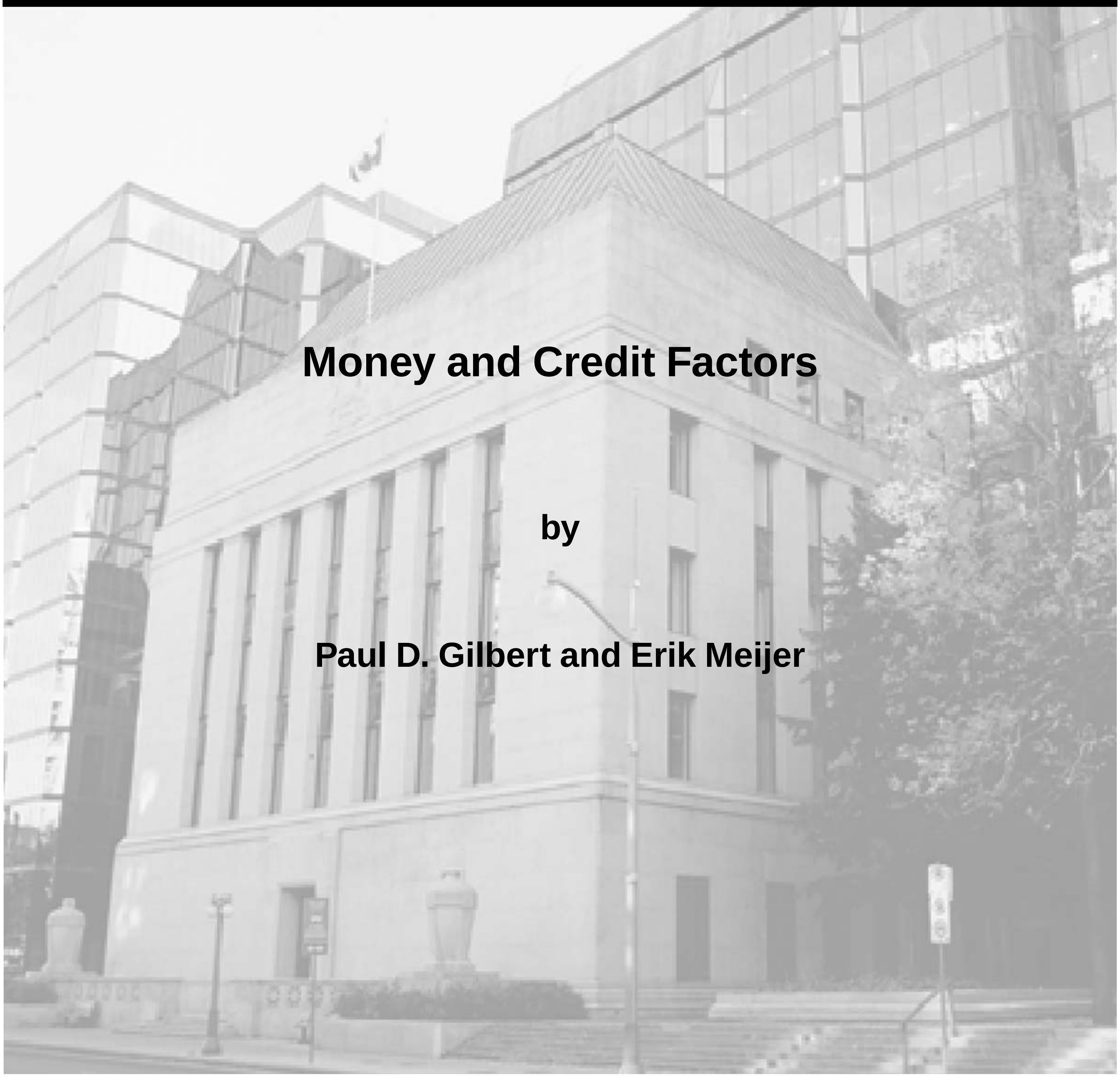
Bank of Canada Banque du Canada

Working Paper 2006-3 / Document de travail 2006-3



Money and Credit Factors

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ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2006-3

March 2006

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Acknowledgements

This paper was prepared in part while the first author was visiting the University of Groningen. We would like to thank Jason Allen, Walter Engert, and Jack Selody for comments on earlier versions of this paper.

Abstract

The authors introduce new measures of important underlying macroeconomic phenomena that affect the financial side of the economy. These measures are calculated using the time-series factor analysis (TSFA) methodology introduced in Gilbert and Meijer (2005). The measures appear to be both more interesting and more robust to the effects of financial innovations than traditional aggregates. The general ideas set out in Gilbert and Pichette (2003) are pursued, but the improved estimation methods of TSFA are used. Furthermore, four credit aggregates are added to the components of the monetary aggregates, resulting in the possibility of extracting more common factors.

This extended data set gives a fairly complete picture of the asset and liability sides of the economy. As might be expected, credit data are largely explained by the same factor that explains investment (since investment provides the capital for credit). Contrary to traditional thinking about monetary aggregates, however, personal chequing deposits do not behave like currency, and thus require a separate factor to explain them. The factor explaining currency reflects current spending, traditionally considered *transactions money*. One interpretation of the factor explaining personal chequing deposits is that it represents potential spending. This may reflect different horizon planning by the same consumers, or it may reflect different consumers with varying spending habits or financial constraints. These conjectures remain subjects for future research.

JEL classification: E51, C43, C82
Bank classification: Credit and credit aggregates; Monetary aggregates; Econometric and statistical methods

Résumé

Les auteurs présentent de nouvelles mesures permettant de saisir d’importants phénomènes macroéconomiques sous-jacents qui se répercutent dans la sphère financière de l’économie. Ces mesures, qui sont établies au moyen de la technique d’analyse factorielle applicable aux séries chronologiques proposée dans Gilbert et Meijer (2005), semblent à la fois plus intéressantes et moins sensibles aux effets des innovations financières que les agrégats classiques. Les auteurs reprennent les idées générales énoncées dans Gilbert et Pichette (2003), mais en faisant appel à la technique améliorée mise au point par la suite. Ils ajoutent aussi quatre agrégats du crédit aux composantes des agrégats monétaires afin de pouvoir extraire un plus grand nombre de facteurs communs.

L'ensemble de données obtenu donne une assez bonne idée des deux côtés du bilan financier de l'économie. Conformément aux attentes, le facteur qui explique l'évolution de l'investissement explique aussi largement celle du crédit (puisque l'investissement sert à financer le crédit). Toutefois, contrairement à ce qu'on suppose généralement au sujet des agrégats monétaires, les dépôts des particuliers transférables par chèque ne se comportent pas de la même façon que la monnaie hors banques, et un facteur distinct doit être introduit pour rendre compte de ceux-ci. Une interprétation possible est que la monnaie hors banques sert aux transactions courantes, d'où son appellation usuelle de « monnaie de transaction », alors que les dépôts transférables par chèque seraient plutôt un indicateur des dépenses potentielles, lesquelles traduiraient la présence de consommateurs ayant des horizons de planification différents ou de consommateurs hétérogènes ayant des habitudes de dépenses ou des contraintes financières dissemblables. Cette hypothèse reste à explorer.

Classification JEL : E51, C43, C82
Classification de la Banque : Crédit et agrégats du crédit; Agrégats monétaires; Méthodes économétriques et statistiques

1. Introduction

This research is motivated by the desire to find good measures of important underlying macroeconomic phenomena that affect the financial side of the economy, without imposing theories about the dynamics of those phenomena. If the underlying phenomena are considered to be data that are to be measured, then imposing an economic theory by the measurement model is not desirable. That is, if possible, the specification of a dynamic economic model should be avoided at the data-measurement stage, which is the focus of this paper.

Technological innovations in the financial industry pose major problems for the measurement of monetary aggregates, which have traditionally been the main measures of the financial economy. Gilbert and Pichette (2003) discuss the general idea of using underlying factors in place of aggregation. This same approach is followed here, but the much improved estimation methods of *time-series factor analysis* (TSFA) are used. Introduced in Gilbert and Meijer (2005), TSFA provides a method to estimate underlying unobserved latent economic variables, the *factors*. It is related to *dynamic factor analysis* (DFA), but different in that there are few assumptions about the dynamic model of the underlying factors. Thus, the underlying factor measurement can be treated purely as a measurement problem, and the economic modelling of the factor dynamics can be done separately. Since a dynamic economic model of the factors is not assumed, this allows for the possibility that many economic models might use the same measured factors.

Also, as discussed in Gilbert and Meijer (2005), the components of the monetary aggregates alone provide enough observed variables (*indicators*) to extract only two factors, which does not appear to be adequate. Adding four credit measures to the money components results in ten indicators and the possibility of extracting more factors.

In this paper, therefore, the indicators are currency, deposit balances, and credit data, while the factors are the intended uses of money and credit (for example, transactions and savings). The objectives are to determine how many factors are important and to measure them. Their possible meaning is also discussed, but their relative importance for economic modelling is a topic for future research.

The methodology and application in this paper are related to Spanos (1984). However, he used only one factor and presumed that it represented liquidity. In contrast, below it is shown that one factor is not enough to explain the financial side of the economy.

Historically, monetary aggregates have been used to measure monetary activity. Their usefulness to forecast economic activity and inflation has been subject to much debate. The problems with these traditional measures are discussed in Gilbert and Pichette (2002, 2003), and in many of the references cited therein. While these traditional measures are largely no longer used, we hope that a better understanding of the financial side of the economy would be useful, and ultimately lead to better models for policy and forecasting. Better measurement of the underlying phenomena is a necessary first step in this process.

This paper is organized as follows. Section 2 gives the mathematical form of the measurement model and describes the data and the 4-factor model estimated with the quartimin rotation, which is thought to be the most interesting. Section 3 discusses models with a different number of factors. Section 4 considers sensitivity to the chosen rotation method. The sensitivity to the sample period is discussed in section 5. Then the factors are compared with the traditional M1+ and M2++ aggregates in section 6. Finally, in section 7 outstanding issues and the next steps for actually using the factors are discussed.

2. Factors from the Money and Credit Data

The TSFA measurement model is as follows. The k unobserved processes of interest (the factors) for a sample of T time periods are indicated by ξ_{jt}, $t = 1, \ldots, T$, $j = 1, \ldots, k$. The M observed processes (the indicators) are denoted by y_{it}, $t = 1, \ldots, T$, $i = 1, \ldots, M$. The factors and indicators for period t are collected in the (column) vectors ξ_t and y_t, respectively. It is assumed that there is a measurement model relating the indicators to the factors, given by

$$y_t = B\xi_t + \varepsilon_t, \tag{1}$$

where B is an $M \times k$ matrix parameter of *factor loadings* or simply *loadings*, and ε_t is a random M-vector of measurement errors, disturbances, and unique or idiosyncratic factors. Additional details and estimation theory are discussed in Gilbert and Meijer (2005). One important intuitive point relative to aggregation is that the loadings are weights that relate the data to the underlying factors, rather than the other way around as in aggregation. If k aggregates were to be calculated from y_t, this would be done by Wy_t, where W is a $k \times M$ matrix.

The TSFA technique extends standard factor analysis (FA) to work with time series. Standard FA (see, for example, Wansbeek and Meijer 2000, chapter 7) does not work directly with typical macroeconomic time series, because the characteristics of the data usually conflict with the underlying FA assumptions. FA was developed for cross-sectional data, where the assumption that observations are independent and identically distributed (i.i.d.) is often reasonable. Macroeconomic data typically trend upwards and are serially dependent.

Gilbert and Meijer's (2005) most important insight is that, if the first differences of equation (1) are taken, an FA model is obtained with the same loadings matrix. The type of data used here (real per capita money and credit) appears to be relatively stable after first differencing (no apparent trend or increasing variance), so that the assumptions of TSFA stated in Gilbert and Meijer (2005) are satisfied. Thus, parameters can be estimated from first differences, after which the undifferenced factor scores can be obtained from the undifferenced data.[1]

2.1 Data

In this paper, TSFA is applied to Canadian money and credit data. The inclusion of credit data extends the application described in Gilbert and Meijer (2005). It is necessary to have more

[1]The term *factor scores* will be used for the factor measures calculated from the estimated parameters and the observed data (indicators). The term *estimated factors* is sometimes used in the literature, but some authors in the FA literature prefer *predicted factors* because, technically, these are not consistent statistical estimates. (The estimation error does not converge asymptotically to zero; see, for example, Wansbeek and Meijer 2000.) The same applies to the state obtained from a non-innovations-form state-space model using a Kalman filter or smoother; however, the term *estimated state* is well established in the time-series literature. In time series, the term *predict* tends to be used interchangeably with *forecast*, but *predicted factors* are not forecasts of the future, so this term is confusing to readers who have a time-series background. In theoretical or simulation contexts, it is possible that *true factors* or *true factor scores* are used for the underlying random variable and its realization. These will always be distinguished with the word *true*. The term *factors* remains generic and somewhat ambiguous.

indicators in order to extract additional and more meaningful factors, since there is a technical limitation on the number of factors that can be extracted from a given number of indicators. The number of factors that can reasonably be obtained with the six money data indicators is two. Three factors is exactly the Ledermann bound (see, for example, Wansbeek and Meijer 2000, chapter 7) with six data series, which means that the covariance structure would be perfectly explained by the factors, regardless of the data (except for anomalous situations). Therefore, it is difficult to assess the fit of the model with three factors if there are only six indicators. Furthermore, results may not be very meaningful due to overfitting. This difficulty is addressed by adding four credit series as additional indicators. Ten indicators relaxes the Ledermann bound, so up to five underlying factors can be reasonably measured. (Six factors satisfies the Ledermann bound exactly in this case.) It is expected that credit data movements will partly be explained by the same factors that explain deposit balances, since these are opposite sides of the balance sheet. Furthermore, the relationship between these opposite sides may be of economic interest. Thus, adding credit data not only allows for the possibility of estimating a more complete set of factors, but also expands the scope of this research to a very large part of the national financial balance sheet data.

