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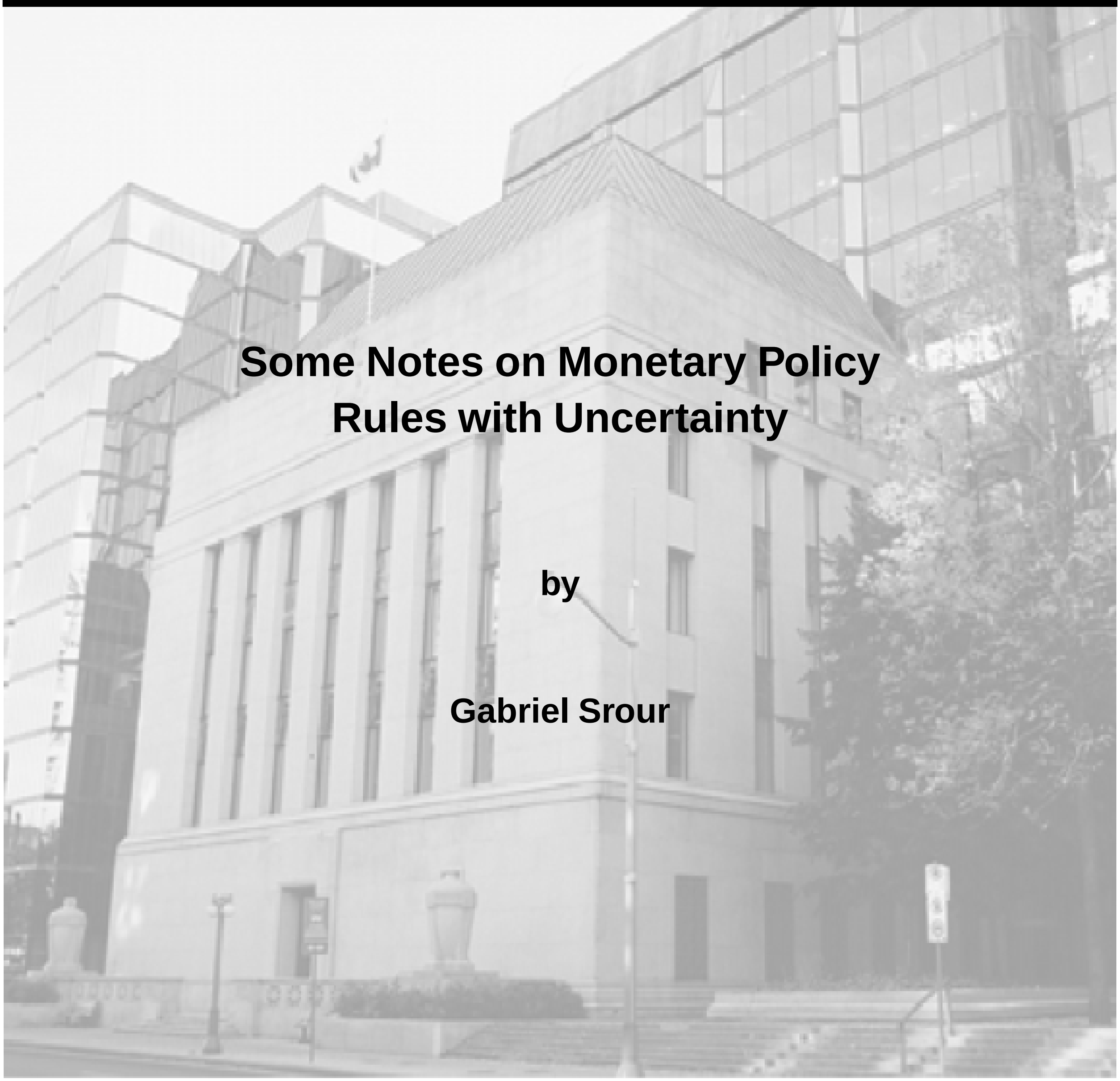

Some Notes on Monetary Policy Rules with Uncertainty

by

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Abstract

The author explores the role that Taylor-type rules can play in monetary policy, given the degree of uncertainty in the economy. The optimal rule is derived from a simple infinite-horizon model of the monetary transmission mechanism, with only additive uncertainty. The author then examines how this rule ought to be modified when there is uncertainty about the parameters, the time lags, and the nature of shocks. Quantitative evaluations are subsequently provided. In particular, it is shown that if the degree of persistence of inflation in the Phillips curve is not high, a simple rule such as the original Taylor rule that offsets demand shocks and puts a relatively small weight on inflation shocks may be an appropriate benchmark for the conduct of monetary policy. Conversely, it is argued that if the degree of persistence of inflation in the Phillips curve is high, then finding a Taylor-type rule that can act as a benchmark for monetary policy is likely to be difficult.

