de Student; ii) les modèles basés sur un processus de Poisson, qui intègrent des effets ARCH ou GARCH et dans lesquels les chocs sont également distribués selon l'une de ces deux lois; et iii) les modèles stationnaires où le prix d'équilibre est incertain.

À l'aide de données quotidiennes, hebdomadaires et mensuelles sur les prix de l'aluminium, les auteurs effectuent une prévision hors échantillon à l'horizon d'une période, puis répètent l'opération en actualisant chaque fois l'estimation des paramètres. Dans le cas des modèles avec saut, où aucune formule analytique ne permet d'obtenir l'espérance conditionnelle des erreurs de prévision, ils mettent au point une méthode de simulation simple pour générer ces erreurs. Les auteurs obtiennent les résultats suivants. Le modèle stationnaire dans lequel le rendement d'opportunité est stochastique l'emporte de loin sur tous les autres à tous les horizons de prévision dans le cas des données de fréquences quotidienne et hebdomadaire; parmi les modèles non stationnaires de type GARCH analysés pour ces deux fréquences, ceux comportant un processus de saut ou des effets asymétriques prédominent, mais ils donnent de moins bons résultats que le modèle stationnaire. Dans le cas des données mensuelles, ce dernier surpasse encore les modèles de marche aléatoire intégrant des effets GARCH; toutefois, selon l'horizon de prévision et les critères d'évaluation retenus, les modèles non stationnaires ayant des effets GARCH-M dominent dans une mesure plus ou moins grande, ce qui laisse croire que l'espérance du risque a un effet non négligeable sur le comportement des prix.

Classification JEL : C52, C53, E37
Classification de la Banque : Méthodes économétriques et statistiques

1. Introduction

Fluctuations in commodity prices are of interest because they affect the decisions taken by producers and consumers; they play a crucial role in commodity-related investments, project appraisals, and strategic planning; and they reflect and influence general economic activity. The ability to accurately forecast the price of these various natural resource products is therefore an important concern in both policy and business circles.

In general, structural models provide valuable insights into the determinants of commodity price movements. Yet, given the multiplicity of known, and especially unknown, factors that affect supply and demand conditions in these markets, such models are not necessarily the best suited for forecasting purposes. Researchers often rely instead on parsimonious representations of price processes for their forecasting needs.

In examining stochastic models for commodity prices, central issues include time-varying trends, convenience yields and volatilities, and mean reversion; see, for example, Gibson and Schwartz (1990), Schwartz (1997), Pindyck (1999), Schwartz and Smith (2000), Cortazar and Schwartz (2003), Beck (2001), Saphores, Khalaf, and Pelletier (2002), Khalaf, Saphores, and Bilodeau (2003), and the references cited therein. In this paper, we assess various approaches that attempt to model these effects, focusing on forecast performance. Three alternative econometric specifications (which cover the recent and popular models in the published literature on commodity prices) are considered: (i) random-walk models with (generalized) autoregressive conditional heteroscedasticity (GARCH) effects, and with normal or student-t innovations, (ii) Poisson-based jump-diffusion models with (G)ARCH effects, and with normal or student-t innovations, and (iii) mean-reverting models that allow for uncertainty in the equilibrium to which prices revert.

Whereas market efficiency may motivate the analysis of stock prices as random walks, demand and supply pressures and non-constant convenience yields in commodity markets suggest mean reversion to long-run equilibrium prices. Intuitively, when prices are higher (or lower) than some equilibrium level, high-cost producers will enter (or exit) the market, which pushes prices downward (or upward). The *convenience yield* can be defined as the flow of goods and services that accrues to the owner of a spot commodity (a physical inventory) but not to the owner of a futures contract (a contract for future delivery). The random-walk hypothesis is consistent with a constant convenience yield. In contrast, mean reversion and the positive correlation between spot price and convenience yield changes is consistent with the theory of storage: when inventories decrease (or increase), the spot price will increase (or decrease) and the convenience yield will also increase (or decrease), because futures prices will not increase (or decrease) as much as the spot prices. Studies such as Schwartz (1997), Pindyck (1999), Schwartz and Smith (2000), and Cortazar and Schwartz (2003) refute the hypothesis of a constant convenience yield and their results suggest mean reversion to a long-run equilibrium that itself can change randomly over time. Note that Pindyck (1999) models long-run prices, whereas Schwartz (1997), Schwartz and Smith (2000), and Cortazar and Schwartz (2003) propose models that incorporate both short- and long-run considerations.

Alternatively, Beck (2001) argues that given the storability of commodities, random-walk specifications that allow for conditional heteroscedasticity are compatible with rational expectations and risk aversion. Accordingly, she suggests that ARCH-in-mean type frameworks present a good modelling choice for these commodity prices.