The money indicators are organized into six categories. Data are measured in a larger number of categories, but shifts between indicators for reasons other than fundamental macroeconomic behaviour will interfere with extracting the factors of interest. Thus it is necessary to combine asset categories among which there have been important shifts for other reasons. For example, Canada Savings Bonds are no longer widely used and people have shifted to mutual funds. Instruments typically thought to be savings are grouped together and called investment. The six money indicators are *currency*, *personal chequing deposits*, *non-bank chequing deposits*, *non-personal demand and notice deposits*, *non-personal term deposits*, and *investment*. Most non-personal accounts belong to businesses. In Canada, trust companies, credit unions, and some other financial institutions are not grouped with banks; the term “non-bank” refers to deposits at these institutions. Gilbert and Pichette (2003) and Kottaras (2003) provide considerably more detail about the money data.

The credit indicators are *consumer credit*, *residential mortgage*, *short-term business credit*, and *other business credit*. The latter is often referred to as long-term business credit. It includes the categories of equity and non-residential mortgages.

To remove the factors that are not of main interest, the data are measured in per capita terms and in real Canadian dollars. The data span the 277 months from November 1981 to November 2004.

Some indicators show no seasonality, while others do. For example, currency has marked seasonal peaks at Christmas and in September. These different patterns in the indicators may reflect differences in the underlying factors. Thus, seasonality may help distinguish the factors of interest and measure them better, as illustrated in a small simulation by Meijer and Gilbert (2005). Therefore, the data are not seasonally adjusted. The factor scores could be seasonally adjusted for economic modelling.

All the analyses reported herein have been performed with the software R (R Development Core Team 2004). The data are available in the R package CDNmoney and the code for the analyses is available in the R package tsfa. The software and the packages are available from the CRAN sites (`http://cran.r-project.org`).

2.2 Model fit and choice of number of factors

Eigenvalues of the sample correlation matrix of differenced indicators give a rough idea of the number of factors to consider. These eigenvalues are 3.26, 1.65, 1.07, 0.87, 0.84, 0.62, 0.51, 0.48, 0.40, and 0.28. A conventional rule of thumb is that the number of factors should be equal to the number of eigenvalues that are larger than 1.0, suggesting three factors. However, the fourth and fifth eigenvalues are close to 1.0, suggesting possibly four or five factors. Figure 1 shows the scree plot, which plots the eigenvalues in descending order against their number in the sequence. Another rule of thumb is that the number of factors is the same as the number of eigenvalues before the kink in the scree plot. This suggests five factors.

Figure 1: Scree plot of the eigenvalues of the differenced indicators' correlation matrix.

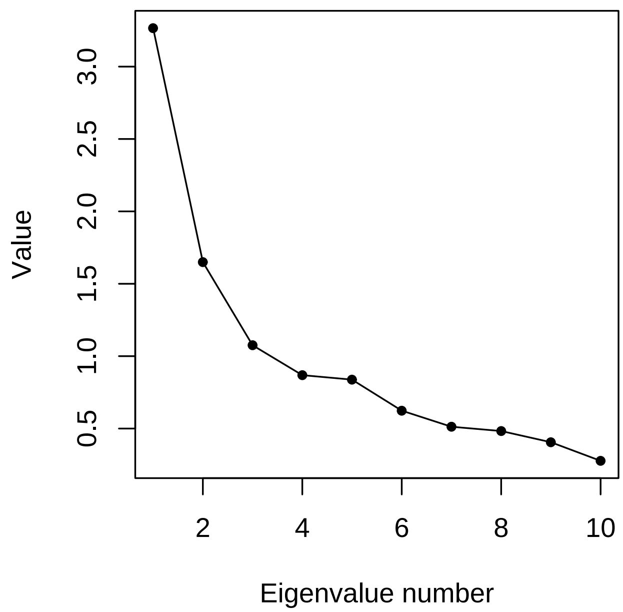


Table 1 shows several fit statistics for measurement models with varying numbers of factors; Wansbeek and Meijer (2000, chapter 10) discuss these fit statistics. Statistics reported below are all based on the multivariate normal likelihood, which requires i.i.d. normally distributed observations. This assumption is clearly violated; therefore, the statistics should not be regarded as formal statistical tests, but as rough indications of model fit.

In the top panel of Table 1, models are compared with the *saturated model* and the *null model*. By definition, the saturated model gives a perfect fit: the number of parameters equals the number of covariances and the model-implied covariance matrix exactly equals the sample covariance matrix, regardless of the data. The saturated model is the model in which the (population) covariance matrix is completely free except, of course, for its symmetry. Usually, the 6-factor model would be a saturated model, because it has the same number of parameters. In some cases, such as ours, a perfect fit is not obtained unless complex valued loadings and/or negative error variances are allowed. Obviously, this does not make sense. For a discussion of this phenomenon see, for example, Dijkstra (1992).

Table 1: Fit Statistics for Different Number of Factors

	Number of factors						
	0 (Null)	1	2	3	4	5	Saturated
Chi-square	721.6	251.6	126.3	37.9	17.8	8.52	0
df	45	35	26	18	11	5	0
p-value	8.9×10^{-123}	1.4×10^{-34}	3.9×10^{-15}	0.004	0.09	0.13	–
RMSEA	0.23	0.15	0.12	0.063	0.047	0.050	–
CFI	0	0.68	0.85	0.97	0.99	0.99	1
Sequential tests:							
Chi-square	470.0	125.3	88.4	20.0	9.33	8.52	
df	10	9	8	7	6	5	
p-value	1.2×10^{-94}	1.1×10^{-22}	9.9×10^{-16}	0.0055	0.16	0.13	

The null model is a very restrictive baseline model. In factor analysis, the usual null model is the independence model; i.e., the model that specifies that all observed variables are independently distributed. This is the same as the zero-factor model.

The chi-square statistic in the top panel of Table 1 is the likelihood ratio statistic of a model compared with the saturated model. The *p*-values for 0-, 1-, and 2-factor models are extremely small, so models with less than three factors fit extremely badly. The 3-factor model still has a very small *p*-value, but the rule of thumb that $\chi^2/\mathrm{df} < 2$ does not entirely rule this model out. The 4- and 5-factor models have *p*-values above the conventional level of 0.05 and are therefore good candidates. Apart from the non-i.i.d. and non-normality problems, however, the 5-factor model has the additional problem that it is a Heywood case. That is, two of the estimated error variances are zero, which makes the *p*-value even less trustworthy and the model less plausible.

The root mean square error of approximation (RMSEA) also compares a model with the saturated model. It is a non-negative number that measures the lack of fit per degree of freedom. Usually, as a rule of thumb, RMSEA < 0.05 is considered a well-fitting model and RMSEA < 0.10 is considered a moderately well-fitting model. According to this criterion, the 4- and 5-factor models fit well and the 3-factor model fits moderately well.

The comparative fit index (CFI) is a pseudo-R^2 that compares a model to the null model. Its value is always between 0 and 1. It is 0 if a model fits as badly as the null model and it is 1 if it fits very well. A general rule of thumb for the CFI is that it should be greater than 0.9 or 0.95, depending on the researcher. In either case, a model of at least three factors is indicated.

In the bottom panel of Table 1, consecutive models are compared sequentially, using sequential chi-squares (likelihood-ratio tests), which show that extremely large improvements are made by adding factors until the 3-factor solution is reached. The difference between the 3-factor solution and the 4-factor solution is still significant, but not as extreme. The differences between the 4- and 5-factor solutions and between the 5-factor solution and the saturated model are not significant, although, again, the Heywood problem of the 5-factor model further diminishes the usefulness of the *p*-value.

All statistics lead to a similar conclusion: the 3- and 5-factor models may be reasonable candidates, but the 4-factor model appears to fit better than the 3-factor model, and not substantially worse than the 5-factor model. Reasons of parsimony and problems with the Heywood case in the 5-factor solution suggest that a 4-factor model is the best choice.

The overriding consideration in FA is that the loadings should allow a meaningful interpretation. Sometimes, the rules of thumb and even the fit statistics do not point at the model with the most interesting substantive interpretation. Extra factors can dilute the measurement of some common movement, or, conversely, remove a different feature that is confounding the phenomena of interest. Section 2.3 discusses the 4-factor model that is supported by the fit statistics, but this model is interesting mainly because of the clearer interpretation it allows.

2.3 The 4-factor model

A quartimin rotation (direct oblimin with tuning parameter $\gamma = 0$) with Kaiser normalization is used to identify a unique solution. This is arguably the most common non-orthogonal rotation method. Non-orthogonal (oblique) rotations are of primary interest in this application, because it does not seem reasonable to impose that transactions and savings money should be uncorrelated. In fact, one would be inclined to believe that these should be correlated. In good economic times people may both spend and save more, and in hard times they may do less of both.

The quartimin objective is to give loadings that weight heavily on one factor or the other. This may be appropriate for currency and investment, at the two ends of the spectrum, but it is not so obvious for some deposit types in-between. Modern deposit accounts can pay good interest rates and allow the convenience of point-of-sale payments, so they may be used for both savings and transactions. However, quartimin gives only a preference for this weighting; it does not necessarily succeed if the data do not support this. Quartimin does give an interesting rotation of these data. Other possibilities are discussed in section 4, but none appear to be especially better, so quartimin seems the best choice. The rotations have been computed with the gpa functions of Bernaards and Jennrich (2005).[2]

Table 2 shows the estimated standardized solution. The standardized solution (scaled so that each series in the factors and data has variance 1.0) is typically easier to interpret, because a larger (in absolute size) loading indicates a stronger relationship. Loadings of the standardized solution are typically between -1 and 1. This is necessarily the case for orthogonal solutions, because then the loadings are the correlations between the indicators and the factors, but, as Table 2 illustrates, for non-orthogonal solutions it is possible that loadings are larger than 1 in absolute value. The last column gives the communalities. The communality of an indicator is its common variance (i.e., the variance explained by the factors) expressed as a fraction of its total variance; thus, it is the R^2 of the regression of the indicator on the factors. The bottom half of the table shows the correlations between the factors (the Φ matrix). Table 3 shows the loadings from the corresponding unstandardized solution.

With a few exceptions, these communalities are higher than those of Gilbert and Meijer (2005), sometimes substantially so. Nevertheless, some communalities are still small compared with communalities typically found in cross-sectional analyses. But, as Gilbert and Meijer (2005) explain, this is normal for this type of data (differenced time series).

The usual rule of thumb is that a standardized factor loading less than 0.3 is not large enough to be considered important. However, sometimes additional factors shift loadings

[2]We would like to thank Coen A. Bernaards and Robert I. Jennrich for extensive discussions about their code.

