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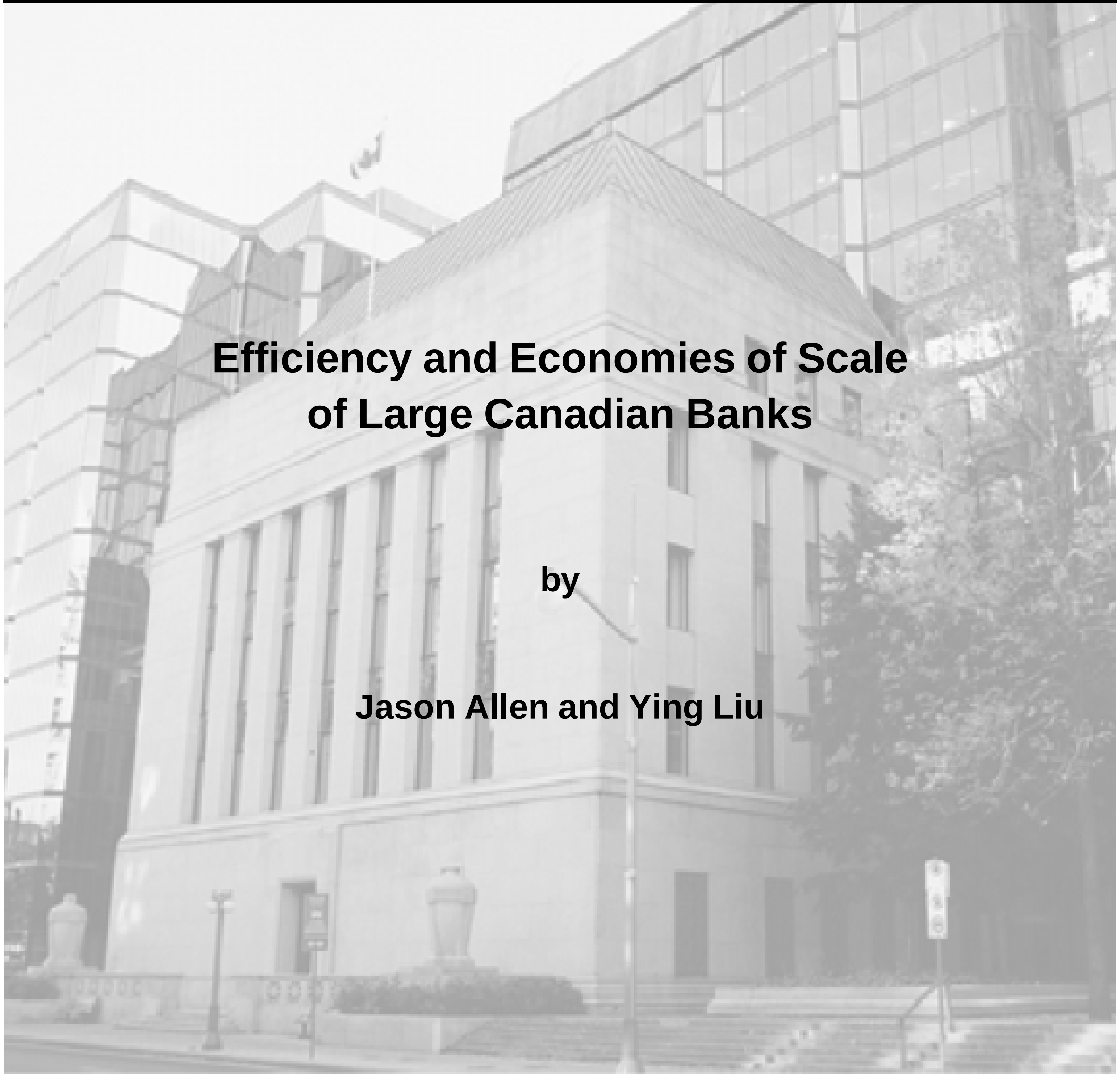

Efficiency and Economies of Scale of Large Canadian Banks

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Abstract

The authors measure the economies of scale of Canada’s six largest banks and their cost-efficiency over time. Using a unique panel data set from 1983 to 2003, they estimate pooled translog cost functions and derive measures of relative efficiency and economies of scale. The disaggregation of the data allows the authors to model Canadian banks as producing multiple outputs, including non-traditional activities. Given the long time span of the data set, they also incorporate technological and regulatory changes in the banks’ cost functions, as well as time-varying bank-specific effects. The authors’ model leads them to reject constant returns to scale. These findings suggest that there are potential scale benefits in the Canadian banking industry. The authors also find that technological and regulatory changes have had significant positive effects on the banks’ cost structure.

JEL classification: G21, D24, C33
Bank classification: Financial institutions

Résumé

Les auteurs mesurent les économies d’échelle et le rapport efficacité-coût dans le temps des six principales banques canadiennes. Pour ce faire, à partir d’un ensemble unique de données longitudinales allant de 1983 à 2003, ils estiment des fonctions de coût regroupées, de type translog, à partir desquelles ils établissent des indices d’efficience relative et d’économie d’échelle. La désagrégation des données leur permet de créer un modèle selon lequel les banques canadiennes sont associées à de multiples produits, notamment dans des branches d’activité non traditionnelles. Ils incorporent aux fonctions de coût des composantes tenant compte des changements d’ordre technologique et réglementaire intervenus durant la longue période couverte par les données analysées ainsi que de leurs effets spécifiques, variables dans le temps, sur les banques. Les auteurs sont amenés à rejeter l’hypothèse de rendements d’échelle constants, invalidée par leur modèle. Leurs conclusions donnent à penser qu’il y a place à des économies d’échelle au sein du secteur bancaire canadien. Les auteurs constatent également que les changements technologiques et réglementaires survenus ont eu une nette incidence positive sur la structure de coûts des banques.

Classification JEL : G21, D24, C33
Classification de la Banque : Institutions financières

1 Introduction

The efficiency and economies of scale of banks are two key issues in the banking literature. They can provide important insights to managers when making operational decisions and also to policy-makers in the debate on regulatory issues. This paper measures the economies of scale and efficiency in the Canadian banking industry. Using a proprietary panel data set of Canada's six largest banks from 1983Q1 to 2003Q3, we estimate pooled translog cost functions and derive measures of relative efficiency and economies of scale. Economies of scale allow us to assess statistically whether "bigger is better," based on existing technology. Relative efficiency allows us to compare banks' cost-effectiveness over time.

The disaggregation of the data is critical and allows us to model Canadian banks as producing multiple outputs. Included as bank output are non-interest-related activities such as deposit account services, security underwriting, and wealth management. Because of data limitations, non-interest activities have rarely been studied in the literature. We proxy these activities by an asset-equivalent measure of non-interest income derived in Boyd and Gertler (1994).