Furthermore, processes with random jumps, and which account for unobservable surprises, have also been considered in the recent literature; Saphores, Khalaf, and Pelletier (2002) and Khalaf, Saphores, and Bilodeau (2003) document evidence of jumps in commodity and natural resource prices.[1]

In this paper, we compare all of these commodity price modelling approaches. The models we consider are non-nested, highly non-linear, and include parameters that, in some cases, are difficult to identify from observable data. Yet the implications of such problems on model evaluation extend far beyond numerical estimation burdens. Indeed, assessing the models' relative empirical fit gives rise to non-standard econometric set-ups. For example, confronting mean-reverting models with random walks raises the well-known unit-root test difficulties in the presence of breaks (Perron 1989, 1993). Another example relates to assessing the significance of jumps; in this case, the parameters that describe the jump process under consideration are not identified under the null (no-jumps) hypothesis (Andrews 2001). In addition, jumps are quite difficult to disentangle from conditional non-normalities and heteroscedasticity (Aït-Sahalia 2004; Drost, Nijman, and Werker 1998). Consequently, while the estimation challenges for each model we consider can be tackled with manageable ease, comparing and contrasting them statistically and with reliable precision remains at the frontier of econometrics. We thus focus on forecast performance for model appraisal. Following notably the arguments of Pindyck (1999), forecasting performance offers further useful practical and economic insights for model selection.

Our empirical analysis focuses on the price of aluminium. The market for metals has features that set it apart from other types of goods, even compared with other natural resource products. In the case of aluminium, price determination is driven mostly by industrial structure (Bird 1990). In fact, transformation, and not extraction, accounts for most of the cost. For all practical purposes, this renders the long-run supply curve essentially flat.[2] Indeed, the slope of the marginal cost curve has been estimated to have a very small negative value (Schwartz 1997).

Since 1979, a futures market has been operating at the London Metal Exchange, and the price of the metal has been quite volatile. Moore and Cullen (1995) explain that metal prices are generally subject to a lot of speculative trade, which accentuates the volatility of the price series. In turn, this generates more speculation, and therefore even more volatility, which is why time-varying volatility, particularly ARCH, has been used by some authors to model the dynamic behaviour of the price of aluminium.[3]

Metal prices go through cycles that feature flat lows and spikes, suggesting that they are prone to business-cycle-type shocks, to which they react in a non-linear fashion (Gilbert 1995). Evidence of this cyclicity is provided by Labys, Lesourd, and Badillo (1998), who show links between economic conditions and the behaviour of precious metal prices. Their

[1]The relevant literature is vast; see Merton (1976), Ball and Torous (1985), Jarrow and Rosenfeld (1984), Ahn and Thompson (1988), Akgiray and Booth (1988), Jorion (1988), Brorsen and Yang (1994), Bates (1996a,b, 2000), Bakshi, Cao, and Chen (1997, 2000), Drost, Nijman, and Werker (1998), Bates (2000), Pan (2002), Das (2002), Chernov, Gallant, Ghysels, and Tauchen (2003), and the references cited therein.

[2]The principal cost, beyond alumina of course, is electricity. Current technology requires 15 MWH per 1 ton of aluminium; at current aluminium prices, this constitutes around 25 per cent of the final price. The underlying technological process has remained fundamentally unchanged, with continuous minor upgrades.

[3]See, for example, Akgiray, Booth, Hatem, and Chowdhury (1991), Labys, Achouch, and Terraza (1999), and Beck (2001).

tests reveal the presence of two cycles for metal prices: one short, which lasts less than a year, and one longer.

Not surprisingly, one aspect of aluminium prices on which there has been no consensus is whether such series are mean reverting.[4] Ahrens and Sharma (1997) indicate that 50 per cent of studies reject the presence of a unit root, while the other 50 per cent do not. For instance, Labys, Achouch, and Terraza (1999) conclude that almost all of the metal price series that they consider have unit roots. Similarly, Beck (2001) concludes that commodity prices are nonstationary, and Moore and Cullen (1995) find a unit root in the futures prices of metals. On the other hand, Schwartz (1997), Schwartz and Smith (2000), and Khalaf, Saphores, and Bilodeau (2003) conclude in favour of mean reversion in these metal price series.

Khalaf, Saphores, and Bilodeau (2003) also provide evidence of jumps in weekly aluminium price series when these are added to (G)ARCH and conditionally normal fundamentals. Furthermore, another feature that metal prices manifest, and that is in common with other financial assets, is conditional non-normality in the fundamental shocks that impact the series. Student-t distributions for the evolution of fundamental shocks, over and above jumps and/or (G)ARCH, have therefore also been suggested in the literature.