Table 2: Standardized 4-factor Solution Using Quartimin Rotation

Indicator	Factor loadings				Communality
Currency	1.01	0.09	-0.09	-0.08	0.98
Personal chequing	-0.02	0.20	0.87	-0.07	0.77
Nonbank chequing	0.20	-0.13	0.36	0.11	0.25
N-P demand and notice	0.55	-0.00	0.20	0.24	0.57
N-P term	0.02	0.10	-0.23	0.33	0.22
Investment	0.15	0.45	-0.23	0.06	0.35
Consumer credit	0.01	0.10	0.10	0.79	0.76
Residential mortgage	0.21	0.49	0.22	0.14	0.53
Short-term business credit	-0.09	0.23	-0.01	0.09	0.09
Other business credit	0.09	0.66	0.23	0.04	0.57
	Factor correlations				
Transactions factor	1.00				
Long-term factor	0.18	1.00			
Potential spending factor	0.30	-0.02	1.00		
Consumer credit factor	0.34	0.57	0.01	1.00	

Table 3: Factor Loadings of the Unstandardized 4-factor Solution Using Quartimin Rotation

Indicator	Factor loadings			
Currency	13.73	1.27	-1.26	-1.07
Personal chequing	-1.24	8.72	37.63	-3.39
Non-bank chequing	2.17	-1.49	3.95	1.28
N-P demand and notice	25.71	-0.15	9.28	11.09
N-P term	1.52	7.27	-16.89	23.95
Investment	17.56	53.68	-28.01	8.02
Consumer credit	0.55	3.15	3.04	23.86
Residential mortgage	10.87	24.83	11.38	7.47
Short-term business credit	-6.49	17.22	-0.98	6.97
Other business credit	5.42	37.35	13.51	2.37

from slightly above to slightly below this cut-off, or the reverse, so the rule of thumb should be kept in context. Currency and non-personal demand and notice deposits load on the first factor, which can therefore be interpreted as the *transactions* factor. Investment, residential mortgage, and other business credit load on the second factor, which can therefore be interpreted as a *long-term* factor. Personal chequing and non-bank chequing load on the third factor, which is called the *potential spending* factor. This factor might capture money that consumers have in reserve—and may save or spend—but are not spending yet. This could be a different sort of spending by the same consumers; for example, more expensive durable goods in contrast to consumable goods (expenditures typically planned on different horizons). Or, it could be different consumers with varying spending habits or financial constraints. Also, personal chequing accounts are influenced to some extent by some small business activity for which they are used.[3] This activity could be very different from typical consumer activity. Consumer credit and, to a much lesser extent, non-personal term, load on the fourth factor. One possible interpretation of this factor is that it indicates consumers' willingness to spend money not yet earned. For lack of a better name, it is called the *consumer credit* factor. The fact that non-personal term also loads on this factor may reflect the way this credit is financed, but this explanation is speculative.

There are moderately positive correlations between some of the factors, as may have been expected. Note, however, that these are correlations between the *differenced* factors. The correlations between the undifferenced factors are much larger.

Figure 2 shows the factor scores in terms of year-over-year growth. The year-2000 effect on the transactions factor is readily apparent. (People held unusually large amounts of currency at the end of December 1999, above the normally large currency holdings at Christmas.) Figure 3 shows the factor scores in level terms. The measurement units of these factor scores are arbitrary, so they should be considered indexes rather than absolute numbers. Note that this does not affect the growth rates. In this figure, a seasonal effect in the transactions factor is clearly visible, whereas the long-term (second) and consumer credit (fourth) factors exhibit steady growth without a seasonal effect. This may have been expected for the long-term factor, but it is quite surprising for the consumer credit factor. The potential spending (third) factor appears to follow a slow cyclical pattern without systematic growth.

The purpose of TSFA is to measure the underlying true factors, not to explain the observed indicators. In particular, an indicator that has little in common with movements in other indicators will typically not be well explained by the factors. Movement in that indicator would be explained by the idiosyncratic term. Nevertheless, as discussed in Gilbert and Meijer (2005), the explained part of the data can be computed as $\hat{y}_t = \hat{B}\hat{\xi}_t$, where $\hat{B}$ is the estimated factor loadings matrix and $\hat{\xi}_t$ is the vector of factor scores at time t. The graphics of the explained data versus the observed (actual) data may provide a useful diagnostic.

Figures 4 and 5 show the growth rates of the data and the explained portion for indicators 1–5 and 6–10, respectively. These figures show that five of the ten series (currency, personal chequing, investment, consumer credit, and other business credit) are explained very well by the factor scores. In four series (non-personal demand and notice, non-personal term, residential mortgage, and short-term business credit), the explained curve is considerably smoother than the observed series. This illustrates the larger idiosyncratic variation of these series, which also follows from their lower communalities. In addition, a more serious deviation is the consistently higher explained growth rate of short-term business credit. The most likely explanation of this phenomenon is the extremely small communality of this series, so that a small estimation error in the loadings can result in relatively large errors in the explained part of this series.

The non-bank chequing series (Figure 4) is not represented well by the explained part. In particular, the "explained" growth rates explode on the right-hand side of the graph. As discussed below, this occurs because the level of the explained series is close to zero. This, in turn, is probably caused by the anomalous negative estimated loading on the second (long-term) factor. Gilbert and Meijer (2005) show that the loadings may not be estimated precisely, which

[3] Sometimes, personal chequing accounts are used by sole proprietor businesses when the individuals need not distinguish their personal accounts from their business accounts; for example, independent consultants.

Figure 2: Year-to-year growth rates (in %) of the factor scores from the 4-factor model using quartimin rotation.

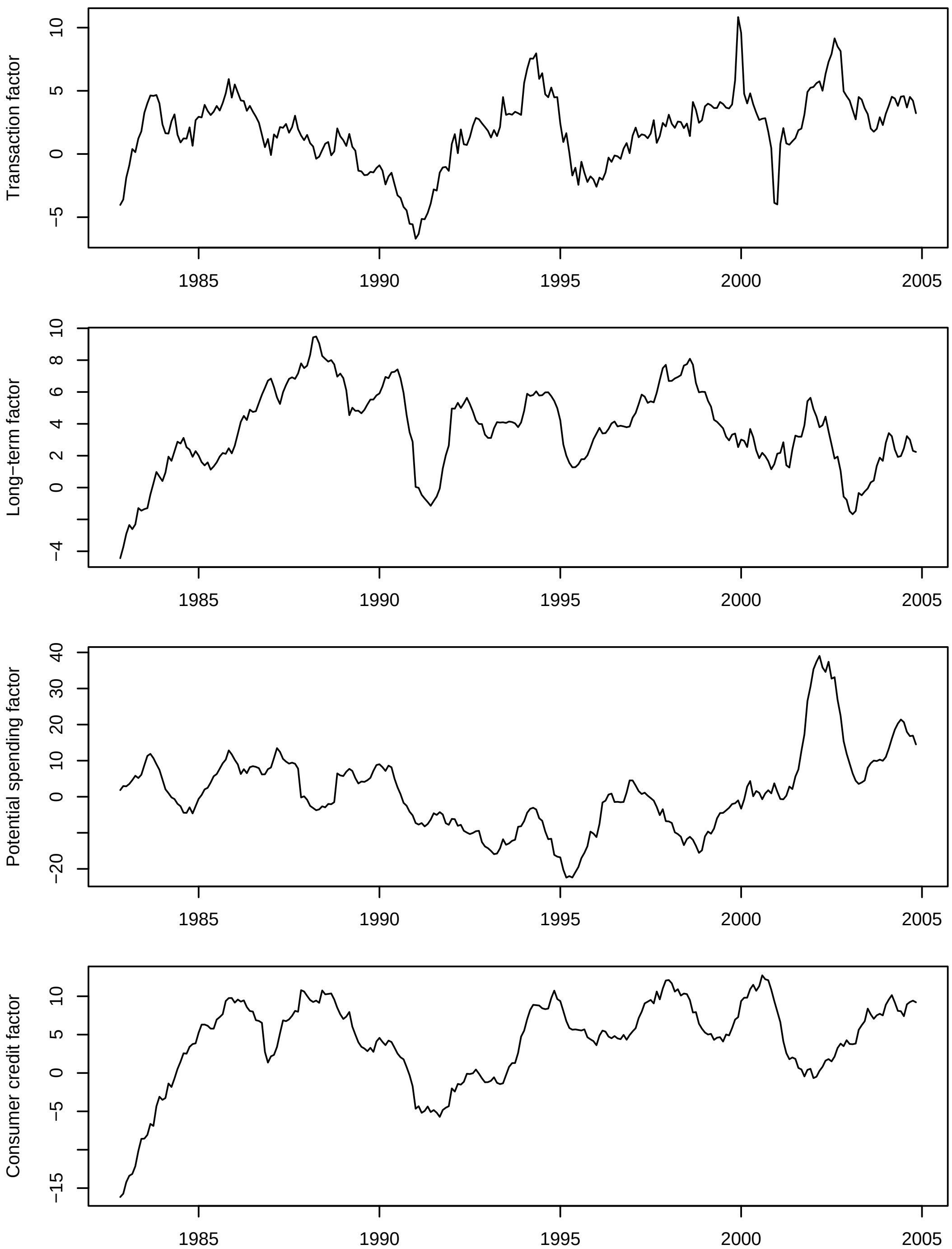
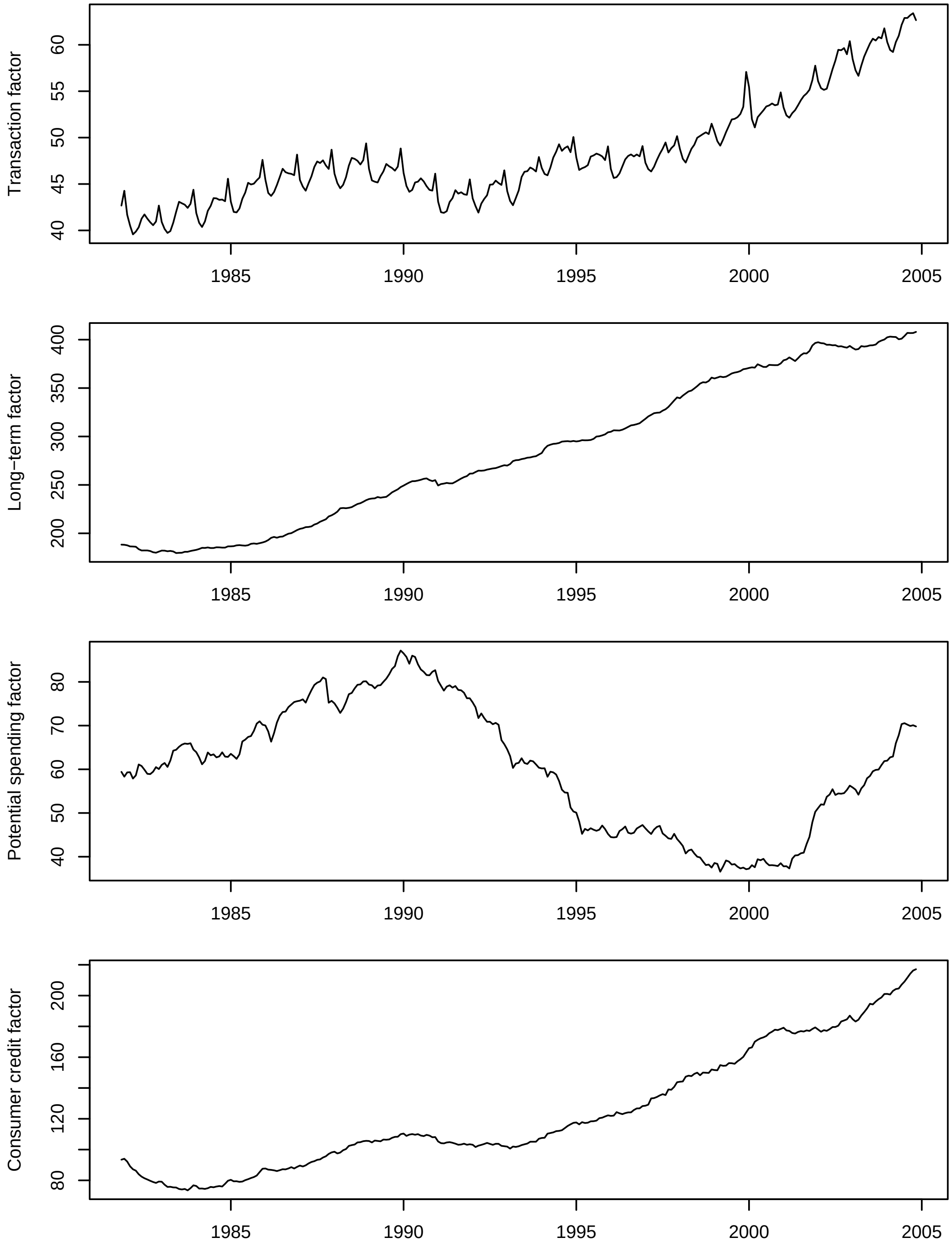


Figure 3: Estimated factor scores from the 4-factor model using quartimin rotation.



may have caused this negative estimate. Quite possibly, this loading may not be significantly different from zero, which opens up the possibility of fixing it at zero in a more "confirmatory" estimation of the loadings. Standard errors for TSFA are not yet available, so this conjecture cannot be confirmed. However, Gilbert and Meijer (2005) mention how standard errors could be obtained in principle. This is an important avenue for further research.

Figures 6 and 7 show the data and the explained portion in level terms for indicators 1–5 and 6–10, respectively. These figures largely corroborate the findings from the analyses of the growth rates. For example, they show that some series, especially currency, are explained very well by the factors. The seasonal pattern in currency is clearly present, whereas for investment neither the observed indicator nor the explained portion exhibits obvious seasonality. Figures 6 and 7 also show the problems with some series, such as the near-zero explained part of non-bank chequing in the right-hand side of Figure 6, which caused the explosion of the growth rates. For some series, there appears to be a systematic discrepancy between the observed data and the explained part, which might erroneously be interpreted as a bias. As Gilbert and Meijer (2005) show, this phenomenon is due to sampling variability (across samples) in the estimation of the factor loadings, and should become less pronounced as more data become available and the loadings are estimated with more precision.