The analytical framework is the flexible translog cost function, assuming the intermediation approach in bank production. That is, banks use deposits, labour, and capital as inputs to produce outputs such as loans. Four econometric models are estimated: (i) a time-varying fixed-effects panel model, (ii) a stochastic cost-efficiency frontier model estimated by maximum likelihood (ML), (iii) a system of seemingly unrelated regressions (SUR) using generalized least squares (GLS), and (iv) a time-varying fixed-effects model including leads and lags of the explanatory variables (dynamic ordinary least squares, DOLS). We also include a fixed effect in the SUR model to represent inefficiency specific to individual banks. Measures of economies of scale are calculated from the derivatives of cost with respect to output. We also generate measures of relative cost-efficiency among banks. The results are robust and do not depend on a particular estimator.

Given the relatively long time-series component to our panel, we are able to identify possible technological changes over time. The term "technological change" is a broad term that includes financial innovations, changes in the competitive nature of banks, and demographically led changes in household portfolios. Freedman and Goodlet (1998) note that banks have been undergoing significant technological changes that affect the way services are provided, the instruments used to provide services, and the nature of the financial service providers. These changes include the adoption of electronic processing of transactions, the development of new instruments and products, the spread of ATMs, and internet banking and better risk-management techniques. We model technological changes in two mutually exclusive ways: (i) by imposing common technological trends on the cost function, and (ii) by allowing technological changes to affect bank-specific progress.

Another important factor that may have affected the cost structure of banks over time is regulatory change. Calmès (2004) suggests that changes to the Bank Act in 1987, 1992, and 1997 may have encouraged the trend towards direct financing; i.e., financing done at financial markets rather than through financial intermediaries. At the same time, banks have been increasingly involved in non-traditional, typically market-oriented, activities. We model the potential impact of such changes in the sources of a bank's income on its cost structure by introducing specific regulatory variables.

Our results suggest that we can reject constant returns to scale. Depending on the model and assumptions, we find that banks can reduce their average cost by 6 to 20 per cent by doubling their output. That said, our preferred model using DOLS suggests that the estimates of economies of scale are closer to 6 per cent. We also conclude that both technological and regulatory changes have had positive effects on the banks' cost structure.

This paper is organized as follows. Section 2 introduces the literature on efficiency and economies of scale in financial institutions. Section 3 presents our model. Section 4 discusses the econometric issues and the models that we estimate. Section 5 provides a detailed description of the data. Section 6 reports our estimation results. Section 7 summarizes our findings.

2 Literature Review

The study of economies of scale in financial institutions has had a long history. While studies have been done for different types of financial institutions in different countries, very few have focused on Canadian institutions. In general, most studies find only small economies of scale in a firm's cost structure. In those studies that find evidence of increasing returns to scale, the measured economies of scale seem to be stronger in small to medium-sized firms than for large firms.[1] More recent studies, however, find stronger evidence of increasing returns to scale in large U.S. commercial banks in the 1990s (Berger and Mester 1999, Stiroh 2000).

Two recent studies on Canadian financial institutions address economies of scale assuming a Cobb-Douglas cost function. Using a panel data set of 25 Canadian trust companies for the years 1985 to 1988, Breslaw and McIntosh (1997) show that the scale function of these trust companies is convex with respect to firm size. Assuming a Cournot oligopoly and using a time-series data set of the "Big Five" banks from 1976 to 1996, McIntosh (2002) finds significant increasing returns to scale among those banks.[2]

An obvious limitation of the Cobb-Douglas framework is that banks are assumed to produce one (composite) output with the same inputs and technology. This assumption is debatable (and rejected in our sample), given that banks are diversified in their business lines. The Cobb-Douglas framework also has an overly restrictive functional form. Lawrence (1989) demonstrates that the non-rejection of the Cobb-Douglas technology results from an *ad hoc* specification that excludes the possibility of multiple-product cost complementarity. In the case of British Columbia Credit Unions, Murray and White (1983) show that none of the restrictive production conditions commonly imposed by researchers using the Cobb-Douglas framework provides a valid representation of the technology of the firms they studied. Instead, the authors propose to use a translog specification that captures the heterogeneous nature of a bank's intermediation activity. Using data from 1976 to 1977, Murray and White (1983) find that most of the credit unions in the sample experience significant increasing returns to scale.

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[1] See Ferrier and Lovell (1990) for U.S. banks, Rime and Stiroh (2003) for Swiss universal banks, and Rezvanian and Mehdian (2002) for Singaporian banks.

[2] The five banks are: Royal Bank Financial Group, Bank of Montreal, Canadian Imperial Bank of Commerce, TD Bank Financial Group, and Bank of Nova Scotia.

The translog cost function was first proposed by Christensen, Jorgenson, and Lau (1971). Schmidt and Lovell (1979) show that, under the cost-minimization assumption, a firm's stochastic production frontier can be written as a cost function. Diewert and Kopp (1982) show that any frontier cost function, such as the translog function, can be derived without knowing its underlying production function. The translog specification is often used to provide a numerical efficiency value, called X-efficiency, and ranking of firms. The X-efficiency of a bank is measured as its cost level compared with that of the best-practice banks of similar size (the frontier firm), controlled for type of banking activity and the input prices it faces. Inferences regarding the scale economies of banks are drawn from the derivative of a bank's cost with respect to its output. The specification is often applied to a set of cross-sectional data on banks and estimated for several years. The parameters for economies of scale are averaged over the sample years. Berger and Humphrey (1997) provide a detailed survey of the literature.

While most studies using the translog cost function examine U.S. or European banks, there are limited studies on Canadian banks. This is because of the small number of banks and partly because of the lack of publicly available data.[3] Nathan and Neave (1989) estimate a translog cost function for a cross-section of Canadian banks. Results on economies of scale depend on whether deposits are assumed to be inputs. Our panel, however, has a long time dimension and a sufficient cross-sectional dimension to conduct a detailed study of the Canadian banking industry.

A major difficulty in dealing with larger time dimensions relative to the size of the panel is that the assumption of constant firm effects over time is most likely violated. Cornwell, Schmidt, and Sickles (1997) propose to solve the problem by replacing the coefficients on firm effects with a flexibly parameterized function of time. Applying this to eight U.S. airlines from 1970 to 1981, the authors construct an efficiency frontier for the firms and find that the firms become more efficient over time. We follow this approach to allow bank-specific effects to vary over time.

Another challenge in modelling a panel with a relatively long time dimension is that variables are likely to be non-stationary. Stationarity tests show that the variables in the cost function are indeed non-stationary. We find, however, that the translog cost function is balanced. Using the sequential limit theory developed in Phillips and Moon (1999), Kao and Chaing (2000) provide evidence that the DOLS estimator is consistent given the cointegrating model. Kao and Chaing (2000) also provide Monte Carlo evidence that DOLS outperforms competing estimators in finite samples. We apply DOLS to our panel by including leads and lags of the explanatory variables.[4]

A further contribution of this study is the inclusion of non-traditional activities. Most studies measure bank output by their traditional activities, such as loan generation and security investment. Banks, however, have been moving into non-traditional activities such as depositor services, underwriting, and wealth management. Excluding these activities will result in a misspecified cost function and potentially lead to incorrect inference about economies of scale.

Clark and Siems (2002) apply the asset-equivalent measure of off-balance-sheet activities proposed by Boyd and Gertler (1994) to measure the impact of off-balance-sheet activities on the efficiency measure of banks. They find that such activities are important determinants in explaining cost-efficiency. The idea of an asset-equivalent measure is to capitalize the bank's non-interest income to proxy the

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[3] As of July 2003, there were 17 domestic chartered banks and 51 foreign banks or subsidiaries in Canada.