Our objective in this paper is to statistically compare the various classes of the foregoing empirical models for the price of aluminium, based on the mean-square forecast errors and non-parametric prediction error statistics, for daily, weekly, and monthly frequencies, and for various forecast horizons. We use one-step-ahead out-of-sample forecasts, where parameter estimates are updated at every step of the procedure. Of course, updating estimates raises an extra burden, yet we argue that it is a worthy effort given our focus on time-varying parameter models. In addition, in models with jumps, where analytical formulae are not readily available for obtaining conditional expected forecast errors, we devise a simple simulation-based procedure to approximate these errors.

Our results reveal the following. First, although unit-root tests favour the random-walk specification, for daily and weekly data, the mean-reverting model with stochastic convenience yield outperforms all other competing models, and for all forecast horizons. Although largely inferior to the mean-reverting model, random-walk models with GARCH and jumps or asymmetries seem the second-best specification for the high-frequency data. The mean-reverting model still performs relatively well with monthly data. Nonetheless, depending on the forecast horizon or the forecast evaluation criteria, the random walk with GARCH-in-mean effects dominates at the monthly frequency, suggesting that expected risk has a non-negligible effect on price behaviour.

This paper is organized as follows. In section 2, we present the various models under consideration. Data and forecasting results are discussed in section 3. In section 4 we offer some conclusions.

2. The Competing Models

Following the models analyzed in the literature and reviewed in section 1, we first consider random-walk-based specifications, with conditional heteroscedasticity and jumps.[5] We con-

[4] This holds true for commodity prices in general.

[5] We performed the standard battery of unit-root tests (the augmented Dickey-Fuller, the Phillips-Perron, and KPSS tests); as expected (and indeed, as reported in the literature), these tests do yield conflicting

sider a number of models that feature ARCH effects, varying from the most basic formulation to ones that allow asymmetric effects of shocks of different sign on conditional variance, even with non-normal fundamentals. Specifically, defining $y_t = ln(Y_t) - ln(Y_{t-1})$, we consider the GARCH(1,1) model:

$$\begin{aligned} y_t &= \mu + \sqrt{h_t} z_t, \\ h_t &= \alpha_0 + \alpha_1 (y_{t-1} - \mu)^2 + \phi h_{t-1}, \end{aligned}$$

where Y_t is the nominal price level. Two distributional hypotheses are considered for z_t:

$$z_t \sim N(0,1), \qquad \text{or} \qquad z_t \sim \text{student-}t \text{ with } \tau \text{ degrees of freedom,}$$

where the degrees-of-freedom parameter is unknown and needs to be estimated from the data. Given its preponderance in the literature, we also consider the ARCH(1) case in our analyses (which is obtained when the ϕ parameter is set to zero in the GARCH model).

Often, the current conditional variance of a series affects its mean. The GARCH-M(1,1) incorporates this feature by including h_t in the mean equation, as follows:

$$\begin{aligned} y_t &= \mu + \sqrt{h_t} z_t + \beta h_t, \\ h_t &= \alpha_0 + \alpha_1 (y_{t-1} - \mu)^2 + \phi h_{t-1}. \end{aligned}$$

In addition, we consider the exponential GARCH (EGARCH) model that allows a differing impact of past positive and negative shocks on conditional volatility:

$$\begin{aligned} y_t &= \mu + \sqrt{h_t} z_t, \\ ln(h_t) &= \alpha_0 + \phi ln(h_{t-1}) + \gamma \left(\frac{y_{t-1} - \mu}{\sqrt{h_{t-1}}} \right) + \eta \left[\frac{|y_{t-1} - \mu|}{\sqrt{h_{t-1}}} - \frac{\sqrt{2}}{\sqrt{\pi}} \right]. \end{aligned}$$

The latter two models are considered by Beck (2001), motivated by various theoretical arguments.

To integrate discontinuities into an ARCH or GARCH process or to account for fatter tails, we add a Poisson process to these formulations, as follows:

$$\begin{aligned} y_t &= \mu + \sqrt{h_t} z_t + \sum_{i=1}^{n_t} ln P_{it}, \\ h_t &= \alpha_0 + \alpha_1 (y_{t-1} - \mu)^2 + \phi h_{t-1}, \end{aligned}$$

where n_t is the number of jumps that occur between t and $t-1$, and P_{it} ($i = 1, ..., n_t$) is the size of the ith jump in the time interval $(t-1; t)$. We assume that jumps follow a Poisson process with arrival rate λ (i.e., there is a jump, on average, every $1/\lambda$ periods), and that the P_{it}'s are (independently) lognormally distributed with mean θ and variance δ^2. Note that n_t is an integer random variable; if $n_t = 0$, there are no jumps. The latter specification is

results (available upon request). Interestingly, we note that the KPSS test rejected stationarity of the data at all the frequencies considered. In addition, Akaike and Bayesian-Schwartz model-selection criteria selected the random-walk model, with no further lags in the mean.