3. The 2-, 3-, and 5-Factor Models

This section describes in more detail the 2-, 3-, and 5-factor models estimated with quartimin. As noted earlier, the principal reason for choosing the 4-factor model is the clear separation of factors in ways that give a more meaningful interpretation. The choice of four factors is also supported by the fit statistics shown in Table 1, but the overriding consideration in FA should be the substantive interpretation; therefore, in this application one would like an economically interesting or meaningful result. Tables and figures are labelled with the factor numbers, because factor names come from interpreting the 4-factor model, but the meanings appear largely unchanged.

Tables 4, 5, and 6, respectively, show the standardized solutions from the 2-, 3-, and 5-factor models. The 2-factor model gives a separation that is similar to the narrow and broad monetary aggregates. Currency, personal chequing, non-bank chequing, and non-personal demand and notice deposits load on the first factor, whereas non-personal term, investment, and all the credit measures load on the second factor. The two factors are moderately positively correlated. This solution differs somewhat from the one reported by Gilbert and Meijer (2005), where some moderately large negative loadings are found and the correlation between the factors is close to zero. It follows that the additional credit indicators, and possibly the longer sample period that is used in this research, lead to a substantively more appealing solution. As was to be expected, most communalities are lower than the corresponding communalities of the 4-factor solution, and sometimes considerably so. The only sizable exception is non-personal demand and notice, which has a much higher communality in the 2-factor solution: the transactions factor is more strongly related to non-personal demand and notice deposits, and less strongly related to currency, than in the 4-factor solution.

Although the interpretation of the 2-factor solution aligns more clearly with traditional money classifications, the scree plot and various fit statistics show that the 2-factor model is not adequate.

Figure 4: Year-to-year growth rates (in %) of the observed money and credit indicators 1–5 (solid) and their explained portion using 4 factors (dashed).

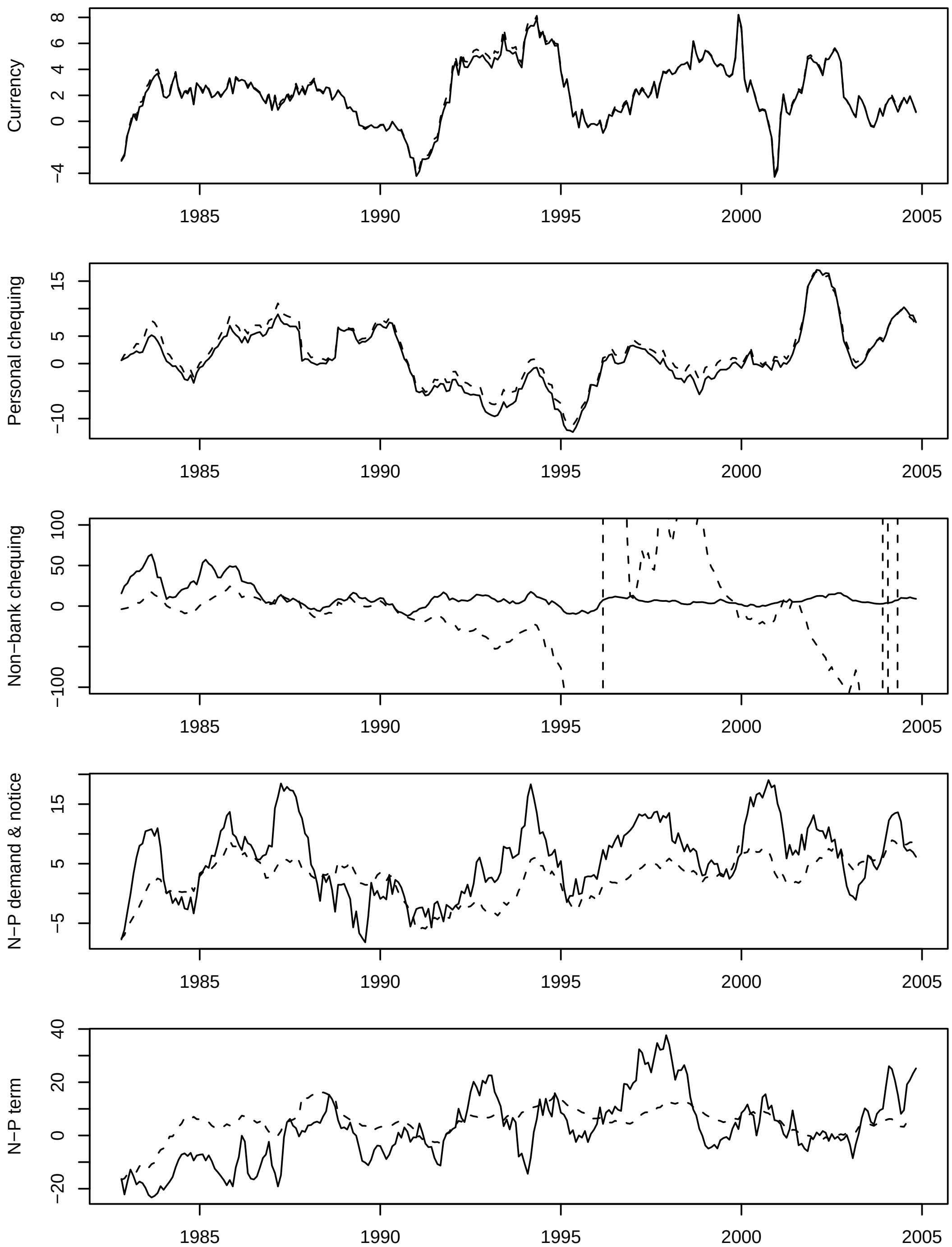


Figure 5: Year-to-year growth rates (in %) of the observed money and credit indicators 6–10 (solid) and their explained portion using 4 factors (dashed).

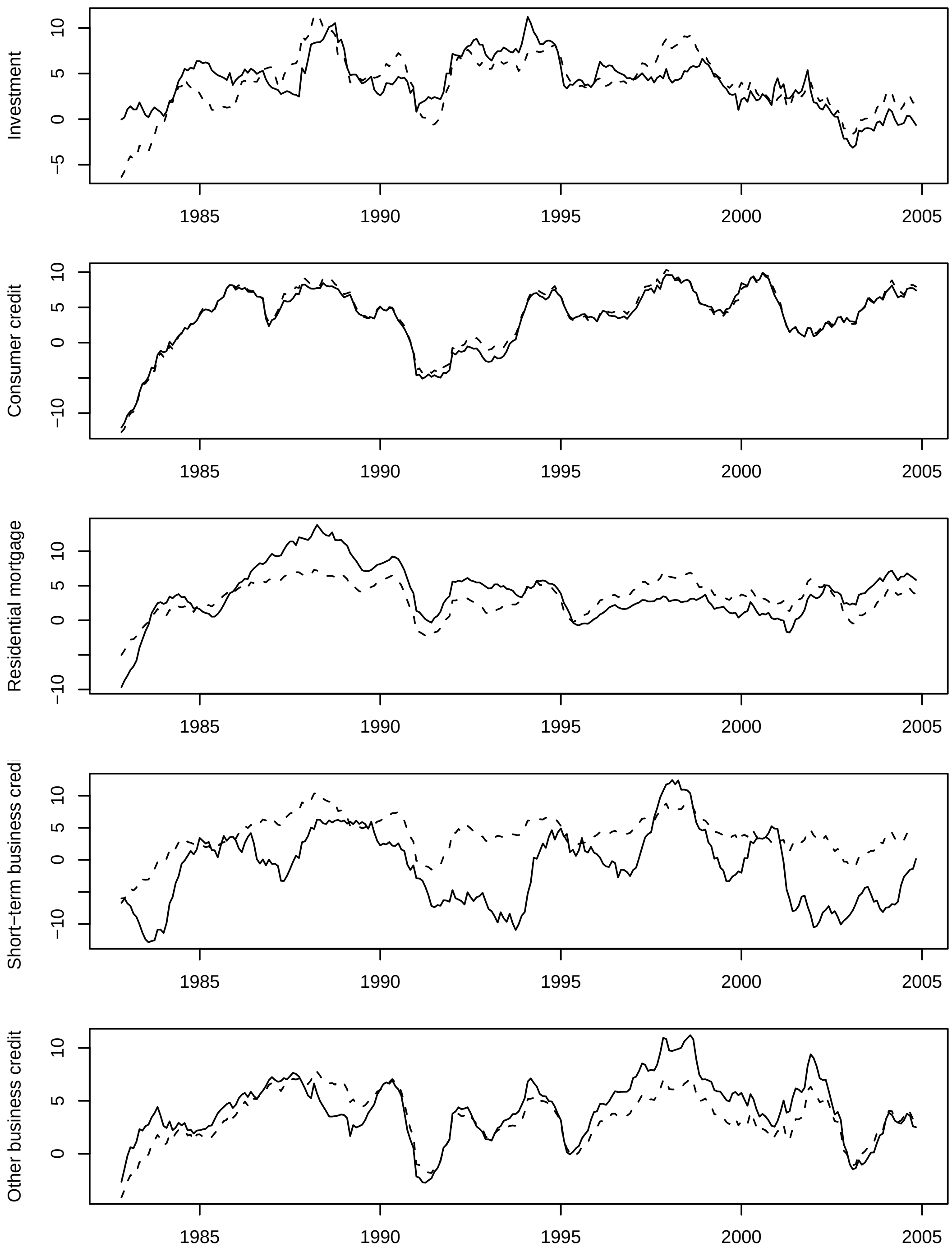


Figure 6: Observed money and credit indicators 1–5 (solid) and their explained portion using 4 factors (dashed).

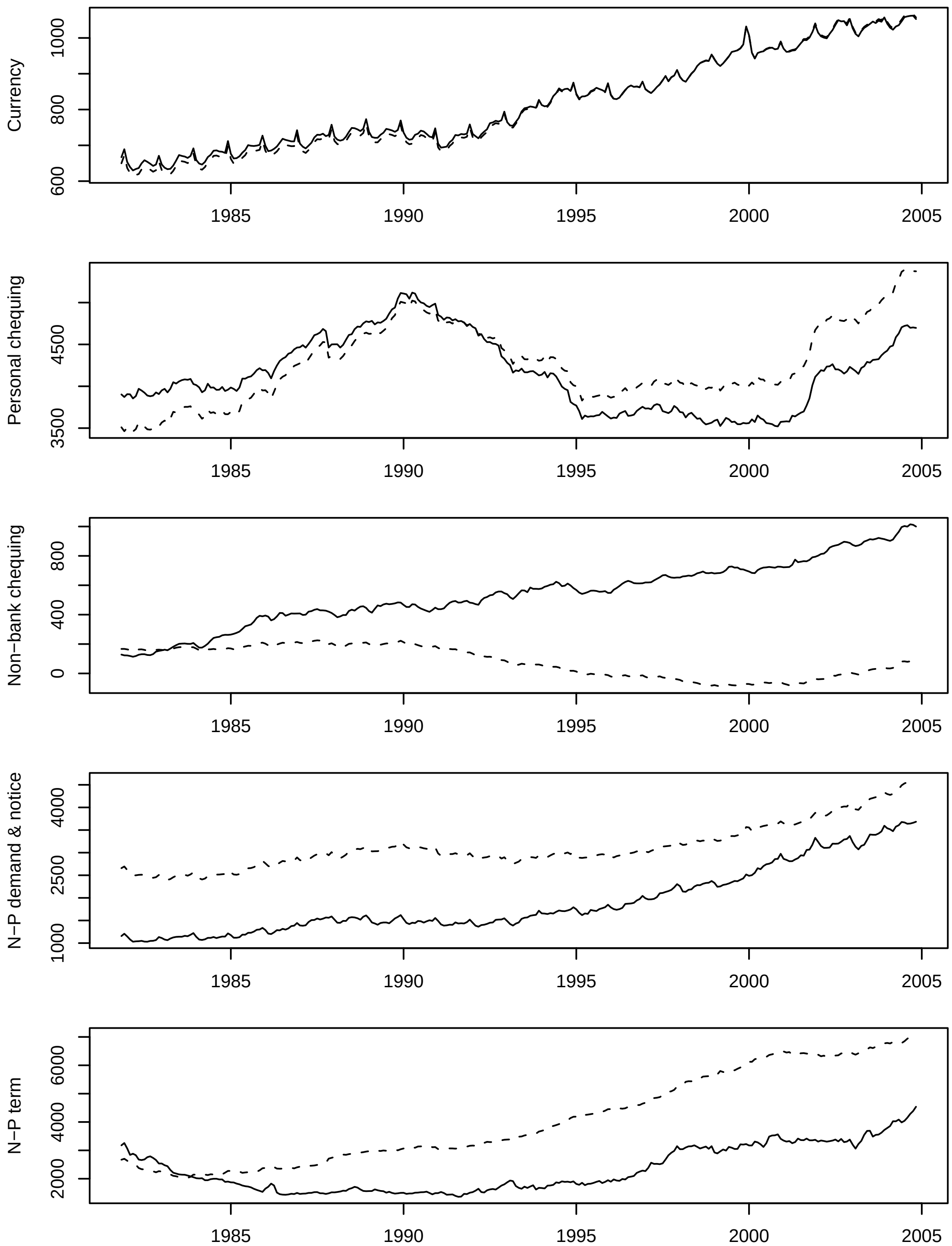


Figure 7: Observed money and credit indicators 6–10 (solid) and their explained portion using 4 factors (dashed).