[4] We thank Chihwa Kao for making his Gauss code available online.

assets required to produce such revenue. Boyd and Gertler (1994) also argue that the credit-equivalent measure proposed by the Basel Committee often underestimates the off-balance-sheet assets of banks.[5] We follow this framework.

3 Models

3.1 Cost minimization

We assume that a bank i $(i = 1,...,N)$ is a cost-minimizing entity that produces output $Q = (Q_1,...,Q_m) \in R^m_+$ using inputs $X = (X_1,...,X_k) \in R^k_+$ at prices $W = (W_1,...,W_k) \in R^k_+$ subject to a production constraint, $F(Q,X)$:[6]

$$\min C = \sum_{j=1}^{k} W_j X_j(Q,W),$$

subject in turn to the following production constraint:

$$F(Q,X) = 0.$$

Possible environmental variables and proxies for technological change are included in $G = (G_1,...,G_L) \in R^L_+$. Consistent with most of the firm-efficiency literature, we estimate a multi-product translog cost function.[7] The function is assumed to be positive for all positive prices and outputs, homogeneous of degree one, monotonic, and concave in prices. A second-order Taylor expansion around the log of output and prices gives the following cost function (all lower case variables are in logarithms):

$$\begin{aligned} c(q,w) = \alpha_0 + \sum_{l=1}^{m} \alpha_k q_l + \sum_{j=1}^{k} \beta_j w_j + \frac{1}{2} \sum_{l=1}^{m} \sum_{j=1}^{m} \sigma_{lj} q_l q_j + \\ \sum_{l=1}^{m} \sum_{j=1}^{k} \gamma_{lj} q_l w_j + \frac{1}{2} \sum_{l=1}^{k} \sum_{j=1}^{k} \delta_{lj} w_l w_j + \sum_{l=1}^{L} \theta_l G_l + \xi + \varepsilon. \end{aligned} \tag{1}$$

We do not subscript the variables across time or cross-section explicitly, because subscripts indicate the statistical method employed. The assumption regarding the distribution of ξ and ε also depends on the method. Economic theory imposes certain restrictions on the parameters: the cost function is homogeneous if $\sum_j^k \beta_j = 1$, $\sum_j^k \gamma_{lj} = 0$, and $\sum_j^k \delta_{lj} = 0$. We also impose $\sigma_{lj} = 0$, due to collinearity. Imposing these restrictions gives,

$$\log(C/W_1) = \sum_{j=2}^{k} \beta_j \log(W_j/W_1) + \sum_{l=1}^{k-1} \sum_{j=l+1}^{k} \delta_{lj} [W_l W_j - 0.5 \times (W_l^2 + W_j^2)]$$

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[5] Under the current reporting requirement, off-balance-sheet activities are allocated into four broad risk categories that carry conversion factors of 100 per cent, 50 per cent, 20 per cent, and 0 per cent.

[6] Shaffer (1993) tests for banking competition in Canada between 1965 and 1989, and he cannot reject input price-taking behaviour. Nathan and Neave (1989) find that the Canadian financial system does not exhibit monopoly power.

[7] Mitchell and Onvural (1996) show that the Fourier functional form performs better than the translog function when there is a wide variety of banks. This is clearly not the case in our sample, and thus we estimate the simpler of the two models.

$$+\sum_{l=1}^{m}\sum_{j=2}^{k}\gamma_{lj}q_l\log(W_j/W_1)+\alpha_0+\sum_{l}^{m}\alpha_l q_l+\sum_{l=1}^{L}\theta_l G_l+\xi+\varepsilon. \tag{2}$$

It is important to note that we gain estimation efficiency by including equations for the input cost share. The cost share of input j is given by S_j and is derived using Shephard's lemma. The equations for the cost share add information without adding parameters to the multivariate regression. Since shares must sum to unity, $j-1$ input share equations are specified for a system of j shares. Results are invariant to which share is dropped:

$$S_j=\frac{\partial\log(C)}{\partial\log(w_j)}=\beta_j+\sum_l\delta_{lj}w_l+\sum_l\gamma_{lj}q_l, \qquad j=1,\ldots,m. \tag{3}$$

Imposing symmetry, $\gamma_{lj}=\gamma_{jl}$, gives,

$$S_j=\beta_j+\sum_{l=2}^{m}\delta_{lj}\log(W_l/W_j)-\sum_{l=1}^{m}(\gamma_{l2}+\gamma_{l3})q_l. \tag{4}$$

3.2 Economies of scale and technological changes

One important focus in this study is the economies of scale of the Canadian banking industry. Economies of scale are measured as,

$$\zeta=\left(\sum_{l=1}^{m}\alpha_l+\sum_{l=1}^{m}\sum_{i=2}^{m}\gamma_{ij}\log(W_l/W_j)\right)^{-1}.$$

There are increasing returns to scale if $\zeta>1$, constant returns to scale if $\zeta=1$, and decreasing returns to scale if $\zeta<1$. Economies of scale inform us of the cost savings/dissavings when a bank increases its output, while keeping the output mix constant.

We think it is essential to capture technological changes over time. We achieve this in two ways. First, we assume that technological change affects the cost function directly; i.e., banks are subject to the same technological shocks over time. We proxy such shocks by including a quadratic time polynomial. The rate of technological change is given by $T^*=-\partial C/\partial t$. Change is progressive if $T^*>0$ and regressive if $T^*<0$. Since it is hard to pinpoint when banks adopt those technologies and when the effects on their cost structure are fully realized, such a non-parametric specification of technological change has the advantage of not requiring knowledge of the dates of those changes.

Second, if technological changes affect banks differently over time, then there should be time-varying effects in the bank-specific terms. Such effects will affect the measure of efficiency. To illustrate, let us redefine the translog cost function (2) as:

$$y_{it}=X_{it}^{\prime}\beta+\xi_i+u_{it}, \tag{5}$$

where $y_{it} = \log(c/w_1)$, $\xi_i = \alpha_0 + \varepsilon_i$, and u_{it}, ε_i are white noise. For the cost function, $X_{it} = [\log(W_j/W_1), (W_l W_j - 0.5 \times (W_l^2 + W_j^2)), q_l \log(W_j/W_1), q_l, G_l]'$, and $\beta = [\beta_j, \gamma_{lj}, \delta_l, \alpha_l, \theta_l]'$ for each bank i.

Assuming time-invariant inefficiency, the cost frontier intercept is given by the minimum of the firm-specific effects, $\xi_i : \hat{\alpha}_0 = \min_j(\hat{\xi}_j)$. Inefficiency is the difference between the frontier, or the "best-practice" firm, and the firm effects: $\hat{\varepsilon}_i = \hat{\xi}_i - \hat{\alpha}_0$. Such deviation from the "best-practice" firm can come from differences in management skills, human inertia, and adoption of technology.