designed to account for unanticipated surprises or large moves attributable to the arrival of unexpected information. The ARCH(1) model with jumps follows from this model by setting ϕ to zero.[6]

We next turn to the mean-reverting class of models. First, we consider the two-factor model of Schwartz and Smith (2000) that formally allows for a time-varying long-run mean and integrates both short- and long-run movements by construction: the long-run equilibrium component follows a Brownian motion, whereas the short-run deviations follow an Ornstein-Uhlenbeck process that reverts towards zero. The model can be written in continuous time for the log level of the spot price as:

$$\begin{aligned} \ln(Y_t) &= \chi_t + \xi_t, \\ d\chi_t &= -\kappa \chi_t dt + \sigma_\chi dz_\chi, \\ d\xi_t &= \mu_\xi + \sigma_\xi dz_\xi, \\ dz_\chi dz_\xi &= \rho_{\chi\xi} dt, \end{aligned} \tag{1}$$

where ξ_t is the log equilibrium price of aluminium at time t, χ_t is the deviation of the log price at time t with respect to the equilibrium price, and dz_χ and dz_ξ are correlated increments of Brownian motions. The mean-reversion coefficient κ represents the rate of speed at which the price reverts to its equilibrium (i.e., the rate at which short-run deviations disappear), μ_ξ is the mean of the equilibrium price, and σ_χ and σ_ξ are the short-run and equilibrium volatilities of the process, respectively. For estimation purposes, (1) can be discretized as follows:

$$\begin{aligned} \ln(Y_t) &= \chi_t + \xi_t, \\ \chi_t &= e^{-\kappa} \chi_{t-1} + \epsilon_t^\chi, \\ \xi_t &= \mu_\xi + \xi_{t-1} + \epsilon_t^\xi. \end{aligned}$$

Besides its intuitive appeal, the latter model formally includes a stochastic convenience yield, and has been recently advocated on empirical and theoretical grounds for commodities and metal prices;[7] see Gibson and Schwartz (1990), Schwartz (1997), Pindyck (1999), and Schwartz and Smith (2000). To explain how a stochastic convenience yield intervenes in this model, Schwartz and Smith (2000) relate (1) to the following model from Schwartz (1997):

$$\begin{aligned} dX_t &= (\mu - \delta_t - \frac{1}{2}\sigma_1^2)dt + \sigma_1 dz_1, \\ d\delta_t &= \kappa(\alpha - \delta_t)dt + \sigma_2 dz_2, \\ dz_1 dz_2 &= \rho dt, \end{aligned} \tag{2}$$

where $X_t = \ln(Y_t)$ (in our notation, X_t gives the log of the current spot price), dz_1 and dz_2 are correlated increments of Brownian motions, and δ_t is the convenience yield, which

[6] For expressions of likelihood functions and further references regarding these models, see Khalaf, Saphores, and Bilodeau (2003).

[7] Schwartz and Smith (2000) note that their short-term/long-term model can be estimated from spot and/or futures prices, and that the accuracy of the estimated state variables depends on the latter choice. In this case, even when state variable estimates are based on spot price observations only, the uncertainty has very little impact on forecasts and on futures prices.

intervenes as a reduction in the drift term of (2). Formally, Schwartz and Smith (2000) show that processes (1) and (2) are equivalent, in the sense that factors of (1) can be written as a linear combination of the factors in (2). In particular, $\chi_t = \frac{1}{\kappa}(\delta_t - \alpha)$; note that κ gives the short-term mean-reversion rate in both versions of the model, which justifies the overlap in notation. Since ξ_t and χ_t are not observable, the model is rewritten in state-space form and estimated using the Kalman filter.

3. Forecast Results

Daily spot prices (in U.S. dollars) are obtained from the London Metal Exchange, over the period January 1989 to December 2003, for one ton of aluminium. From these, we construct weekly and monthly prices, the former using Wednesday values and the latter using the price on the Wednesday that is closest to the 15th day of that month. For the few cases where the Wednesday value is not available, the Tuesday value closest to the 15th day of that month is used.

Prices are analyzed in logarithms for the mean-reverting models, or in log-difference for the random-walk based models. Following our notation in section 2, Y_t is the nominal price of one ton of aluminium, and the differenced series is given by $y_t = ln(Y_t) - ln(Y_{t-1})$.