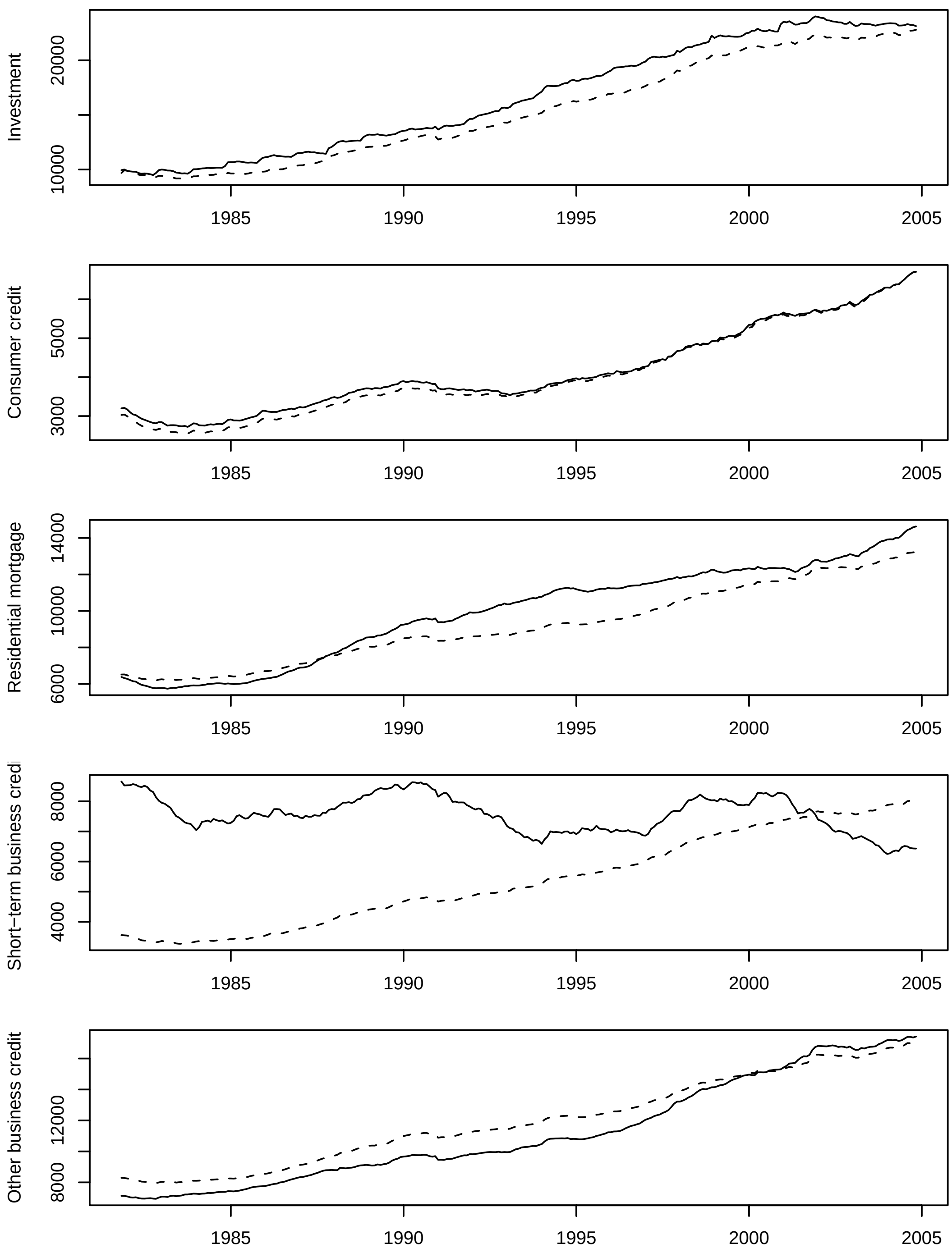


Table 4: Standardized 2-Factor Solution Using Quartimin Rotation

Indicator	Factor loadings		Communality
Currency	0.71	0.11	0.57
Personal chequing	0.37	0.04	0.15
Non-bank chequing	0.43	-0.09	0.17
N-P demand and notice	0.82	0.08	0.73
N-P term	-0.03	0.37	0.14
Investment	0.01	0.49	0.25
Consumer credit	0.24	0.61	0.51
Residential mortgage	0.31	0.57	0.52
Short-term business credit	-0.12	0.34	0.11
Other business credit	0.21	0.60	0.47
	Factor correlations		
Factor 1	1.00		
Factor 2	0.26	1.00	

The 3-factor model is the one that is suggested by the rule of eigenvalues greater than 1.0. In this model, currency, non-bank chequing, and non-personal demand and notice deposits load on the first factor. Non-personal term, investment, and all the credit measures load on the second factor, as in the 2-factor model. The third factor is strongly related to personal chequing, but note that the loading of non-bank chequing on this factor is 0.24, whereas in the 4-factor model it is 0.36, and so the shift is not too large.

It is interesting to note that adding the third factor separates personal chequing from currency, which suggests that this difference is more substantial than the differences among the credit measures and investment. The implication is that personal chequing accounts are not used very much like currency, which is at odds with traditional thinking about monetary aggregates. One possibility is that the first factor explaining currency reflects current activity, whereas the third factor reflects potential future activity. In that case, the first would be what most people would think of as "transactions money." It is not obvious that it is more important for forecasting future activity, but it should also not be assumed that potential spending is eventually spent; money in chequing accounts may eventually be moved into longer-term investments. Non-personal demand and notice deposits also load on the first factor, which may indicate that the increased activity resulting in more currency use also means that businesses turn over more cash and keep higher balances in their demand and notice accounts.

In this solution, the first factor has moderately large positive correlations with the other two factors, but the second and third factors are virtually uncorrelated. This seems to be neither obvious nor problematic. Compared with the 2- and 4-factor solutions, there are some differences in the absolute sizes of corresponding factor loadings. This results in the communalities not always being bracketed by the communalities of the 2- and 4-factor solutions, but we do not consider this to be problematic, either.

The 5-factor model is suggested by the kink in the scree plot. Currency and non-personal demand and notice deposits load on the first factor. Investment and some of the credit indi-

Table 5: Standardized 3-Factor Solution Using Quartimin Rotation.

Indicator	Factor loadings			Communality
Currency	0.81	0.07	-0.10	0.67
Personal chequing	0.05	0.10	0.95	0.95
Non-bank chequing	0.33	-0.06	0.24	0.20
N-P demand and notice	0.78	0.09	0.05	0.69
N-P term	0.04	0.36	-0.23	0.20
Investment	0.08	0.48	-0.18	0.30
Consumer credit	0.22	0.61	0.01	0.50
Residential mortgage	0.23	0.59	0.20	0.54
Short-term business credit	-0.13	0.33	0.02	0.11
Other business credit	0.10	0.64	0.24	0.52
	Factor correlations			
Factor 1	1.00			
Factor 2	0.27	1.00		
Factor 3	0.26	-0.05	1.00	

cators, especially long-term credit, load on the second factor. Personal chequing, non-bank chequing, and other business credit load on the third factor. The fourth factor is very strongly related to consumer credit, whereas the fifth factor almost coincides with non-personal term. The communalities of consumer credit and non-personal term are 1.0, which means that the estimated variances of their error terms are zero.[4] These are the Heywood cases noted earlier. Typically, Heywood cases are considered problematic. In the current situation, we interpret this as an indication of overfitting.

Figure 8 shows the growth rate of the transactions factor score and the long-term factor score for all of these models, and the growth rate of the potential spending factor score for the 3-, 4-, and 5-factor models. The figure suggests that the measurement of the long-term (second) and potential spending (third) factors is not overly sensitive to the number of factors in the model. The growth rates of the transactions (first) factor score, however, are higher and more variable for the 2- and 3-factor models than for the 4- and 5-factor models. We perceive this watershed as an affirmation of the need for a fourth factor.

4. Sensitivity to the Rotation Method Choice

Quartimin is probably the most often used oblique (i.e., non-orthogonal) rotation. This reason alone suggests that it is a good starting choice of rotation method, and should be preferred if

[4]This may look strange, but it is not. Although the communality is an R^2, it is quite different from the R^2 of a typical regression meant to explain one economic variable using other economic variables. In FA, the R^2 is from an equation to explain the observed data by its assumed true underlying concept. Economists often treat observed variables as perfect measures of underlying concepts that they are trying to measure, which is an implicit assumption that the communality is 1.0; i.e., the R^2 from regressing the observed data on a perfect measure of the concept would be 1.0.

Table 6: Standardized 5-Factor Solution Using Quartimin Rotation.

Indicator	Factor loadings					Communality
Currency	1.00	0.07	-0.08	-0.04	0.01	0.96
Personal chequing	-0.01	0.03	0.80	-0.01	-0.07	0.65
Non-bank chequing	0.20	-0.17	0.33	0.11	-0.07	0.26
N-P demand and notice	0.57	-0.01	0.17	0.17	0.05	0.56
N-P term	0.02	-0.06	-0.06	0.00	1.00	1.00
Investment	0.15	0.57	-0.22	0.06	-0.04	0.42
Consumer credit	-0.00	0.05	-0.06	0.97	0.03	1.00
Residential mortgage	0.22	0.41	0.29	0.12	0.08	0.52
Short-term business credit	-0.08	0.24	0.01	0.08	0.02	0.09
Other business credit	0.09	0.57	0.37	0.01	0.11	0.59
	Factor correlations					
Factor 1	1.00					
Factor 2	0.14	1.00				
Factor 3	0.35	0.02	1.00			
Factor 4	0.36	0.46	0.28	1.00		
Factor 5	0.07	0.28	-0.08	0.30	1.00	

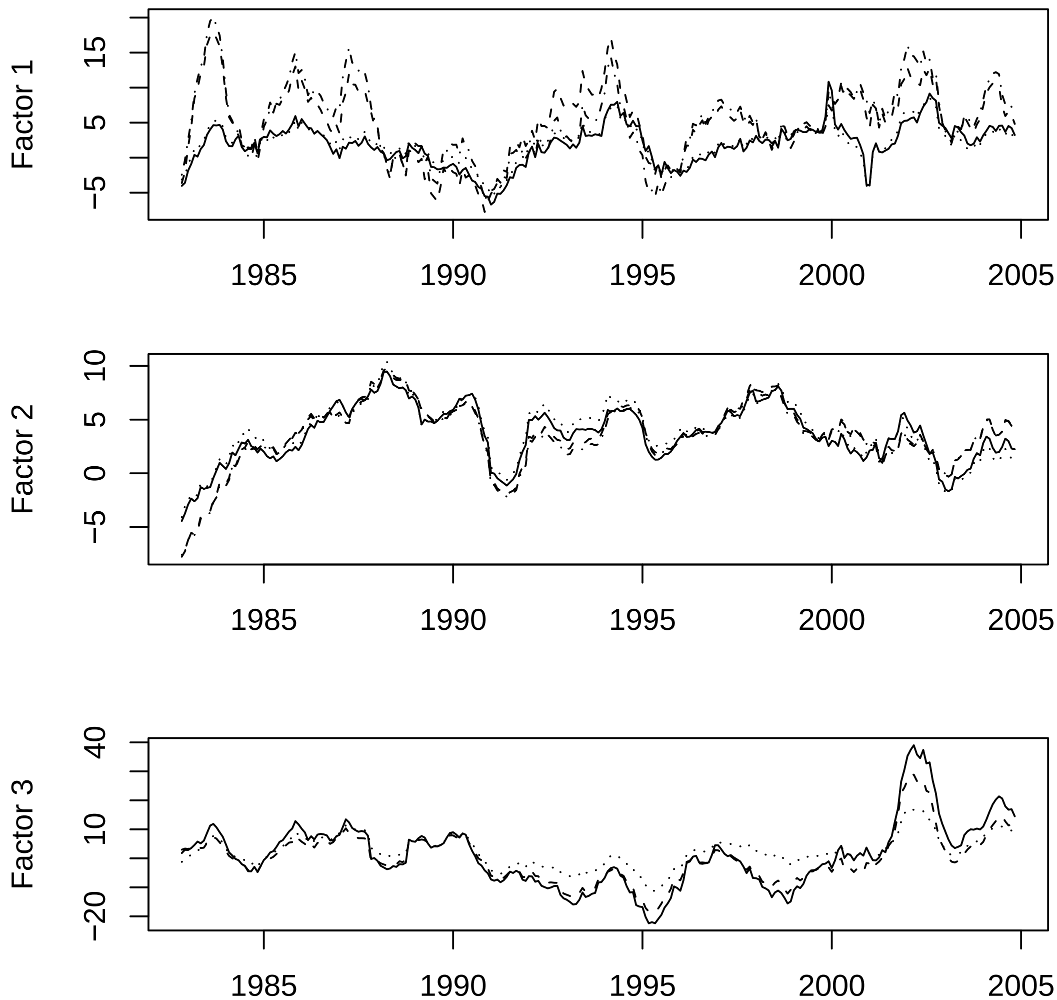
another rotation does not result in a substantially better interpretation of the loadings. Many alternative rotation methods have been proposed in the literature, however, and different rotation methods have different virtues. Browne (2001) provides a recent overview of different rotation methods and discusses their merits. Bernaards and Jennrich (2005) also provide an extensive overview, focusing on a new algorithm for computation of the various rotation methods. This section considers the extent to which results obtained thus far are affected by the choice of rotation method. These comparisons are restricted to the 4-factor solution.