Assuming that cost-efficiency is constant over time may seem implausible, given the long time dimension of the panel. We allow time-varying cost-efficiency. Consider a fixed-effects approach. For time-varying firm inefficiency,

$$\hat{u}_{it} = \Omega_{i1} + \Omega_{i2} t + \Omega_{i3} t^2,$$

and the time-varying fixed effect is

$$\hat{\xi}_{it} = \hat{\Omega}_{i1} + \hat{\Omega}_{i2} t + \hat{\Omega}_{i3} t^2.$$

This specification allows cost-efficiency to be time-varying as well as different for each bank. The time-varying costs frontier intercept is given by $\hat{\alpha}_t = \min_j(\hat{\xi}_{jt})$. Time-varying inefficiency is given by $\hat{\varepsilon}_{it} = \hat{\xi}_{it} - \hat{\alpha}_t$. The time-invariant case is nested if the same firm is selected for all t.

Cost-efficiency is derived as

$$CE_{it} = \exp\{-\hat{\varepsilon}_{it}\}.$$

Two components of efficiency can be distinguished: technical efficiency, the ability to obtain maximum output from a given set of inputs, and allocative efficiency, the skill to use the inputs in optimal proportions, given their respective prices and the production technology. The two can be combined to provide a measure of economic efficiency, or, when cost instead of production is considered, cost-efficiency. Measures of cost-efficiency allow us to rank the banks over time from the most to the least cost-efficient. The most efficient bank has a measure of cost-efficiency equal to one, and less efficient banks have measures below one.

4 Statistical Method

In this section, we discuss alternative ways used to estimate the cost function. The unique panel data set that we have consists of 6 banks and 83 quarterly observations. The availability of a long time series and short cross-section leads to several natural parametric estimators. In each case, the cost function is concave with respect to prices at the parameter estimates. All other theoretical restrictions are imposed prior to estimation.

4.1 Fixed-effects model

Recall equation (5), a generic unobserved-effects model. The fixed-effects model assumes that we can capture differences across banks in the intercept terms, ξ_i. Although these effects can be correlated with X_{it}, we require $E[u_{it}|X_{it},\xi_i] = 0$ for consistent estimation. An alternative assumption would be that ξ_i comes from some orthogonal distribution. In our case, the cross-section draws are not random and it would be inappropriate to estimate the model using a random-effects estimator.[8] The fixed-effects estimator of β and the individual effect $(\xi_i + \mu_{it})$ are consistent. The large time-series component allows us to evaluate cost inefficiencies apart from the statistical noise.

4.2 Stochastic frontier model

The stochastic frontier method decomposes the fixed effect in equation (5) into a constant α_0 and a firm-specific inefficiency variable, ε_i. This framework allows us to calculate the efficiency level of each Canadian bank relative to the "best-practice" firm in the sample, rather than to the absolutely efficient firm. Inefficiencies are assumed to follow the truncated normal distribution, while the random errors follow a standard normal distribution. The logic is that the inefficiencies must have a truncated distribution because inefficiencies cannot be negative. Both the inefficiencies and the errors are assumed to be orthogonal to the explanatory variables in the model. Relative to the fixed-effects estimator, the truncated normal assumption may be overly restrictive, because it clusters near full efficiency. We estimate this model using maximum likelihood (ML), which is consistent when $u_{it} \sim iidN(0,\sigma_u^2)$, $\xi_i \sim iidN^+(\mu,\sigma_\xi^2)$, and u_{it} and ξ_i are independently distributed.

The log likelihood is given by:

$$\log L = -\frac{I(T-1)}{2}\log\sigma_u^2 - \frac{I}{2}\log(\sigma_u^2 + T\sigma_\xi^2) + \sum_i \log\left[1 - \Phi\left(-\frac{\mu_i^*}{\sigma^*}\right)\right]$$

$$-\left(\frac{e'e}{2\sigma_u^2}\right) + \frac{1}{2}\sum_i\left(\frac{\mu_i^*}{\sigma^*}\right)^2,$$

where $\mu_i^* = \frac{T\sigma_\xi\bar{e_i}}{(\sigma_u^2+T\sigma_\xi^2)}$, $\sigma^* = \left(\frac{\sigma_\xi^2\sigma_u^2}{(\sigma_u^2+T\sigma_\xi^2)}\right)^{1/2}$, and $\bar{e_i} = (1/T)\sum_i(\xi_i + u_{it})$. See Kumbhakar (2002) for details.

4.3 Seemingly unrelated regressions (SUR)

The cost specifications so far have not taken into account the share equations. Imposing cross-equation restrictions can improve estimator efficiency. The system of equations is estimated using GLS:

$$C_{it} = A_{it}\Theta + \xi_i, + u_{it}$$

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[8] For completeness, we did estimate the model assuming random effects. The Breusch and Pagan Lagrange multiplier test for random effects rejects that assumption.

$$\begin{aligned} S_{Kit} &= B_{it}\Gamma + \eta_{Ki} + \nu_{Kit} \\ S_{Lit} &= D_{it}\Phi + \eta_{Li} + \nu_{Lit} \quad i = 1,\dots,N \quad t = 1,\dots,T. \end{aligned} \tag{6}$$

The share equation with SUR estimation provides sufficient structure to identify the coefficients. Consistent estimation of each equation separately leads to poorly identified parameter estimates, due to multicollinearity. As expected, the assumption of zero contemporaneous correlation is strongly rejected.

4.4 Dynamic ordinary least squares (DOLS)

Given the long time dimension of our panel, we investigate the stationarity of the data with unit root tests. As McIntosh (2002) notes, valid inference of such a long panel data set requires stationarity. The test statistics do not reject the null hypothesis that the variables are non-stationary. By conducting unit root tests on the residuals from the cost function (2), we do find, however, that the cost function is balanced. Table 1 reports the Fisher test and modified augmented Dickey-Fuller (MADF) test, introduced by Maddala and Wu (1999) and Sarno and Taylor (1998), respectively. The tests were performed on cost function (2), including a time polynomial.

Table 1
Unit Root Tests on the Cost Function Residuals

	Fisher test	MADF
Test-statistic	26.07	47.00
p-value	0.0105	0.0000

Note: The Fisher test uses the least-squares estimator and an augmented Dickey-Fuller test with four lags and is distributed χ^2_{12}. Under the null hypothesis, each cross-section is non-stationary. The MADF test also has a null hypothesis of non-stationarity. Estimation is done using the SUR estimator, and the distribution of the test statistic is achieved via simulation.

The hypothesis of no-cointegration is rejected at the 5 per cent significance level. In this case, Kao and Chaing (2000) argue that DOLS is the best estimator to use. Consider equation (5) and that the regressors follow a common unit root process:

$$X_{it} = X_{it-1} + \nu_{it}.$$

We rewrite equation (5) to estimate β consistently:

$$y_{it} = \xi_i + X'_{it}\beta + \sum_{j=-q}^{q} c_{ij}\Delta X_{it+j} + \omega_{it}. \tag{7}$$

The DOLS estimator is:

$$\hat{\beta}_{DOLS} = \Big[\sum_{i=1}^{N}\sum_{t=1}^{T}(X_{it} - \overline{X}_i)(X_{it} - \overline{X}_i)'\Big]^{-1}\Big[\sum_{i=1}^{N}\sum_{t=1}^{T}(X_{it} - \overline{X}_i)(y_{it} - \overline{y}_i)\Big],$$

where $\overline{X_i} = 1/T \sum_t X_{it}$ and $\overline{y_i} = 1/T \sum_t y_{it}$. Kao and Chaing (2000) provide details on the limiting distribution and Wald test for parameter restrictions.