Although many statistical goodness-of-fit measures exist to evaluate the relative merits of econometric models, forecasting provides another avenue for judging a model's ability to adequately describe a given set of data. In this section, we present the summary statistics that are used in this paper and that are based on one-step-ahead forecasts for each different model and frequency. We first rely on the usual mean absolute prediction error (MAPE) and the mean square prediction error (MSPE). We supplement the latter with a sign test on the difference between the forecast error of the model that yields the smallest MAPE or MSPE, and, in turn, the forecast error from each of the remaining available models. The sign test informs us whether the two forecast errors are statistically distinguishable from one another, and is given by:

$$S = \frac{2}{\sqrt{K}} \sum_{k=1}^{K} (I[d_k > 0] - \frac{1}{2}) \overset{asy}{\sim} N(0,1),$$

where d_k is the difference in forecast errors at the kth forecasting point, and K is the forecast horizon. Underlying parameter estimation may distort the size of the latter test; hence, results are interpreted with caution.[8]

The out-of-sample one-step-ahead prediction errors are obtained as follows: given a sample of size $T+K$, we first remove K observations at the end of the sample and that correspond to the forecast horizon considered. The model is then estimated on the remaining sample (i.e., until T); the dependent variable's value is forecast for period $T+1$ and denoted $ln(\hat{Y}_{T+1|T})$. The $T+1$ forecast error resulting from the comparison of $ln(\hat{Y}_{T+1|T})$ and $ln(Y_{T+1})$ is computed. Next, the $T+1$ observed value of the dependent variable is added to our sample, and the model is re-estimated. The $T+2$ observation is then forecast and denoted $ln(\hat{Y}_{T+2|T+1})$, the

[8]Alternative tests for forecasting accuracy, such as Diebold and Mariano's procedure, are not immune to size problems in our context. Given the statistical complications of the models we consider, conducting formal statistical tests on forecasts is beyond the scope of this paper.

$T+2$ forecast error is computed, and so on, until all K observations are covered. The MAPE and the MSPE are then defined as:

$$
\begin{aligned}
MAPE &= \frac{1}{K}\sum_{k=1}^{K} |ln(\hat{Y}_{T+k|T+k-1}) - ln(Y_{T+k})|, \\
MSPE &= \frac{1}{K}\sum_{k=1}^{K} \Big[ln(\hat{Y}_{T+k|T+k-1}) - ln(Y_{T+k})\Big]^2.
\end{aligned}
$$

For the models that include jump features, it is not possible to obtain forecast errors in a straightforward manner: there is no analytical form for the forecast equation in such cases. As in Khalaf, Saphores, and Bilodeau (2003), our solution is to rely on simulation (see also Bilodeau 1998). Thus, for a given model with jumps, we first estimate the parameters of the conditional mean, the conditional variance, and the jump parameters $(\lambda, \theta, \delta)$, over the sample of size T. Then, drawing from a normal or t-distribution for the residuals, a Poisson distribution with estimated mean $\hat{\lambda}$ for the arrivals of the jumps, and a normal distribution with mean $\hat{\theta}$ and variance $\hat{\sigma^2}$ for the amplitude of each jump, we generate 1,000 simulated values of the dependent variable $\widetilde{Y}_{T+1}$. The forecast value of Y_{T+1} is then taken to be the average value of these 1,000 $\widetilde{Y}_{T+1}$, and the $T+1$ forecast error is computed. At this point, the observed value of the dependent variable, Y_{T+1}, is added to the sample, the model is re-estimated, and the entire simulation process is repeated. Thus, $\widetilde{Y}_{T+2}$ is obtained, as well as the forecast error for $T+2$. The above steps are repeated until $T+K$ forecast errors are obtained, which are then used to construct the MAPE, MSPE, and sign statistics.

For each frequency, we conduct out-of-sample one-step-ahead dynamic forecasts for three forecast horizons: one, three, and five years. The corresponding results on spot prices are reported in Tables 1, 2, and 3. The minimum MAPE and MSPE are shown in bold. The sign statistic is significant in all cases, although we reiterate the need to interpret this result with caution, given that recursive parameter estimation may distort the test's asymptotic null distribution.

Table 1: Daily Frequency, spot prices

Forecast horizon	One year		Three years		Five years	
	MSPE	MAPE	MSPE	MAPE	MSPE	MAPE
ARCH(1)	0.0088	0.0799	0.0072	0.0716	0.0560	0.2218
GARCH(1,1)	0.0095	0.0831	0.0065	0.0679	0.0605	0.2306
GARCH-M(1,1)	0.0087	0.0799	0.008	0.0759	0.0611	0.2321
EGARCH(1,1)	0.0102	0.0866	0.0061	0.0659	0.0738	0.2553
GARCH-T(1,1)	0.0144	0.0144	0.0080	0.0716	0.1170	0.3195
ARCH(1) with jumps	0.0095	0.0831	0.0061	0.0658	0.0756	0.2588
GARCH(1,1) with jumps	0.0086	0.0796	0.0083	0.0784	0.0535	0.2166
GARCH-T(1,1) with jumps	0.0150	0.1049	0.0096	0.0753	0.1316	0.1537
Schwartz and Smith	**0.0001***	**0.0067***	**0.0001***	**0.0072***	**0.0001***	**0.0083***