An obvious alternative to the quartimin rotation is another oblimin method with a different tuning parameter, γ. According to Loehlin (1987, 164), negative values of γ tend to penalize non-zero correlations between the factor scores, whereas positive values of γ do the opposite and “tend to produce overoblique and often problematic solutions.” Another alternative mentioned by Bernaards and Jennrich (2005) is Bentler’s maximum “invariant pattern simplicity” method for oblique rotation.

Figure 9 shows the year-to-year growth rates (in per cent) of the factor scores from oblimin with γ equal to 0.0 (quartimin), 0.5 (biquartimin), -0.5, and -1.0, respectively, and for Bentler’s oblique rotation. This shows that the factor score values are not greatly affected by these choices. In an attempt to use oblimin with $\gamma = 1.0$ (covarimin), convergence was not obtained and estimates of loadings diverged. Such problems are common for this value of γ. For this reason, SPSS allows only $\gamma \leq 0.8$ (SPSS 2003). Jennrich (1979) proves that the oblimin criterion always has a proper minimum if $\gamma \leq 0$, but that for every positive value of γ there are circumstances in which the oblimin criterion function is unbounded below.

The interpretation of the factor loadings is not improved by these other rotations. Three of the four give very similar results. For the fourth, oblimin with $\gamma = 0.5$, none of the indicators

Figure 8: Year-to-year growth rates (in %) of first and second factor scores from the 2- to 5-factor models, and of the third factor from the 3-, 4-, and 5-factor models, in all cases using quartimin rotation. (2-factor model, dot-dash; 3-factor model, dashed; 4-factor model, solid; 5-factor model, dotted)

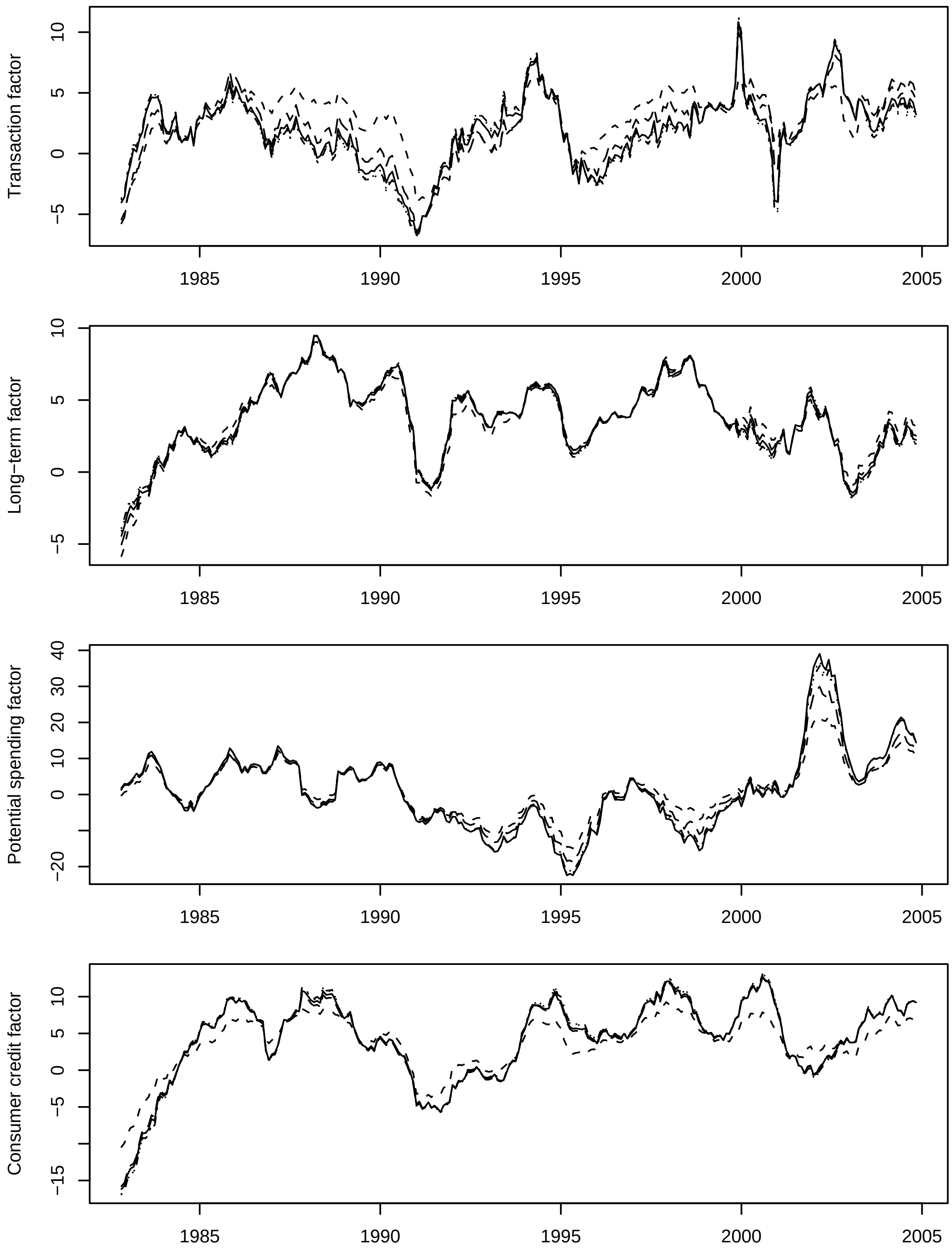


loads substantially on factors three and four, and as a result this rotation does not have a meaningful economic interpretation.

Browne's (2001) adaptation of Yates' geomin rotation gives different results, however. Browne found good solutions in many situations, including ones for which no simple clustered solution exists. He therefore recommends that this rotation method be tried in addition to others, such as quartimin.

Figure 10 shows the year-to-year growth rates of the resulting factor scores against those obtained from quartimin. Factors 2 and 4 are almost the same as for the quartimin solution.

Figure 9: Year-to-year growth rates (in %) of the factor scores from 4-factor models using oblimin rotations with $\gamma = 0$ (quartimin; solid), 0.5 (biquartimin; dashed), -0.5 (dotted), and -1.0 (dot-dash), and using Bentler's oblique rotation (long dash).



The third factor is quite different, however, and the first factor of the geomin rotation inherits a peak growth rate around the year 2002 from the third factor of the quartimin rotation.

Table 7 reports the estimated loadings with geomin rotation. The table shows what happens with an unsuccesful rotation. Although the loadings pattern is fairly similar to the pattern of the corresponding quartimin solution, the absolute sizes of the loadings are different in a way that causes difficulties in the interpretation, especially with regard to the third factor. The indicators that have substantial loadings on this factor also have substantial loadings, often of a similar magnitude, on another factor. Thus, the factors are not clearly separated. Moreover, the most important loadings on the third factor do not all have the same sign, which is substantively problematic. Therefore, it is difficult to interpret the third factor and the solution as a whole.

Table 7: Loadings of the Estimated Standardized 4-Factor Solution Using Geomin Rotation

Indicator	Loadings			
Currency	1.04	0.00	-0.44	-0.00
Personal chequing	0.01	0.58	0.81	-0.03
Non-bank chequing	0.28	-0.00	0.29	0.18
N-P demand and notice	0.61	0.04	-0.01	0.30
N-P term	-0.01	0.00	-0.28	0.29
Investment	0.02	0.37	-0.37	-0.00
Consumer credit	0.02	0.14	-0.00	0.77
Residential mortgage	0.13	0.60	0.03	0.10
Short-term business credit	-0.15	0.25	-0.03	0.04
Other business credit	-0.02	0.79	0.06	-0.03

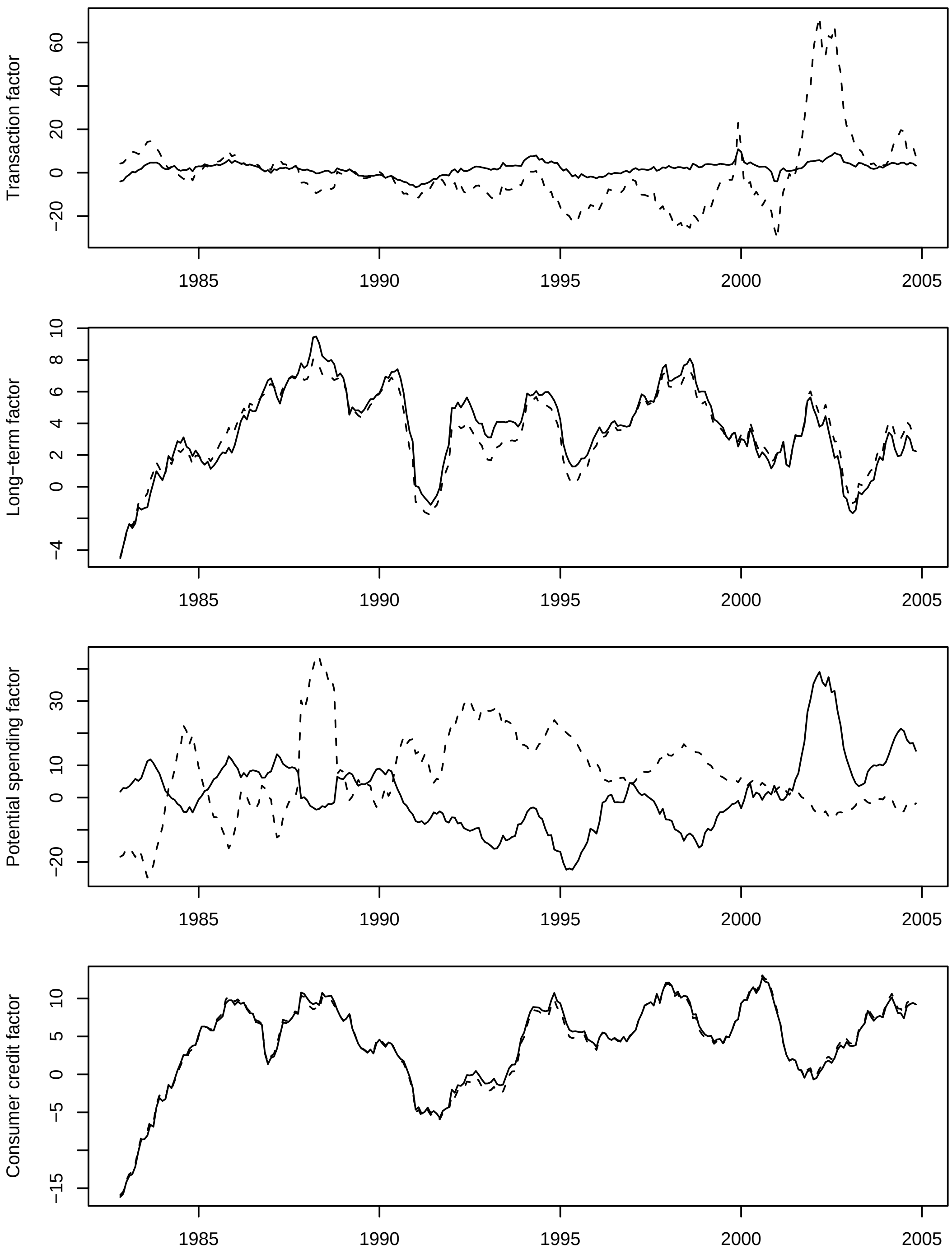
Kaiser normalization scales the rows of the factor loadings matrix, so that squared row entries sum to 1.0 before rotation, and then undoes this scaling after rotation. Kaiser normalization has been used in all rotations in this paper. Not doing this has a very small effect on the rotation of the 4-factor quartimin model, but does nothing that would alter any of the conclusions. The most noticeable effect on the factor scores is that some of the extremes in the third factor are attenuated.

In summary, the quartimin rotation gives substantively interesting results. Other rotations give either very similar or anomalous results. Therefore, we conclude that the quartimin solution is the preferred solution.