We present four estimation techniques: a fixed-effects model estimated by least squares; a single-equation stochastic frontier model and a SUR, both estimated by maximum likelihood; and a fixed-effects model estimated by DOLS. Each approach has its advantages and disadvantages. The fixed-effects model has the advantage that only weak distributional assumptions are necessary to estimate cost-efficiency consistently. However, the estimates are less efficient than a likelihood-based approach. Maximum likelihood makes strong distributional assumptions about cost-efficiency and statistical noise, and with these assumptions we gain efficiency. Maximum likelihood is usually less robust to model misspecification than least squares. While the point estimates from these three models are valid, we cannot interpret their statistical significance, because of non-stationarity in the data. The fourth method, DOLS, produces consistent estimates, the standard errors of which are valid. The DOLS estimator also makes endogeneity a second-order effect. The interpretation of our findings is based on all four models.

5 Data

The data are quarterly observations of Canada's six largest banks starting from 1983Q1 to 2003Q3, deflated by the GDP deflator (1997 = 100). The number of banks is limited by data availability. The six banks are: Royal Bank Financial Group, Bank of Montreal, Canadian Imperial Bank of Commerce, TD Bank Financial Group, Bank of Nova Scotia, and National Bank. They are the only banks for which data are available in the entire sample period. The majority of the rest of the domestic chartered banks did not start reporting until after 2000. The Big Six account for approximately 90 per cent of the Canadian banking industry in terms of total assets. The data set is from the chartered banks' consolidated monthly balance sheet and quarterly consolidated statement of income, collected by the Office of the Superintendent of Financial Institutions (OSFI). The consolidated monthly balance sheet data at the aggregate level are published in Tables C1 and C2 in the Bank of Canada *Banking and Financial Statistics*. Large categories of the consolidated statement of income at the aggregate level are available in Table K2 in the same publication with an annual frequency. Disaggregate data are confidential. All balance sheet data are end-of-month values and are converted to quarterly series by taking the quarterly average. Appendix A provides a detailed definition of all variables used in this study.

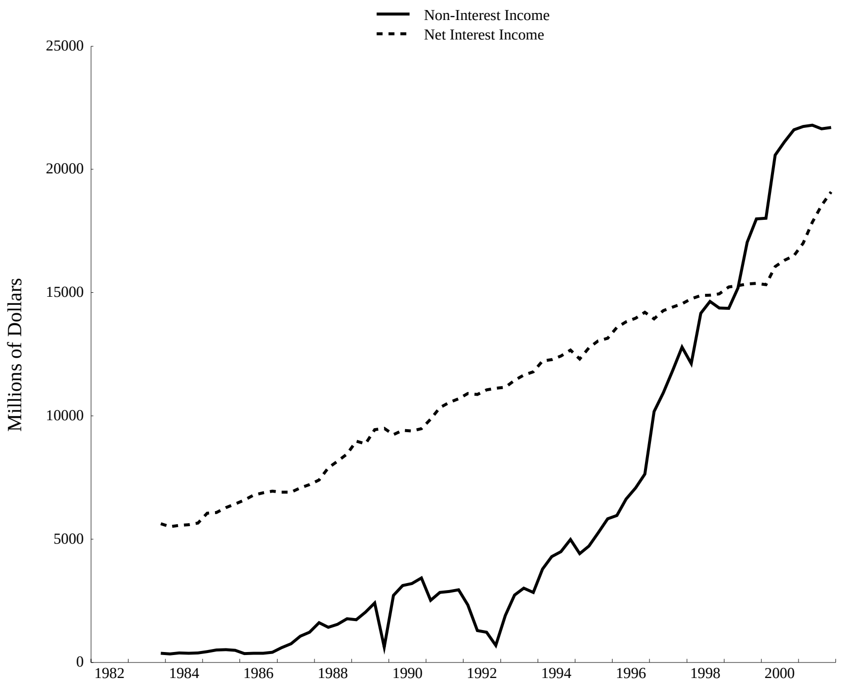
The definition of a bank's inputs and outputs is a matter of ongoing debate. We have opted for the intermediation approach. It is commonly used in the conventional cost-function literature. In the intermediation approach, a bank is assumed to use labour, capital, and deposits to produce earning assets. Deposits are treated as an input. On the other hand, the production approach postulates that banks also provide value-added in their deposit services. Deposits are treated as an output under the production approach.

There are three input prices. L is the hourly wage of a bank's full-time equivalent employee. K is capital cost, measured by the expense on premises, and computer and equipment divided by the total stock of premises and fixed assets on the bank's balance sheet. *D* is the price of deposits, measured by the total interest expense on total deposits divided by total deposits.

We identify five output categories: Y1, consumer loans; Y2, non-mortgage loans; Y3, mortgage loans; Y4, security investment; and Y5, non-traditional banking activities. The first four categories are taken from the asset side of a bank's balance sheet.[9]

Non-traditional banking activities are often overlooked in the literature on bank efficiency and scale economies. Non-interest-related activities include depositor services, underwriting, foreign exchange trading, and wealth management. Figure 1 shows that non-interest income has increased substantially since the late 1980s and has exceeded net interest income. Banks have been shifting away from traditional lending and investment activities to non-traditional activities, a trend that is also observed in U.S. banks (see Stiroh 2000).

Figure 1. Canadian Chartered Bank Total Net Interest Income and Non-Interest Income



Non-traditional activities can be divided into three categories, according to their underlying asset or liability position. The first category concerns on-balance-sheet assets that are captured in the output measures noted above. Examples include mortgage loan fees, gains and losses from trading and investment activities, and insurance income. The second category concerns the liability side of the bal-

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[9] All assets are reported in book value, except securities that are categorized under the trading account.

ance sheet, such as deposit account fees and payroll processing fees. Because of data limitations, the latter type of output items are omitted in studies using the intermediary approach. The third category concerns off-balance-sheet activities like securities underwriting, wealth management, the provision of loan guarantees, and letters of credit. These activities are also unavailable under the conventional approach to measuring bank output.