JEL classification: E52
Bank classification: Uncertainty and monetary policy

Résumé

L'auteur explore le rôle que les règles à la Taylor peuvent jouer dans la conduite de la politique monétaire compte tenu du degré d'incertitude de l'économie. Il tire la règle optimale d'un modèle simple à horizon infini du mécanisme de transmission monétaire, où l'incertitude intervient sous une forme additive. Il examine comment cette règle doit être modifiée quand il y a incertitude au sujet des paramètres, des retards et de la nature des chocs et procède ensuite à des évaluations quantitatives de la nouvelle règle. L'auteur montre en particulier que, si le degré de persistance de l'inflation dans la courbe de Phillips n'est pas élevé, une règle simple telle que la règle initiale de Taylor — où la politique monétaire contrecarre l'effet des chocs de demande mais réagit relativement peu aux chocs d'inflation — pourrait servir de modèle de référence pour la conduite de cette politique. À l'inverse, il pourrait être difficile de trouver une règle à la Taylor pouvant jouer ce rôle si l'inflation est très persistante.

Classification JEL : E52
Classification de la Banque : Incertitude et politique monétaire

1. Introduction

While a consensus has yet to be reached regarding the conduct of monetary policy, there is now general agreement with Friedman (1960) that there is too much uncertainty about the economy to fine-tune monetary policy responses to every shock. At the same time, fixing the growth rate of the money supply once and for all, as Friedman advocated, is widely seen as unrealistic. Rather, attention in recent years has turned towards simple reaction functions, such as the well-known Taylor rules (Taylor 1998), which specify how the instrument of policy ought to be adjusted when certain state variables deviate from equilibrium. These types of rules can be viewed as a compromise in that while they prescribe exactly how the policy instrument ought to respond to certain shocks, they also allow the responses to differ for different kinds of shocks. Still, one may remain skeptical about the use of such rules in practice, either because they would have to involve too many variables, i.e., too much fine-tuning, or they would be overly rigid. This paper will explore what role, if any, Taylor-type rules can play, given the degree of uncertainty in the economy.

Clearly, some type of rule is helpful and perhaps necessary as a guide and benchmark for the conduct of monetary policy. A rule promotes credibility and facilitates communication, both externally with the public, and internally within a central bank during the decision-making process. It conveys to the public not just the objectives of monetary policy, but also the way in which the objectives will be achieved while providing policy-makers with a reference point.

A large body of work has sought to derive efficient, simple Taylor-type rules within the context of large macroeconometric models of the economy.[1] Our approach is different in that we examine how policy should respond to primary shocks in the context of small models that incorporate a few broad stylized facts about the transmission mechanism. This approach implicitly concedes that, at best, a rule can act as a benchmark for policy, and that judgment must be applied in every period on a case-by-case basis to adjust the response to the particular characteristics of that period.

Some stylized facts regarding the monetary transmission mechanism are widely accepted. For example, interest rates affect output with a lag, and output, in turn, affects prices with a lag. Others, however, are still hotly debated. This is especially the case with regard to the degree of persistence of inflation, or the degree to which inflation expectations are backward-looking in the Phillips curve. On the one hand, inflation appears to have been highly persistent over certain

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1. See, for instance, Levin, Wieland, and Williams (1999); Armour, Fung, and Maclean (2002); or Côté, Kuszczak, Lam, Liu, and St-Amant (2002).

periods. On the other, there are indications that the degree of persistence has trended downwards over time, perhaps because of improvement in the conduct of monetary policy. The degree of persistence of inflation plays a pivotal role in our context, because the appropriate monetary responses are sensitive to the calibrations and to uncertainty when this degree is close to 1, but quite robust otherwise. The reason is that, with a degree of persistence that is close to 1, inflation is almost non-stationary. Hence, given the lag between monetary actions and their effects on inflation, small shocks to the transmission mechanism, or alternatively, small changes in the specification of the model or the policy objective, are quickly magnified and have non-trivial consequences. Accordingly, we single out the role of this parameter in our discussion.