5. Sensitivity to Sample Period

This section indicates the extent to which sample size and the selected sample period may influence results. By most accounts, because of financial innovations, the most recent data, starting in the mid-1990s, have been problematic for econometric and policy purposes. Therefore, this section provides results for several subperiods. Figures in this section show the results estimated on the following subsamples: (i) November 1981–December 1989, (ii) November 1981–December 1994, (iii) January 1995–November 2004, (iv) January 2000–November 2004, and (v) January 1990–January 2000. Estimates are made using a 4-factor model and

Figure 10: Year-to-year growth rates (in %) of the factor scores from 4-factor models with geomin rotation (dashed) versus quartimin (solid).



quartimin rotation. Of course, parameter estimators based on smaller subsamples are more variable than estimators based on the whole sample, so differences are expected.

Figure 11 shows the growth rates of factor scores. The results for the fourth factor (consumer credit) appear to be very insensitive to the sample period. The same holds for the second factor (long term), with the exception of the results for the "before 1990" subsample. This subsample produces near-zero levels of this factor score, so that the growth rates explode. For the third factor (potential spending), the "before 1995" estimates are highly volatile, and the "post-2000" results differ from the whole sample results to a lesser extent. Finally, for the first factor, the overall patterns, as obtained from the different subperiods, show considerable similarities, but are far from identical.

If the observed differences were due to structural breaks in the measurement model, we would expect the resulting explained indicators from the subsamples to fit the observed indicators much better, whereas if the observed differences were due only to (larger) sampling variability, we would expect this to be not so apparent (although not completely absent).

To investigate this matter, the year-to-year growth rates of the observed indicators and their explained parts are plotted in Figures 12 and 13. For most series, the explained parts using the whole sample are better than the explained parts using a subsample. While in some cases the whole-sample explanations are slightly worse, they are still very good. This gives a strong indication that there are no structural breaks in the measurement model.

An exception to this general finding is non-personal demand and notice, where subsamples are clearly better, whereas the whole sample oversmooths, although it is still reasonable. As shown earlier, for non-bank chequing, the explained growth rates based on the whole sample explode. Using subsamples, this problem does not occur, and, consequently, the results from the subsamples are much better, especially for the post-1991 period. It may be that for this series a structural change has occurred.[5] But it may also be that the problems for this indicator that result from using the whole sample do not show up in the subsamples because of the relatively larger freedom of fitting (i.e., a smaller number of points need to be fitted with the same number of parameters). This issue needs further investigation.

In our view, this one anomalous series does not cast serious doubts on the method used and/or the validity of the factor scores. On the whole, we believe that there is no convincing evidence that the factor scores from the subsamples are noticeably better than those from the complete sample, which indicates that the measurement model does not vary over time. This is extremely encouraging, given that so much earlier work with the monetary aggregates over this period suggests that structural change is an important issue.

6. Comparison with Aggregates

The factors are intended to be used as alternatives to traditional aggregates in economic models. Therefore, it is interesting to compare the factor scores with these existing measures. In particular, the transactions factor appears to be a substitute for traditional narrow measures of money, such as M1+, and the long-term factor appears to be a substitute for traditional broad measures of money, such as M2++. Because the factor scores are derived from real per capita series, they will be compared with real per capita versions of M1+ and M2++. Because the

[5] There is a break in this series due to a poor estimate of the split between non-bank non-chequing and non-bank term deposits for one institution. Reporting changes meant that the estimate was no longer needed after April 1996.

Figure 11: Year-to-year growth rates (in %) of the factor scores from 4-factor models with quartimin rotation, obtained from different subsamples.

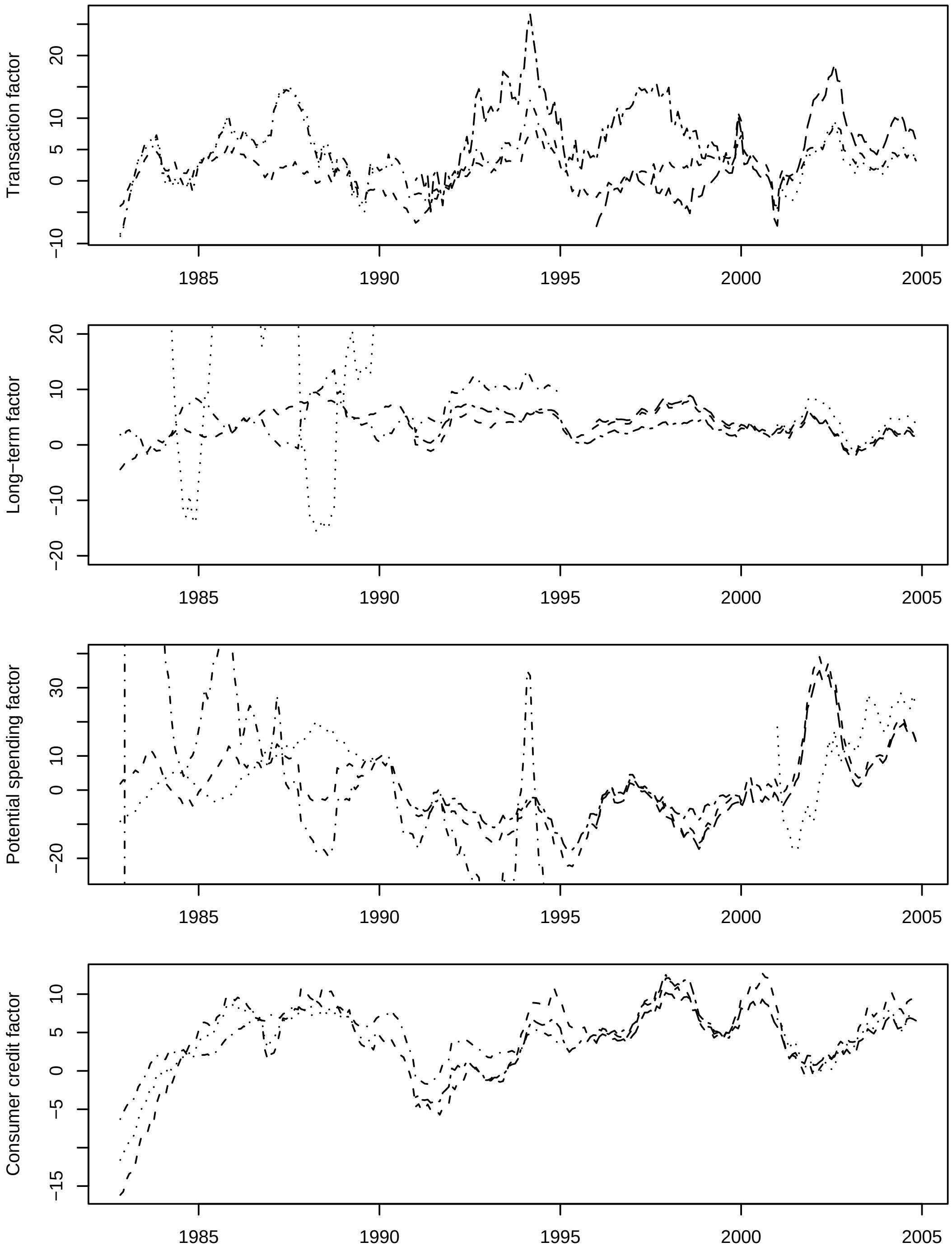


Figure 12: Year-to-year growth rates (in %) of the observed money and credit indicators 1–5 (solid) and their explained parts from using 4-factor models obtained from different subsamples (dashed, dotted, etc.).

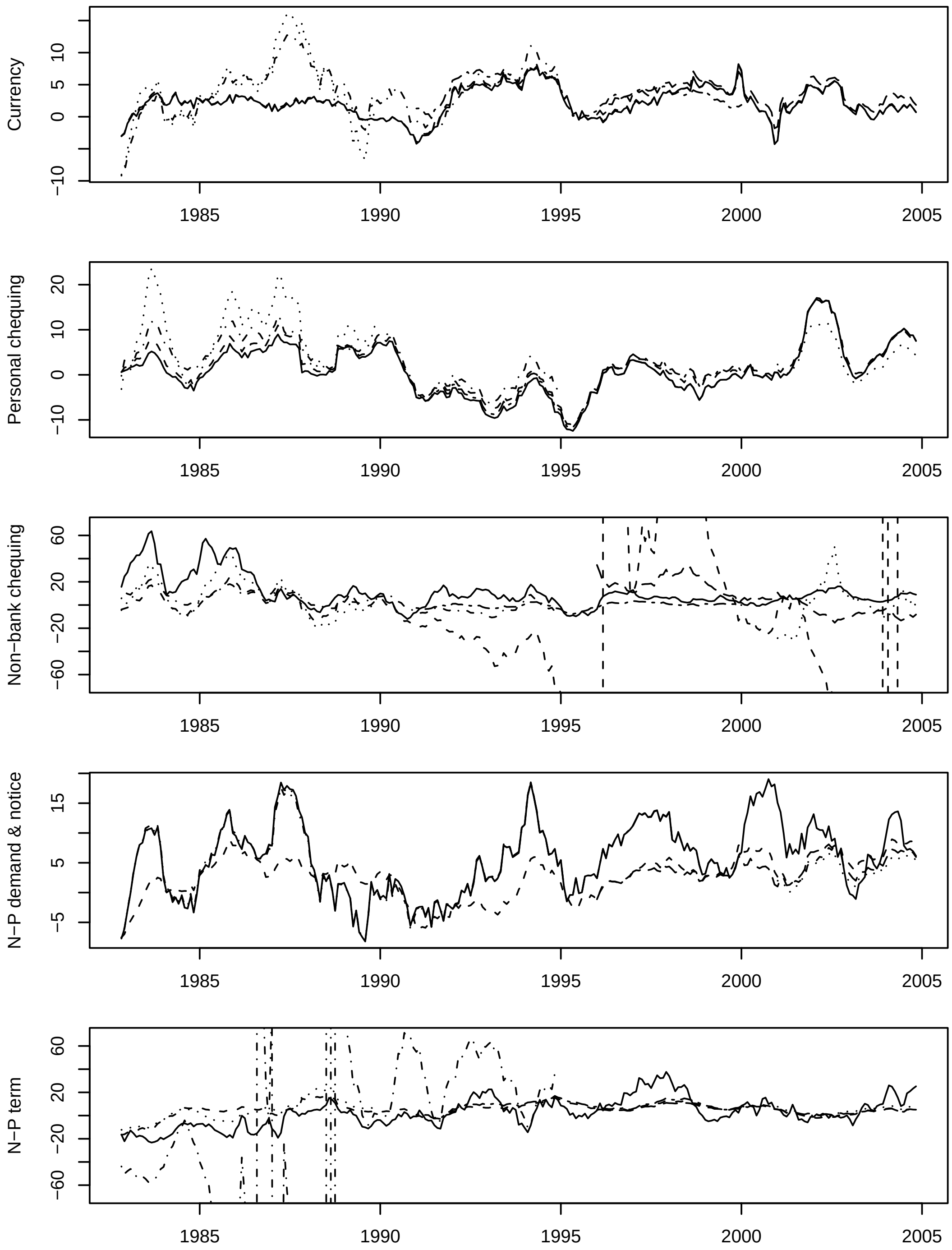
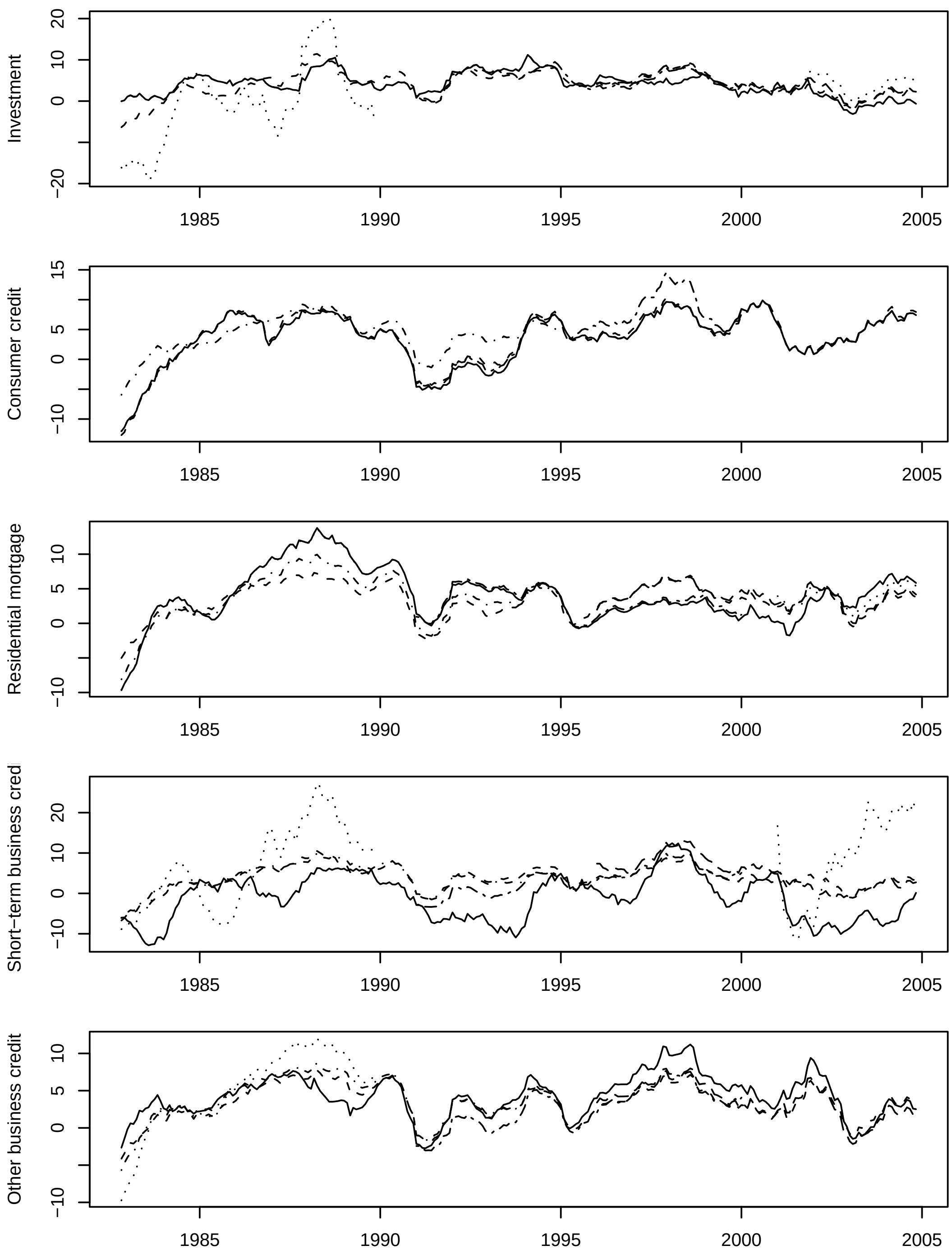