Rogers (1998) suggests that non-interest income be included in the cost function as a measure of non-traditional activities. The resulting cost function, however, mixes stocks (asset) and flows (revenue). A more internally consistent approach is introduced by Boyd and Gertler (1994). They introduce an asset-equivalent measure (AEM) of these non-traditional activities. Boyd and Gertler (1994) assume that all non-interest income is generated from off-balance-sheet assets. Assuming that these non-traditional activities yield the same rate of return on assets (ROA) as traditional activities, the assets that are required to produce non-interest income can be calculated by dividing non-interest income by the ROA of traditional activities. Bank profits can be defined as follows:

$$\pi = II - IE - PROV - NE + NII, \tag{8}$$

where II is total interest income; IE is total interest expense; $PROV$ is loan-loss provisioning; NE is non-interest expense; and NII is non-interest income. Boyd and Gertler (1994) assume that (i) non-interest income is generated by some hypothetical off-balance-sheet asset, A_o, and that (ii) using the same capital and liabilities, A_o generates the same rate of return as on-balance-sheet asset A_b. The AEM of non-traditional activities is given by,

$$AEM = A_o = A_b[NII/II - IE - PROV]. \tag{9}$$

This measure can be viewed as the hypothetical asset holdings that would be required to generate non-interest income. It has the convenience of using available data and it is easy to understand. There are some limitations. AEM includes elements that have been captured by other measures of output. As noted earlier, some components of non-interest income are generated from on-balance-sheet assets. Ideally, we would like to be able to subtract those components from non-interest income. Unfortunately, disaggregate data of non-interest income are available only from 1997 onwards.[10] We consider this the best available proxy for such activities. A second deficiency is the assumption that off-balance-sheet assets yield the same rate of return as on-balance-sheet assets. This ignores the fact that some off-balance-sheet activities, such as derivatives, may be used for hedging on- or off-balance-sheet risks and therefore may not yield the same profitability as on-balance-sheet assets.

A bank's cost function can be influenced by exogenous factors, such as changes in the regulatory environment. Regulatory changes may not have been aiming at reducing costs or improving efficiency. Rather, they are often the product of a convolution of forces, such as technological advances, demographic changes, and global trends. Therefore, changing the regulatory environment in which banks operate may not have helped banks in their cost-minimizing/profit-maximizing objectives.

Three notable changes to the Bank Act took place in our sample period. The most significant change occurred in 1987, when Canadian banks were permitted to invest in corporate securities, as

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[10] Non-interest income generated from on-balance-sheet assets accounts for approximately 20 per cent of total non-interest income from 1997 to 2003.

well as distribute government bonds. Banks are allowed to purchase control of investment dealers and invest in the securities business. As a result, banks have substantially increased their financial-market-based activities. On the demand side, as bank customers began to invest in the financial market directly through their banks, the amount of direct financing (for example, financing done through financial markets rather than through financial intermediaries) also increased.

In 1992, banks were given the right to establish or acquire trust companies in Canada. In subsequent years, the major banks bought most of the trust companies in Canada. They were also allowed to offer a number of in-house activities, such as portfolio management and investment advice. These changes may have attracted a larger fraction of depositors to invest in financial markets directly through their banks. The 1992 amendments were updated and refined in 1997.

Figure 1 shows a gradual launch of the banks' non-interest income in 1987, reflecting a trend away from loan-oriented activities. This trend grew after the amendments in 1992 and continued throughout the 1990s.

Besides regulatory changes that permitted banks to diversify their business mix, developments in legal reserve or capital requirements that aimed to ensure the financial soundness of banks may have affected the banks' cost structure. In 1989, Canadian banks adopted the minimum capital requirement proposed by the Basel Committee on Banking Supervision. This may have occasionally affected a bank's output decisions if the capital requirements were binding. In addition, the removal of the legal reserve requirement in 1991, and the complete phasing out of reserve requirements by banks in 1994, likely had an impact on the banks' production decisions.

To investigate whether such institutional changes in a bank's revenue source would have a significant statistical impact on a bank's cost structure, we examine six regulatory dummies in our cost function: 1987Q2, 1989Q1, 1991Q1, 1992Q1, 1994Q1, and 1997Q1. The dummies are zero before these dates and one afterwards.

6 Findings

The translog cost function (2) is first estimated by fixed-effects GLS and by single-equation ML. Then the system of equations with the share equations (4) is estimated by SUR. Finally, the cost function with the lags and leads of the explanatory variables (equation (7)) is estimated by DOLS. For each method, we estimate two models. The first model, labelled Model REG, includes statistically significant regulatory changes. The second model, labelled Model T, includes a time polynomial, which is assumed to proxy technological change.[11] Complete parameter estimates of each model are provided in Appendix B.

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[11] When both types of dummies are included in the model, the cost function is non-concave everywhere.

6.1 Model evaluation

Before we discuss the results for economies of scale and relative efficiency, we conduct two model evaluation checks. We first check the sense of the own- and cross-price elasticity estimates of the cost function. Table 2 reports the own- and cross-price elasticities for the SUR specification. All own-price elasticities are negative and their absolute values are less than one, implying that all inputs are price-inelastic. Capital has the highest own-price elasticity among the three inputs, which is interesting because capital expense makes up the smallest share of input. Deposits, by far the largest component in the input mix, have the lowest price sensitivity. This is reasonable, because banks are likely to shift any price changes in their sources of funding to their customers by charging a higher rate of interest on loans or demanding a higher rate of return on their securities. Changes in capital expenses, however, may be much harder to absorb. The cross-price elasticities between labour and capital, and between labour and deposits, are roughly equal, which suggests a similar substitutability between the two pairs of inputs. The cross-price elasticity between capital and deposits is negative but statistically insignificant at conventional levels.

Table 2
Own- and Cross-Price Elasticities

	Capital	Labour	Deposits
Capital	-0.8709 (0.0159)		
Labour	0.6862 (0.1241)	-0.8000 (0.0970)	
Deposits	-0.2746 (0.2862)	0.7709 (0.1442)	-0.3391 (0.1233)

Note: Standard errors are in parentheses.

The second check of the model specification tests restrictions on the production structure. Table 3 shows the results of testing restrictions of homotheticity and homogeneity against the unrestricted cost function. The production function is homothetic if $\gamma_{lj} = 0, \forall\, l, j$, and homogeneous of degree one if $\sum_l \alpha_l = 1$. The homothetic or homogeneous production structure restrictions typically imposed by researchers using a Cobb-Douglas framework are strongly rejected for the Canadian banking sector.[12]

6.2 Economies of scale

Results on economies of scale are reported in Table 4. We test the null hypothesis of constant returns to scale (CRS) and present test statistics. We report only the *p*-values of the test statistics for the DOLS estimator. The likelihood ratio test of CRS is distributed chi-squared with one degree of freedom under the null hypothesis. Overall, the results from all eight models are in favour of increasing returns to scale. All eight economies-of-scale measures (ζ) are greater than one and statistically significantly different from one. The economies-of-scale measures are smaller under Model T than under Model REG. Evaluated at the sample mean, the measured economies of scale under Model T range between 6 per cent and 13 per cent. That is, a 1 per cent increase in each of the five outputs simultaneously will raise production costs by 0.87 per cent to 0.94 per cent, depending on the methodology. The implied

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[12] We report results only for the stochastic frontier model. Results for the fixed-effects model are similar.

Table 3
Tests of Production Structure

Model	Log likelihood	Test statistic	Degrees of freedom	*P*-value
Unrestricted model:				
Model REG	999.56		31	
Model T	1021.96		31	
Homotheticity:				
Model REG	886.31	226.38	16	0.0000
Model T	901.21	241.50	16	0.0000
Homogeneity:				
Model REG	978.95	41.22	24	0.0000
Model T	990.28	63.36	24	0.0000

cost savings are even higher under Model REG, ranging between 6 and 20 per cent. Since only the statistical significances of the DOLS estimates are valid, we place more emphasis on those estimates. The DOLS estimates for both Model REG and Model T are around 6.5 per cent. This magnitude is larger than those typically found in the literature on U.S. banks and smaller than those of McIntosh (2002) for Canadian banks in a different model.