The gist of our results is that, under the conditions of uncertainty usually encountered, if the degree of persistence of inflation in the Phillips curve is not high,[2] then a simple rule (such as the original Taylor rule) that mostly offsets demand shocks and puts a relatively small weight on inflation shocks may be appropriate.[3]

This conclusion stands in contrast with the results usually found in the context of large macroeconometric models. Armour, Fung, and Maclean (2002), for example, find that efficient Taylor-type rules usually involve a weight on inflation shocks that is substantially larger than the value of 0.5 in the original Taylor rule (e.g., around 2.0). Levin, Wieland, and Williams (1999) find that efficient rules typically incorporate a strong persistence in interest rate movements. One should note, however, that in these studies the efficient rules are derived under the assumption that policy-makers *commit* forever to the given rule, whereas the efficient rules derived in our study are better compared with *time-consistent* rules, in the usual sense that policy-makers re-optimize policy in every period, letting bygones be bygones. We would submit that commitment to time-*in*consistent rules is unrealistic. It is not easy to justify monetary policy actions to the public if these actions are dictated in some complicated fashion by objectives decided in the past. And since digressions from the announced rule are inevitable given the complexity of the economy, they are not easy to justify without the monetary authorities losing credibility. In these respects, time-consistent rules are much more attractive.[4]

2. Let us say it is below 0.5 annually in the context of a fully backward-looking model.
3. Although we do consider the case of open economies, our focus will be on monetary responses to domestic shocks.
4. An interesting alternative approach explored in the literature is to show that the optimal rule with commitment is equivalent to a time-consistent rule that is optimal with regard to some redefined objective. Under some conditions, for instance, it can be shown that inflation targeting with commitment is equivalent to price-level targeting without commitment (see Srour 2001).

Conversely, it is shown that if the degree of persistence of inflation in the Phillips curve is high, then finding a Taylor-type rule that can act as a benchmark for monetary policy may be difficult.

Our analysis consists of five parts. First, we examine the optimal policy rule that obtains in a small, stylized, closed-economy model of the transmission mechanism, with only additive white noise shocks. Second, we examine the case where the model incorporates diverse exogenous variables and shocks. Under these circumstances, it would be hopeless to design a practical rule that purports to describe monetary policy responses to every possible shock. We show, however, that the optimal responses to changes in the *endogenous* variables are independent of the nature and behaviour of exogenous variables. In this sense, the simple rule found in the context of the small model that incorporates few primary shocks can still act as a benchmark reaction function, with the understanding that monetary responses must be assessed on a case-by-case basis to consider exogenous shocks.

Third, we examine the case where the model's parameters may vary in time[5] and are unknown. It can then be shown that the optimal policy again has the form of a Taylor-type rule, but its coefficients may also vary in time.[6] Time-varying parameters, therefore, can seriously hinder the use of a rule for monetary policy. Clearly, if the economy is in a state of transition, or if it is frequently subject to permanent structural changes that alter the key relationships, then finding a benchmark rule would be virtually impossible.

If the model's parameters are relatively stable, however, even though they are time-varying and uncertain, a benchmark rule might still be appropriate. Thus, it can be shown that if the parameters' data-generating process is stationary, then so are the coefficients of the optimal policy rule. If the model's parameters can be considered to be i.i.d., then the optimal response coefficients are functions solely of the unconditional means and variances of the parameters' distributions and are, therefore, constant. Moreover, in this last case, we show that if the degree of inflation persistence in the Phillips curve is not high and under plausible calibrations otherwise, the effect of this type of uncertainty on monetary policy is small. In light of these results, we argue that the optimal rule that obtains when parameter uncertainty is ignored can be used as a benchmark under plausible conditions.