Table 2: Weekly Frequency, spot prices

Forecast horizon	One year		Three years		Five years	
	MSPE	MAPE	MSPE	MAPE	MSPE	MAPE
ARCH(1)	0.0056	0.0605	0.0058	0.0643	0.1166	0.3218
GARCH(1,1)	0.0047	0.0555	0.0062	0.0658	0.1078	0.3101
GARCH-M(1,1)	0.0030	0.0464	0.0101	0.0866	0.0544	0.2198
EGARCH(1,1)	0.0027	0.0419	0.0137	0.1040	0.0466	0.2009
GARCH-T(1,1)	0.0061	0.0635	0.0063	0.0668	0.1097	0.3113
ARCH(1) with jumps	0.0047	0.0553	0.0071	0.0703	0.0894	0.2811
GARCH(1,1) with jumps	0.0041	0.0515	0.0086	0.0786	0.0777	0.2630
GARCH-T(1,1) with jumps	0.0056	0.0607	0.0064	0.0675	0.1036	0.3027
Schwartz and Smith	**0.0003***	**0.0152***	**0.0005***	**0.0163***	**0.0005***	**0.0165***

Our results for estimation with spot prices can be summarized as follows. For daily and weekly frequencies (Tables 1 and 2, respectively), the mean-reverting model of Schwartz and Smith (2000), with stochastic convenience yield, emerges as the best model for all of the forecast horizons considered.[9] Random-walk models with GARCH and jumps or asymmetries seem the second-best specification for the high-frequency data, where allowing for student-t innovations seems to pay off only at longer horizons. Nevertheless, these second-best models perform dramatically worse than the mean-reverting one.

Evidence in favour of mean reversion is less conclusive at the monthly frequency (Table 3). The mean-reverting model still dominates for the three-year horizon. However, given a one-year forecast horizon, the GARCH-in-mean random-walk model outperforms Schwartz and Smith's (2000) specification. For the five-year forecast horizon, the mean-reverting model minimizes the MSPE criterion, while the random walk with GARCH-in-mean minimizes the

[9]Recall that the classical unit-root tests we ran all rejected mean reversion. These results agree with Pindyck's (1999) arguments, which advocate time-varying parameter models as an alternative to unit-root testing in models of commodity prices.

Table 3: Monthly Frequency, spot prices

Forecast horizon	One year		Three years		Five years	
	MSPE	MAPE	MSPE	MAPE	MSPE	MAPE
ARCH(1)	0.0030	0.0438	0.0149	0.1103	0.0299	0.1586
GARCH(1,1)	0.0041	0.0516	0.0099	0.0869	0.0304	0.1599
GARCH-M(1,1)	**0.0008***	**0.0215***	0.0508	0.2123	0.0291	**0.1560***
EGARCH(1,1)	0.0033	0.0461	0.0127	0.1006	0.0301	0.1592
GARCH-T(1,1)	0.0055	0.0599	0.0068	0.0694	0.0309	0.1615
ARCH(1) with jumps	0.0028	0.0419	0.0154	0.1115	0.0299	0.1585
GARCH(1,1) with jumps	0.0042	0.0521	0.0164	0.1161	0.0298	0.1583
GARCH-T(1,1) with jumps	0.0056	0.0612	0.0063	0.0663	0.0311	0.1619
Schwartz and Smith	**0.0008***	0.0242	**0.0009***	**0.0255***	**0.0017***	0.1638

MAPE. Interestingly, Beck (2001), using yearly data on a large sample of commodities, fails to identify ARCH-M effects, despite the fact that her underlying theoretical model links ARCH-in-mean effects in storable commodities to risk aversion and rational expectation. Beck's self-acknowledged counterintuitive empirical results can therefore perhaps be explained by her particular choice of sample frequency and span. We find support for Beck's theoretical arguments for the case of aluminium with monthly data, suggesting that expected price risk does indeed seem to affect price behaviour.

Note that these results were obtained using forecasts with model estimates being updated at every step. In its short-run/long-run form, and because of its well-defined state-space form (where forecast errors are easily obtained from the Kalman filter), Schwartz and Smith's (2000) model is straightforward to interpret and to use for forecasting purposes with high-frequency data. Our comparative analysis underscores the merit of this model, and motivates further improvements to it. As Schwartz and Smith (2000) outline, these include improving the long-run equation (incorporating, for example, formulations as in Pindyck 1999), or the short-run one, by adding discrete jumps.