Table 4
Economies of Scale

Model	ζ	$H_0 : \zeta = 1$	
	Statistic	Statistic	*P*-value
Model REG			
FE	1.197	63.52	
MLE	1.176	50.08	
SUR	1.181	67.47	
DOLS	1.065	5.79	0.0161
Model T			
FE	1.126	19.73	
MLE	1.083	7.50	
SUR	1.064	9.48	
DOLS	1.061	5.21	0.0225

Note: The restriction imposed on equation (2) is actually $\zeta^{-1} = 1$ and $\sum_j \delta_{lj} = 0 \ \forall \ l$, since returns to scale is defined as $\frac{\partial C}{\partial q_l} = \sum_l \alpha_l + \sum\sum \delta_{lj} \log(\bar{W}_j / \bar{W}_1)$, where $\bar{\cdot}$ is the sample mean. We report only *p*-values for DOLS because of the non-stationary data.

6.3 Technological and regulatory changes

Technological change is proxied by the change in costs with respect to time (T^*). Tests regarding T^* are conducted using likelihood-ratio tests under the null hypothesis that $T^* = 0$. We find significant evidence of increasing cost-efficiency, on the order of 1 per cent per quarter over the sample period in all three models.

We include six regulatory dummies in our cost function: 1987, 1989, 1991, 1992, 1994, and 1997. As expected, the dummies for 1987 and 1997 are highly significant in the cost function, which suggests that regulatory changes to the Bank Act in those years, to allow banks to be more diversified, did have an impact on the banks' cost structure. The other dummies, however, were not significant, and they are not included in the regressions. All models have negative estimated coefficients for the dummies, which implies that regulatory changes led to a decrease in costs. The results are similar across models, suggesting roughly a 5 per cent cost savings following either regulatory change.

These particular results should be taken with some caution. We are unable to identify the separate effects of technological and regulatory change in a nested model. The individual estimates are therefore upper-bounds. Taken together, however, the results do suggest that technological and regulatory changes have played an important role in banking.

6.4 Relative efficiency

Measures of time-invariant and time-varying relative efficiency are calculated from the fixed-effects model and stochastic frontier model for the Big Six Canadian banks. For reasons of confidentiality, we cannot provide point estimates. However, the range of inefficiency of the six banks is approximately 20 per cent, and the average is about 10 per cent. These results are similar to those found in the literature on U.S. banks and bank holding companies.

Our calculations of time-varying cost-efficiency from the stochastic frontier model are almost identical to those from the fixed-effects model. Overall, there is little change in the ranking of relative efficiency in our sample period. The average of the measures of inefficiency is very similar to those from the time-invariant model. The dispersion of inefficiency between banks seems to have narrowed over time. That said, there is some change in the ranking of banks based on cost-efficiency that we would not have captured with a time-invariant model.

7 Conclusion

We have applied the flexible translog cost-function framework to study the cost-efficiency and economies of scale of Canada's six largest banks. Using a unique panel data set from 1983 to 2003, we estimated four econometric models based on this framework. Given the long time dimension of the data set, we added a time trend and a time trend squared in the cost function to capture any technological change over time. Our results show that banks have experienced technological progress and that regulatory changes have helped to reduce the production cost of banks.

Overall, we reject the hypothesis of constant returns to scale. Depending on the model and assumptions, we find that banks can enjoy cost savings of 6 to 20 per cent by increasing their scale of production, while our preferred model using DOLS suggests that the estimates of economies of scale are closer to 6 per cent. This finding is similar to recent studies of large U.S. banks, although our measure of economies is somewhat larger. Our measures are smaller than those by McIntosh (2002) in his study of Canadian banks.

Measures of efficiency are derived from the efficiency frontier estimated by the stochastic frontier model. On average, the inefficiency of Canadian banks is around 10 per cent, close to what is typically found in the literature on U.S. banks using the cost-function approach. The ranking of efficiency also suggests that larger banks seem to be more cost-efficient than smaller banks. Given that scale economies are already accounted for in our model, such heterogeneous effects may derive from differences in other factors, such as management skills and the adoption of technology. Our time-varying fixed-effects panel specification allows us to trace the changes in efficiency levels over time. The results suggest that there has been some change in the relative efficiency level of banks, although the relative dispersion of efficiency among banks seems to have narrowed.

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Appendix A: Data Description

Table A1: Data Description

Variable Definitions		
Y1	Consumer loans	Dollar value of personal loans for non-business purposes
Y2	Non-mortgage loans	Dollar value of secured call and other loans to investment dealers and brokers + loans to regulated financial institutions + loans to domestic and foreign governments + lease receivables + reverse repurchase agreements + loans to individuals and others for business purposes
Y3	Mortgage loans	Dollar value of residential and non-residential mortgage loans
Y4	Other	Dollar value of other financial assets on a bank's balance sheet
Y5	OBS	Asset-equivalent measure of off-balance-sheet activities
L	Price of labour	Total salaries, pensions, and other staff benefits divided by the number of full-time equivalent employees and hours in a year
K	Price of capital	Rental expense on real estate and depreciation on premises, furniture, fixture, computer and equipment divided by total stock of land, buildings, and equipment, less accumulated depreciation
D	Price of deposits	Total interest expense on deposits divided by the total dollar amount of deposits
C	Total costs	Interest cost + labour expenses + capital costs

Appendix B: Parameter Estimates of the Models

Table B1: Fixed Effects

	Model REG		Model T	
Variable	Coefficient	Std. error	Coefficient	Std. error
α_1	-1.37128*	(0.54683)	-1.09263*	(0.52506)
α_2	3.13151**	(0.4852)	2.59261**	(0.46146)
α_3	-0.92259*	(0.35976)	-1.28914**	(0.34680)
α_4	0.38785	(0.61813)	0.56971	(0.59401)
α_5	-0.56202†	(0.29758)	-0.18192	(0.28848)
β_2	4.20985**	(0.82177)	4.15805**	(0.79775)
β_3	-0.20342	(0.30928)	-0.65401*	(0.29332)
δ_{12}	0.33198**	(0.05356)	0.33826**	(0.05099)
δ_{13}	-0.17716**	(0.02256)	-0.18761**	(0.02154)
δ_{23}	0.03414	(0.02387)	0.04373†	(0.02266)
γ_{12}	0.12243**	(0.0456)	0.09445*	(0.04385)
γ_{13}	-0.04647†	(0.02543)	-0.07026**	(0.02379)
γ_{22}	-0.21692**	(0.0404)	-0.18215**	(0.03829)
γ_{23}	0.14395**	(0.01901)	0.11644**	(0.01948)
γ_{32}	0.07832**	(0.02987)	0.12281**	(0.02894)
γ_{33}	-0.09132**	(0.01827)	-0.07344**	(0.01781)
γ_{42}	-0.0243	(0.0517)	-0.03226	(0.04965)
γ_{43}	-0.04092	(0.02559)	0.00149	(0.0248)
γ_{52}	0.06105*	(0.02389)	0.03021	(0.0232)
γ_{53}	0.06413**	(0.01625)	0.06698**	(0.01552)
α_0	-20.84538*	(8.50474)	-20.92548*	(8.20029)
D1	-0.04903**	(0.01065)		
D2	-0.05208**	(0.01087)		
θ_1			-0.00944**	(0.00102)
θ_2			0.00005**	(0.00001)

N	498	498
R^2	0.9907	0.9914
$F_{(27,470)}$	2265.9	2464.4

Note: **, *, † denote significance at the 1, 5, and 10 per cent levels, respectively. Standard errors are in parentheses.