5. Time-varying parameters could be a consequence of learning about the economy, of structural changes in the economy, or of omitted variables.
6. For this claim to hold, learning must be assumed to be passive. Also, strictly speaking, the optimal rule is not linear, since the response coefficients may be correlated with the state variables. (See Chow 1975.)

Finally, we examine the implications of the exchange rate channel in the case of a small open economy such as Canada. One difficulty that arises in this context is the large uncertainty regarding the pass-through effects of the exchange rate to domestic inflation and the uncertainty regarding the interest-exchange rate relationship. We show, however, that under reasonable calibrations, these uncertainties can, to some extent, be ignored. The reason is that, on the one hand, in Canada, the pass-through effect of changes in the exchange rate to domestic inflation is small and transitory. On the other hand, when determining monetary policy responses to domestic demand shocks, what needs to be known is the combined effect of the interest rate and the exchange rate on demand—knowledge of the exact interest-exchange rate relationship is not necessary.

This paper extends the analysis in Srour (1999, henceforth referred to as [S]). While in [S] we made use of strong simplifying assumptions, such as two-period horizons and strict inflation targeting, this paper derives optimal solutions under general conditions and provides quantitative evaluations of policy rules. We continue to use calibrated models, as in [S], since there are strong indications that certain parameters of the transmission mechanism have changed over time, and since calibrated models are amenable to running comparative exercises and investigating the implications of uncertainty. However, we also verify some of the conclusions with the help of estimated models.

In addition, we continue to use reduced-form, i.e., backward-looking, models, and we assume that the objective of monetary policy is to minimize in every period the expected discounted sum of (weighted squared) deviations of output from potential and inflation from the target. One reason for using such models is tractability of the effects of uncertainty. Another reason is that no one forward-looking model has yet been agreed on. Besides, in many cases, one can judge the implications of forward-looking elements by examining a reduced-form model with alternative values of the parameters.[7] Nonetheless, the results are subject to the Lucas critique and must be taken with caution.

The paper is organized as follows. Section 2 describes the baseline model and derives the optimal rule when uncertainty in the model enters only in the form of white noise additive shocks. We run comparative static exercises with respect to the model's parameters, paying special attention to the degree of persistence of inflation in the Phillips curve.

7. For example, a hybrid Phillips curve that involves backward- and forward-looking inflation expectations can for some purposes be approximated with a Phillips curve that involves backward-looking elements and a constant.

In section 3, we discuss the implications of other types of uncertainty, including parameter uncertainty, model uncertainty, and data uncertainty. In section 4, the analysis is extended to a small open economy with a flexible exchange rate. Some of the results are verified in estimated vector autoregression (VAR) models in section 5. Section 6 concludes and suggests directions for further research. It would be particularly useful to discuss in some detail how the introduction of forward-looking elements in our baseline model affect efficient rules. In this context, it is also important to come to some conclusion regarding which rules are more appropriate: time-consistent efficient rules or efficient rules that obtain under commitment.

2. The Baseline Model

We consider the following model of the transmission mechanism in a closed economy as a benchmark:[8]

$$\pi_{t+1} - \pi^* = a(\pi_t - \pi^*) + d(y_t - y^*) + \varepsilon_{t+1} \tag{1}$$

$$y_{t+1} - y^* = b(y_t - y^*) - c(r_t - r^*) + \eta_{t+1}, \tag{2}$$

where y_t is the log of aggregate output; y^* is the log of potential output (assumed for now to be constant); π_t is the inflation rate; π^* is the inflation target; i_t is the instrument of monetary policy (here identified with the one-period nominal interest rate); r_t is the real interest rate, $r_t \equiv i_t - E_t\pi_{t+1}$;[9] r^* is the equilibrium real interest rate (assumed for now to be constant); a, *b, c,* and *d* are positive constants, $b < 1$; and ε_t and η_t are white noise random shocks. Of course, equations (1) and (2) stand for a Phillips curve and an IS curve, respectively.

The main feature of this model is that the instrument of monetary policy acts on output with a one-period lag. In turn, aggregate demand acts on inflation with a one-period lag, so that monetary actions affect inflation only after two periods. This is roughly consistent with the empirical facts in Canada if periods are chosen to be annual. The form of the Phillips curve implies that there is a trade-off between output and inflation: an increase in inflation requires a temporary demand contraction to bring inflation back to its initial level relatively quickly, that is, more quickly than would follow from the mean-reverting character of inflation witnessed by the coefficient *a*. The coefficient *a*, which measures the degree of persistence of inflation, can be thought of as a

8. This is the same model used by Ball (1997), Svensson (1997), or [S], except that their Phillips curve is accelerationist, i.e., $a = 1$.