4. Conclusion

In this paper, we compare several stochastic models for aluminium prices, focusing on forecast performance. The models differ regarding the assumptions related to mean reversion and structural discontinuities (time-varying first and second moments). Three empirical results emerge from our work: (i) there are both jump and (G)ARCH effects in random-walk specifications estimated with high-frequency data, (ii) random-walk formulations with (G)ARCH-M effects dominate to some extent at monthly frequencies, and (iii) the mean-reverting short-run/long-run model of Schwartz and Smith (2000) performs markedly better than all of the other models at daily and weekly frequencies. With respect to the latter category of models, we concur with Cortazar and Schwartz (2003) that it is unfortunate that practitioners have been slow to adopt such specifications. Indeed, our results provide clear motivation for increased practical and theoretical work regarding these kinds of multifactor time-varying parameter models.

References

Ahn, C.M. and H.E. Thompson. 1988. "Jump-Diffusion Processes and the Term Structure of Interest." *Journal of Finance* 43: 155–74.

Ahrens, W.A. and V.R. Sharma. 1997. "Trends in Natural Resource Commodity Prices: Deterministic or Stochastic?" *Journal of Environmental Economics and Management* 33: 59–74.

Aït-Sahalia, Y. 2004. "Disentangling Diffusion from Jumps." *Journal of Financial Economics* 74: 487–528.

Akgiray, V. and G. Booth. 1988. "Mixed Diffusion-Jump Process Modelling of Exchange Rate Movements." *Review of Economics and Statistics* 70: 631–37.

Akgiray, V., G. Booth, J.J. Hatem, and M. Chowdhury. 1991. "Conditional Dependence in Precious Metal Prices." *Financial Review* 26: 367–86.

Andrews, D.W.K. 2001. "Testing When a Parameter is on the Boundary of the Maintained Hypothesis." *Econometrica* 69: 683–733.

Bakshi, G., C. Cao, and Z. Chen. 1997. "Empirical Performance of Alternative Option Pricing Models." *Journal of Finance* 52: 2003–49.

———. 2000. "Pricing and Hedging Long-Term Options Models." *Journal of Econometrics* 94: 277–318.

Ball, C. and W. Torous. 1985. "On Jumps in Common Stock Prices and their Impact on Call Option Pricing." *Journal of Finance* 40: 155–73.

Bates, D.S. 1996a. "Dollar Jump Fears, 1984-1992. Distributional Abnormalities Implicit in Currency Futures Options." *Journal of International Money and Finance* 15: 65–93.

———. 1996b. "Jumps and Stochastic Volatility. Exchange Rate Processes Implicit in Deutsche Mark Options." *Review of Financial Studies* 9: 69–107.

———. 2000. "Post-87 Crash Fears in S&P 500 Futures Options." *Journal of Econometrics* 94: 181–238.

Beck, S. 2001. "Autoregressive Conditional Heteroscedasticity in Commodity Spot Prices." *Journal of Applied Econometrics* 16: 115–32.

Bilodeau, J.F. 1998. "'Analyse de Processus de Sauts Dans les Prix du Pétrole Brut." Masters Thesis, Université Laval.

Bird, A. 1990. "Price Determination in the Market for Aluminium." *Primary Commodity Prices: Economic Models and Policy*. Cambridge, New York and Melbourne: Cambridge University Press.

Brorsen, B.W. and S.R. Yang. 1994. "Nonlinear Dynamics and the Distribution of Daily Stock Index Returns." *Journal of Financial Research* 17: 187–203.

Chernov, M., A.R. Gallant, E. Ghysels, and G. Tauchen. 2003. "Alternative Models for Stock Price Dynamics." *Journal of Econometrics* 116: 225–57.

Cortazar, G. and E.S. Schwartz. 2003. "Implementing a Stochastic Model for Oil Future Prices." *Energy Economics* 25: 215–38.

Das, S.R. 2002. "The Surprise Element: Jumps in Interest Rates." *Journal of Econometrics* 106: 27–65.

Drost, F.C., T.E. Nijman, and B.J.M. Werker. 1998. "Estimation and Testing in Models Containing Both Jump and Conditional Heteroskedasticity." *Journal of Business and Economic Statistics* 16: 237–43.

Gibson, R. and E.S. Schwartz. 1990. "Stochastic Convenience Yield and the Pricing of Oil Contingent Claims." *Journal of Finance* 45: 959–76.

Gilbert, C.L. 1995. "Modelling Market Fundamentals: A Model of the Aluminium Market." *Journal of Applied Econometrics* 10: 385–410.

Jarrow, R.A. and E.R. Rosenfeld. 1984. "Jump Risks and the Intertemporal Capital Asset Pricing Model." *Journal of Business* 57: 337–51.

Jorion, P. 1988. "On Jump Processes in the Foreign Exchange and Stock Markets." *Review of Financial Studies* 1: 427–45.