Table B2: Stochastic Frontier Model

	Model REG		Model T	
Variable	Coefficient	Std. error	Coefficient	Std. error
α_1	-1.3446*	(0.53479)	-1.01653*	(0.51572)
α_2	3.22947**	(0.47598)	2.73162**	(0.45644)
α_3	-0.89810*	(0.35182)	-1.31962**	(0.33982)
α_4	0.44000	(0.60459)	0.59944	(0.58132)
α_5	-0.62384*	(0.29171)	-0.21008	(0.2821)
β_2	4.25144**	(0.80359)	4.12266**	(0.77941)
β_3	-0.23031	(0.30256)	-0.70444*	(0.28788)
δ_{12}	0.31716**	(0.05271)	0.31407**	(0.05082)
δ_{13}	-0.17143**	(0.02217)	-0.17941**	(0.02136)
δ_{23}	0.03165	(0.02336)	0.03974†	(0.02226)
γ_{12}	0.12183**	(0.04459)	0.09109*	(0.04299)
γ_{13}	-0.03984	(0.02499)	-0.05723*	(0.02401)
γ_{22}	-0.22353**	(0.03958)	-0.19136**	(0.03774)
γ_{23}	0.14899**	(0.0187)	0.12593**	(0.0194)
γ_{32}	0.07591**	(0.02922)	0.12573**	(0.02836)
γ_{33}	-0.09451**	(0.01791)	-0.08028**	(0.0177)
γ_{42}	-0.02919	(0.05057)	-0.03567	(0.04858)
γ_{43}	-0.04268†	(0.02503)	-0.00366	(0.0244)
γ_{52}	0.06525**	(0.02339)	0.03103	(0.02269)
γ_{53}	0.05976**	(0.01599)	0.0605**	(0.0153)
α_0	-23.08205**	(8.34915)	-23.49573**	(8.10225)
D1	-0.0491**	(0.01042)		
D2	-0.05591**	(0.01075)		
θ_1			-0.00987**	(0.001)
θ_2			0.00005**	(0.00001)
N	498		498	
Log-likelihood	938		961.3	
$\chi^2_{(22)}$	51834		55467	

Note: **, *, † denote significance at the 1, 5, and 10 per cent levels, respectively. Standard errors are in parentheses.

Table B3: SUR: Model REG

Variable	Equations		
	Equation $\log(c)$	Equation S_k	Equation S_l
D_1	-0.09742** (0.01046)		
D_2	-0.09887** (0.01094)		
α_1	-0.23412** (0.08385)		
α_2	0.42090** (0.07851)		
α_3	0.05055 (0.05574)		
α_4	0.36156** (0.10125)		
α_5	0.17454** (0.05351)		
β_1		-0.24733** (0.03077)	
β_2	-1.21306** (0.06539)		-1.21306** (0.06539)
β_3	2.46039** (0.07454)		
δ_{12}	0.03101** (0.00171)	0.03101** (0.00171)	0.03101** (0.00171)
δ_{13}	-0.04859** (0.00132)	-0.04859** (0.00132)	
δ_{23}	-0.12333** (0.00305)		-0.12333** (0.00305)
γ_{11}		0.00335 (0.00286)	
γ_{12}	0.03074** (0.00597)		0.03074** (0.00597)
γ_{13}	-0.03408** (0.00704)		
γ_{21}		-0.0102** (0.00261)	
γ_{22}	-0.00618 (0.0054)		-0.00618 (0.0054)
γ_{23}	0.01637** (0.00625)		
γ_{31}		0.01437**	

Continued

Table B3 (concluded)

Variable	Equations		
		(0.00192)	
γ_{32}	0.00496 (0.004)		0.00496 (0.004)
γ_{33}	-0.01933** (0.00472)		
γ_{41}		0.0036 (0.00352)	
γ_{42}	-0.01254† (0.00725)		-0.01254† (0.00725)
γ_{43}	0.00894 (0.00839)		
γ_{51}		-0.01593** (0.0018)	
γ_{52}	-0.01085** (0.0037)		-0.01085** (0.0037)
γ_{53}	0.02678** (0.00442)		
Log-likelihood		3656.7	

Note: **, *, † denote significance at the 1, 5, and 10 per cent levels, respectively. Standard errors are in parentheses.

Table B4: DOLS

	Model REG		Model T	
Variable	Coefficient	Std. error	Coefficient	Std. error
α_1	-0.5604	(0.53812)	0.5190	(0.56530)
α_2	3.2680^{**}	(0.46244)	3.7936^{**}	(0.47331)
α_3	-1.0407^{**}	(0.38237)	-2.4795^{**}	(0.40343)
α_4	1.9226^{**}	(0.54690)	0.0238	(0.57627)
α_5	-0.8576^{**}	(0.27646)	0.2637	(0.28810)
β_2	5.0629^{**}	(0.77108)	7.8579^{**}	(0.81019)
β_3	$0.4920^{\dagger}$	(0.31454)	-1.1258^{**}	(0.33007)
δ_{12}	0.1670^{**}	(0.05052)	0.3931^{**}	(0.05318)
δ_{13}	-0.1837^{**}	(0.02105)	-0.2334^{**}	(0.02217)
δ_{23}	-0.0541^{*}	(0.02636)	0.0472^{*}	(0.02783)
γ_{12}	0.0455	(0.04408)	-0.0365	(0.04639)
γ_{13}	-0.0901^{**}	(0.02671)	-0.0193	(0.02774)
γ_{22}	-0.2413^{**}	(0.03821)	-0.2971^{**}	(0.03926)
γ_{23}	0.0903^{**}	(0.01904)	0.0767^{**}	(0.01972)
γ_{32}	0.0939^{**}	(0.03130)	0.2185^{**}	(0.03301)
γ_{33}	-0.0844^{**}	(0.02277)	-0.1160^{**}	(0.02394)
γ_{42}	-0.1333^{**}	(0.04600)	0.0167	(0.04846)
γ_{43}	0.0577^{**}	(0.02314)	-0.0247	(0.02425)
γ_{52}	0.0894^{**}	(0.02221)	0.0066	(0.02300)
γ_{53}	0.0764^{**}	(0.01438)	0.1446^{**}	(0.01498)
D1	-0.1014^{**}	(0.00930)		
D2	-0.1050^{**}	(0.01056)		
θ_1			-0.0128^{**}	(0.01493)
θ_2			0.0001^{**}	(0.02314)

Note: **, *, † denote significance at the 1, 5, and 10 per cent levels, respectively. Standard errors are in parentheses.

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