9. E_t denotes the expectational operator conditional on information at time *t*.

measure of the degree to which the public is backward-looking with respect to prices or, alternatively, as the degree of (lack of) credibility the public has in the inflation target.

2.1 The optimal rule

The policy-maker is assumed to minimize in each period *t* a discounted (weighted) sum of expected deviations of output and inflation from target

$$E_t \sum_{i=0}^{\infty} \delta^i L(\pi_{t+i}, y_{t+i}), \tag{3}$$

where

$$L(\pi, y) \equiv \alpha(y - y^*)^2 + (1-\alpha)(\pi - \pi^*)^2, \tag{4}$$

δ is the discount rate, $0 < \delta \le 1$, and α is the relative weight placed on output and inflation stability, $0 \le \alpha \le 1$. The larger is δ, the greater is the weight placed on long-run costs. At the limit, $\delta = 1$, only the long-run costs matter, in which case equation (3) is identified with the unconditional expectation $EL(\pi_t, y_t)$. The smaller is α, the more concerned is the policy-maker with inflation stability, the case $\alpha = 0$ corresponding to what Svensson calls strict inflation targeting.

If the central bank could control output directly, then equation (2) would be redundant, and it can be shown that the optimal policy rule would have the form

$$y_t - y^* = -k_1(\pi_t - \pi^*),$$

where k_1 is a constant that depends on δ, α, and the parameters *a* and *d*, which measure the trade-off between output and inflation witnessed in the Phillips curve.[10] However, the central bank can only affect output with a one-period lag. It follows that the optimal rule must, in fact, be expressed as

$$E_t(y_{t+1} - y^*) = -k_1 E_t(\pi_{t+1} - \pi^*)$$

or equivalently, substituting that equation back into the Phillips curve, as

$$E_t\pi_{t+2} - \pi^* = kE_t(\pi_{t+1} - \pi^*), \tag{5}$$

10. $k_1 = \frac{-\alpha + \delta\alpha a^2 - \delta d^2(1-\alpha) + \sqrt{\Delta}}{2\delta\alpha d a}$ and $\Delta = [\alpha - \delta\alpha a^2 + \delta d^2(1-\alpha)]^2 + 4\delta^2\alpha d^2 a^2(1-\alpha)$.

where $k = a - dk_1$ measures the optimal speed at which the central bank ought to bring inflation back to the target following a shock.

Using equations (1), (2), and (5), the optimum rule can also be written as

$$r_t - r^* = A(y_t - y^*) + B(\pi_t - \pi^*) \tag{6}$$

where $A = \dfrac{a-k+b}{c} > 0$, and $B = \dfrac{a(a-k)}{cd} \geq 0$.[11]

Elementary algebraic manipulation shows that k is a constant between 0 and a, and that it increases with a. Likewise, the response coefficients A and B in the optimal instrument rule (6) can be shown to increase with a: a larger coefficient a means that inflation is more persistent, and therefore would return more slowly to the target level, ceteris paribus, and hence k will be larger. For the same reason, a larger a requires sharper monetary responses to reduce deviations in inflation, hence larger coefficients A and B.

Similarly, one can examine the behaviour of k, and A and B, with respect to the model's other parameters. A detailed discussion is provided in Appendix A.

Unless otherwise stated, δ is assumed to equal 1 from now on.