Figure 13: Year-to-year growth rates (in %) of the observed money and credit indicators 6–10 (solid) and their explained parts from using 4-factor models obtained from different subsamples (dashed, dotted, etc.).

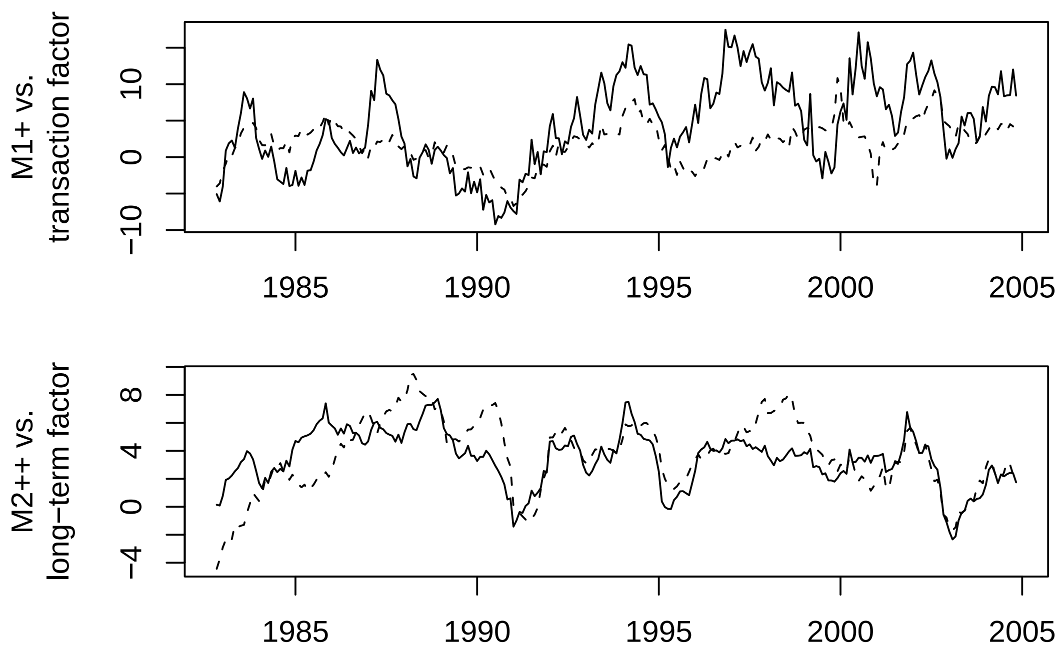


factor scores are intended as improvements over the aggregates, it is expected that they will not be too similar.

Figure 14 plots the year-to-year growth rates of the transactions and long-term factor scores against the corresponding growth rates of real per capita M1+ and M2++. The long-term factor follows the path of M2++ relatively closely, although the growth rate of the factor score is a little bit higher in the late 1990s. The long-term factor is also roughly consistent with what one might expect for savings, given the Canadian business cycle.

The transactions factor has less pronounced growth than M1+ in the second half of the sample period. If one thinks that the growth rate of transactions money should be correlated with inflation, then this less pronounced growth is certainly consistent with observed inflation. Furthermore, the transactions factor exhibits more stable, less volatile, growth than M1+.

Figure 14: Year-to-year growth rates (in %) of real per capita M1+ and M2++ (solid) and transactions and long-term factors (dashed).



Interesting as these comparisons may be, there is another interesting and potentially useful result from the above analyses: the emergence of a third factor. This factor does not correspond closely to any traditional aggregate (although it explains the personal chequing series often included in narrow aggregates). It may be a substitute for traditional aggregates in economic models, or it may capture behaviour not previously measured. These conjectures still need to be investigated.

7. Discussion

These research results suggest that the money and credit data, which are the main data describing the balance sheet of the financial side of the economy, can be modelled by four unobserved underlying factors. Fewer than four factors would not be fully adequate, and more seem unnecessary. Time-series factor analysis (TSFA) provides a method to obtain these factors in a way that emphasizes their interpretation. TSFA extends factor analysis (FA) techniques to time-series data. This technique provides a way to extract factors while making as few assumptions as possible about their dynamics. In particular, with TSFA, factors are established and measured before their dynamics are modelled. This allows different economic models to be estimated using the same factor scores.

The result is that one factor influences currency and non-personal demand and notice deposits, and is related to traditional narrow measures of money. We have interpreted this as a *transactions* factor. The second factor influences investment and thus is related to traditional broad measures of money, but also influences the long-term credit series (residential mortgage and other business credit). Thus we have interpreted this as a *long-term* factor.

The third factor, called the *potential spending* factor, influences personal chequing accounts and, to a lesser extent, non-bank chequing. This factor is perhaps the most novel finding. Personal chequing has usually been considered a substitute for currency, and has been included in many narrower aggregates for this reason. Our analysis, however, suggests that the difference between the behaviour of personal chequing accounts and the behaviour of currency is greater than that between investment and long-term credit. One interpretation is that currency is used for current transactions while some personal chequing balances are held in reserve for a future decision about spending or saving. This factor may measure different activities by the same agents, or it may measure the behaviour of different agents; for example, agents with different financial constraints or agents with small business activities. These speculations have not been investigated, but the possibility that this factor has information for forecasting is enticing.

The fourth factor influences mainly consumer credit, but also, to a lesser extent, non-personal term. The initial reason for adding a fourth factor is that it gives a clearer interpretation of the other factors by removing confounding influences. Consumer credit, however, may indicate an underlying economic phenomenon of interest, such as peoples' readiness to spend money before they have actually earned it.

From the clearly interpretable results, we conclude that the 4-factor model, using the quartimin rotation method with Kaiser normalization, is a very satisfactory measurement model for the money and credit data. Moreover, extensive sensitivity analyses show that alternative modelling options lead either to very similar results or to clearly inferior solutions.

Further research is needed on many aspects of these factors and how they relate to other economic variables. More comprehensive tests of the suitability of the measurement model are the subject of ongoing research. Inevitably, there will be continued adjustments and improvements in the measured factors. Questions concerning structural breaks have not been fully addresssed. Casual consideration suggests that structural breaks may not be a problem, but research on the monetary aggregates over the past 15 years has usually been frustrated by model breakdowns that are often attributed to structural breaks, so further investigation of this matter would be appropriate.

The importance of the inflation measure used to calculate the real data needs to be verified. It would not be especially satisfying if the factor scores are sensitive to the use of other inflation measures. We do not expect them to be very sensitive to different choices, because the data from which the factor scores have been derived employ the price index, rather than the inflation rate, in the denominator. Evidently, the relative differences between price indexes are much less pronounced than the relative differences between inflation measures.

The money data have been examined at great length over many years, and are about as good as possible. The credit data, however, have been added only recently to this analysis, and have therefore not had the same detailed examination. In particular, the other business credit series includes both debt and equity. It seems unlikely that these behave in the same way over the business cycle. Better factor measurement might be obtained by separating this series into two indicators. On closer examination, it will probably be found that the other credit indicators also can be improved.

As in most FA applications, the scale of the factors is arbitrary. In the analysis, the scale has been set by fixing the variance of the differenced factors to unity, which is consistent with the common choice for standard exploratory FA, given that differenced data are analyzed. However, other arbitrary choices for the scales are equally valid. Thus, the factor scores should be viewed as indexes. As such, the scales may be fixed in a more appealing way; for example, by fixing the values for January 2000 at 100. When the scale of a factor score is altered by multiplying it by a number c (e.g., $c = 100/\hat{\xi}_{\text{Jan. 2000}}$, where $\hat{\xi}_{\text{Jan. 2000}}$ is the original value for January 2000), then the corresponding column of the loadings matrix of the unstandardized solution must be divided by the same number c to keep the results consistent. Note, however, that another choice of scale does not affect the growth rates, which is one of the reasons why many results have been reported in this form.

Unlike in typical FA applications, an arbitrary choice of an additive constant (mean) is not allowed, since the measurement model does not contain an intercept. This is also why growth rates of the factor scores can be meaningful.

The next step in this research is to estimate economic models using the factor scores. While the reported factor scores appear interesting, the real question is whether they result in economic models that have good interpretations, provide good forecasting, and result in useful policy analysis.

A related theoretical question is how to handle the remaining measurement errors in the factor scores. The computed factor scores are arguably the best possible, but they are not perfect. There are two sources of error. The first is the estimation of the parameters, especially the loadings. This comes with estimation error, which implies that the weights with which the factor scores are computed are not equal to the theoretically optimal (but unknown) weights. This source of error diminishes (at a rate of root-T, where T is the number of time points in the sample) as data from more time points become available.

The second source of error is the inherent impossibility of estimating the true factor scores consistently, even if the true factor loadings are known, since additional observations do not give additional information about factors at other points in time. Additional observations only improve the estimate of the loadings matrix. Even with the true loadings matrix known, there is still an error associated with measuring the unobserved underlying true factors. Usually, it is assumed that this second source of error is the most important, because it does not vanish asymptotically. The results of Gilbert and Meijer (2005), however, show that the second source of error is very small for the money data, and that the first source of error dominates.

Traditional estimates of the precision (variance) of the factor scores have been asymptotic and thus have neglected parameter uncertainty; see, for example, De Haan et al. (2003) for an application using this approach. It follows from the above that such an approach is unsatisfactory for time-series factor scores derived from the money and credit data. Therefore, theoretical research is required to incorporate uncertainty of parameter estimation into an expression for the factor scores precision. Hamilton (1986) has done such an analysis for state-space models, which possibly can be adapted to TSFA. Note, however, that the precision of the parameter estimators will be necessary input for this. Gilbert and Meijer (2005) mention in general terms how standard errors for parameter estimators in TSFA can be obtained, but specific formulas have not yet been derived and implemented in software.

Once the precision of the factor scores has been assessed satisfactorily, the next question is how this information can be used in subsequent economic modelling. It is well known (e.g., Wansbeek and Meijer 2000) that if a regression analysis uses explanatory variables that are subject to measurement error, then estimators of regression coefficients tend to be biased towards zero ("attenuated"). Furthermore, estimators of regression coefficients of explanatory variables that are not subject to measurement error are also biased if other explanatory variables contain measurement errors. If the variances of the measurement errors are known (or can be estimated consistently), then estimated regression coefficients can be corrected to give consistent estimators. This type of estimator is called consistent adjusted least squares (CALS; see Wansbeek and Meijer 2000, chapter 5). This theory, however, has not been developed for integrating time series, and may therefore have to be adapted to this case. For nonlinear models, additional problems will emerge.

The CALS theory aims to find consistent estimators of regression coefficients. As with other methods of consistent estimation in measurement-error models (such as instrumental variables), CALS is able to do so without requiring a consistent estimator of the true (i.e., without measurement error) explanatory variable. It may be, however, that forecasts based on consistent estimators of the parameters combined with error-ridden explanatory variables are worse than forecasts based on simple OLS estimators combined with these explanatory variables; i.e., by simply ignoring the measurement error. This happens, for example, when all variables are jointly normally distributed (Wansbeek and Meijer 2000, 28–29).

These issues are important questions that need to be addressed in order to use the factor scores optimally in economic modelling and forecasting.

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