Khalaf, L., J.D. Saphores, and J.F. Bilodeau. 2003. "Simulation-Based Exact Jump Tests in Models with Conditional Heteroskedasticity." *Journal of Economic Dynamic and Control* 28: 531–53.

Labys, W.C., A. Achouch, and M. Terraza. 1999. "Metal Prices and the Business Cycle." *Resources Policy* 25: 229–38.

Labys, W.C., J.B. Lesourd, and D. Badillo. 1998. "The Existence of Metal Price Cycles." *Resources Policy* 24: 147–55.

Merton, R.C. 1976. "Option Pricing When Underlying Stock Returns are Discontinuous." *Journal of Financial Economics* 3: 125–44.

Moore, M.J. and U. Cullen. 1995. "Speculative Efficiency on the London Metal Exchange." *The Manchester School* 63: 235–56.

Pan, J. 2002. "The Jump-Risk Premia Implicit in Options: Evidence from an Integrated Time-Series Study." *Journal of Financial Economics* 63: 3–50.

Perron, P. 1989. "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis." *Econometrica* 57: 1361–401.

———. 1993. "ERRATUM: The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis." *Econometrica* 61: 248–49.

Pindyck, R.S. 1999. "The Long Run Evolution of Energy Prices." *Energy Journal* 20: 1–27.

Saphores, J.D., L. Khalaf, and D. Pelletier. 2002. "On Jumps and ARCH Effects in Natural Resource Prices: An Application to Stumpage Prices from Pacific Northwest National Forests." *American Journal of Agriculture Economics* 84: 387–400.

Schwartz, E.S. 1997. "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging." *Journal of Finance* 52: 923–73.

Schwartz, E.S. and J.E. Smith. 2000. "Short-Term Variations and Long-Term Dynamics in Commodity Prices." *Management Science* 46: 893–911.

Bank of Canada Working Papers
Documents de travail de la Banque du Canada

Working papers are generally published in the language of the author, with an abstract in both official languages, and are available on the Bank's website (see bottom of page). ***Les documents de travail sont généralement publiés dans la langue utilisée par les auteurs; ils sont cependant précédés d'un résumé bilingue. On peut les consulter dans le site Web de la Banque du Canada, dont l'adresse est indiquée au bas de la page.***

2006

2006-13	Guarding Against Large Policy Errors under Model Uncertainty	G. Cateau
2006-12	The Welfare Implications of Inflation versus Price-Level Targeting in a Two-Sector, Small Open Economy	E. Ortega and N. Rebei
2006-11	The Federal Reserve's Dual Mandate: A Time-Varying Monetary Policy Priority Index for the United States	R. Lalonde and N. Parent
2006-10	An Evaluation of Core Inflation Measures	J. Armour
2006-9	Monetary Policy in an Estimated DSGE Model with a Financial Accelerator	I. Christensen and A. Dib
2006-8	A Structural Error-Correction Model of Best Prices and Depths in the Foreign Exchange Limit Order Market	I. Lo and S.G. Sapp
2006-7	Ownership Concentration and Competition in Banking Markets	A. Lai and R. Solomon
2006-6	Regime Shifts in the Indicator Properties of Narrow Money in Canada	T. Chan, R. Djoudad, and J. Loi
2006-5	Are Currency Crises Low-State Equilibria? An Empirical, Three-Interest-Rate Model	C.M. Cornell and R.H. Solomon
2006-4	Forecasting Canadian Time Series with the New Keynesian Model	A. Dib, M. Gammoudi, and K. Moran
2006-3	Money and Credit Factors	P.D. Gilbert and E. Meijer
2006-2	Structural Change in Covariance and Exchange Rate Pass-Through: The Case of Canada	L. Khalaf and M. Kichian
2006-1	The Institutional and Political Determinants of Fiscal Adjustment	R. Lavigne

2005

2005-45	An Evaluation of MLE in a Model of the Nonlinear Continuous-Time Short-Term Interest Rate	I. Lo
2005-44	Forecasting Core Inflation in Canada: Should We Forecast the Aggregate or the Components?	F. Demers and A. De Champlain

Copies and a complete list of working papers are available from:
Pour obtenir des exemplaires et une liste complète des documents de travail, prière de s'adresser à :

Publications Distribution, Bank of Canada
234 Wellington Street, Ottawa, Ontario K1A 0G9
Telephone: 1 877 782-8248
(toll free in North America)
Email: publications@bankofcanada.ca
Website: http://www.bankofcanada.ca

Diffusion des publications, Banque du Canada
234, rue Wellington, Ottawa (Ontario) K1A 0G9
Téléphone : 1 877 782-8248 (sans frais en Amérique du Nord)
Adresse électronique : publications@banqueducanada.ca
Site Web : http://www.banqueducanada.ca