2.2 Efficiency frontiers

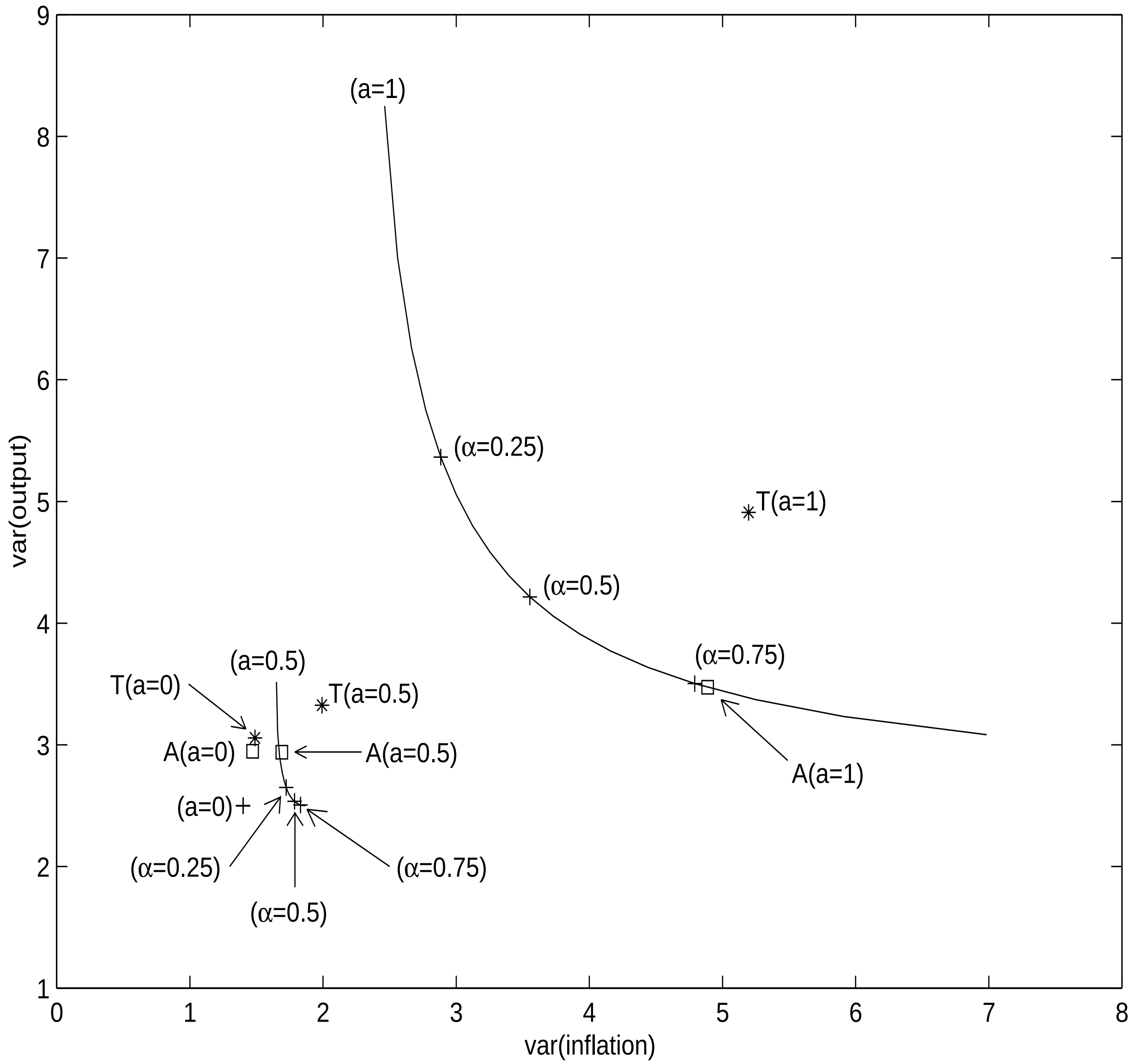
By definition, drawn in $var(y) - var(\pi)$ space, the variances of output and inflation under any policy rule are bounded (to the southwest) by the variances associated with the optimum rules. The latter trace an efficiency frontier as the relative weight α in the loss function ranges between 0 and 1.

Figure 1 plots the efficiency frontier under the assumption that

$$b = 0.8 \qquad c = 1 \qquad d = 0.4,$$

11. Alternatively, the rule can be expressed in terms of the nominal interest rate $i_t - i^* = A'(y_t - y^*) + B'(\pi_t - \pi^*)$, where $A' = A + d > 0$, $B' = B + 1 \geq 1$, and $i^* = \pi^* + r^*$.

Figure 1: Efficiency frontiers[12]



Notes: A = Alternative rule; T = Taylor rule.

12. Owing to limited space, the efficiency frontier in the case $a = 1$ is not shown fully.

and the shocks to inflation and output, ε_t and η_t, are uncorrelated and have variances equal to 1.0 and 2.5, respectively. This parameterization is roughly consistent with the Canadian data at an annual frequency.[13] The figure shows the efficiency frontiers when *a* takes the values 1, .5, and 0. It also shows the outcomes for the standard Taylor rule (T),

$$r_t - r^* = 0.5(y_t - y^*) + 0.5(\pi_t - \pi^*)*, \tag{7}$$

and the Alternative rule (A),

$$r_t - r^* = 1(y_t - y^*) + 0.5(\pi_t - \pi^*). \tag{8}$$

For example, when $a = 1$ and an equal weight is placed on output and price stability, i.e., $\alpha = 0.5$, the optimum rule is

$$r_t - r^* = 1.13(y_t - y^*) + 0.82(\pi_t - \pi^*); \tag{9}$$

the variances of inflation and output under this policy are 3.55 and 4.22, respectively; whereas, under the Taylor rule, the variances of inflation and output are 5.2 and 4.91; and under the Alternative rule, 4.89 and 3.47.

When considering the desirability of a particular rule, the policy-maker is concerned with its relative efficiency (i.e., its position relative to the efficiency frontier) and the trade-off it involves between output and inflation variability. Although the present model does not specify a value of the relative cost of output and inflation variability (i.e., the weight α in the loss function), one may deem unsuitable outcomes that involve a *large* trade-off between output and inflation variability.[14] One may therefore want to rule out, on practical grounds, those outcomes on the efficiency frontier associated with too large or too small values of α, e.g., $\alpha > 0.75$ or $\alpha < 0.25$, for which a small increase in the variance of output can produce a large reduction in inflation

13. The covariance matrix of the shocks does not affect the coefficients of an optimal rule. Moreover, for any linear policy rule such as equation (6), the variances of the state variables under that policy are proportional to the variances of the shocks. So the variances in Figure 1 should be understood only up to a constant of proportionality, say equal to 10^{-4}. Of course, the relative variances of output and inflation under one given rule, as well as the relative efficiency (as measured by the loss function) of two different rules, may change if the relative variances of the two types of shocks change. For example, a rule that responds little to output innovations can be more or less efficient, depending on whether output shocks are small or large. What is pertinent in the calibration, therefore, is that the variance of output shocks is assumed to be 2.5 times larger than that of inflation shocks.
14. Arguably, this claim is less clear if the variances of output and inflation are small to begin with (also see next paragraph).

variability, or vice versa. This has important consequences, since as will be seen, certain claims are reasonable for mid-range values of α, but unreasonable for extreme values.

Excluding the lower and upper tails of α amounts to excluding the more vertical and more horizontal segments of the efficiency frontier. For example, for $a = 1$, the variance of output under optimum rules varies between 2.68 and 11.25 when α ranges over the whole interval $[0,1]$, and the variance of inflation varies between 2.4 and 19.12; whereas, if one restricts α to the interval $[0.25,0.75]$, the variance of output varies between 3.5 and 5.36, and the variance of inflation varies between 2.88 and 4.79.

As the degree of persistence of inflation, *a,* decreases, the range of the efficiency frontier shrinks. In other words, for lower values of *a,* the exact weight, α, placed on the relative cost of output and inflation variability makes less difference. For example, as already noted, when $a = 1$, the variance of output under optimum rules varies between 2.68 and 11.25, and the variance of inflation varies between 2.4 and 19.12. When $a = 0.5$, the variance of output varies between 2.5 and 3.52, and the variance of inflation varies between 1.65 and 1.87. The reason is that the smaller is the degree of persistence of inflation, *a,* the less inflation shocks feed into future inflation and the more the overall objective becomes identified with the objective of achieving output stability. This is also reflected in a greater weight accorded to demand shocks relative to inflation shocks in the optimal rule. For instance, when $a = 0.5$ and $\alpha = 0.5$, the optimum rule is

$$r_t - r^* = 0.88(y_t - y^*) + 0.1(\pi_t - \pi^*) .$$

Of particular interest is that, for values of *a* below 0.5, which are the values sometimes found in empirical studies, the optimal response coefficients, as well as the variances of output and inflation, do not change significantly with α. In the limit case, $a = 0$, the efficient frontier reduces to a single point, and the optimal rule always consists in bringing output back to potential next period, whatever the value of α, e.g.,

$$r_t - r^* = 0.8(y_t - y^*) .$$

Given the policy rule and the variances of output and inflation, one can derive the variance of the real interest rate. In the case of optimal rules, this variance is larger, the greater the relative weight on inflation stability (i.e., the smaller is α), since this requires a larger monetary response to inflationary shocks: for $a = 1$, the variance of the real interest rate ranges between approximately 1.78 and 19.95 over all values of α, and it ranges between 2.79 and 5.95 when one restricts α to range between 0.25 and 0.75. As before, that range shrinks as *a* decreases.

Interestingly, the Alternative Taylor rule is found to lie almost on the efficiency frontier, and to be unambiguously more efficient than the standard Taylor rule. As noted in footnote 12, however, this is dependent on the calibration of the shocks' variances. If the variances were actually specified to be equal, then the Alternative rule, which calls for an equal reaction to inflation shocks and a stronger reaction to output shocks than the Taylor rule, can be shown to lead to a higher variance in inflation than the Taylor rule. The reason is that, following an inflation shock, although both rules respond equally initially, the Alternative rule subsequently slows the convergence of inflation towards the target, since it responds more strongly to the induced output gap. If inflation shocks are in large part responsible for the movements in inflation, i.e., if the variance of inflation shocks is large enough relative to demand shocks, then the variability of inflation will be greater under the Alternative rule than the Taylor rule. The relationship between the magnitude of the response coefficients in a monetary rule and the relative variability of output and inflation is therefore not obvious.

3. Uncertainty

The analysis in the previous section assumes that, except for white noise, all the components of the transmission mechanism and the loss function are known with certainty. The data are assumed to be complete and reliable, and the nature of the shocks, the magnitude of non-observable variables such as the output gap and the equilibrium interest rate, and the manner in which the shocks and monetary actions are transmitted to the rest of the economy are all assumed to be known with certainty. But this is hardly realistic.

To investigate the implications of uncertainty on the conduct of monetary policy, we assume that the state variables are not observed, and the elasticities in the transmission mechanism, the nature of shocks, and the weight in the loss function are uncertain. Formally, we suppose that the model of the transmission mechanism has the form

$$\pi_{t+1} = a_{t+1}\pi_t + d_{t+1}y_t + \Phi_{t+1}X_t + \varepsilon_{t+1} \tag{10}$$

$$y_{t+1} = b_{t+1}y_t - c_{t+1}r_t + \Psi_{t+1}X_t + \eta_{t+1} \tag{11}$$

$$X_{t+1} = \Gamma_{t+1}X_t + \theta_{t+1}, \tag{12}$$

and the periodic loss function has the form

$$L_t(\pi, y) \equiv \alpha_t(y)^2 + (1-\alpha_t)(\pi)^2, \tag{13}$$

where all state variables are measured as deviations from their equilibrium values (which themselves can be varying in time); the parameters a_t, b_t, c_t, d_t, Φ_t, Ψ_t, Γ_t, and α_t are random variables; X_t is a vector of diverse autonomous variables and shocks affecting the economy; and ε_t, η_t, and θ_t are white noise shocks, $E_t\varepsilon_{t+1} = E_t\eta_{t+1} = E_t\theta_{t+1} = 0$.

Note that any innovation ξ_{t+1} in the economy can be decomposed into a component $E_t\xi_{t+1}$ forecasted at time *t*, which can be incorporated into the vector X_t, and a white noise shock that can be incorporated into the shocks ε_{t+1}, η_{t+1}, and θ_{t+1}. Thus, through the terms $\Phi_{t+1}X_t$ and $\Psi_{t+1}X_t$, the model can incorporate the fact that different shocks may propagate differently through the economy, and their effects are uncertain.

Decision-making is assumed to proceed as follows. At the beginning of every period *T*, policy-makers gather data and form their beliefs about the state of the economy and its future outlook based on all information available. More specifically, they form beliefs about the nature of the shocks, the state variables y_t, π_t, r_t, and X_t at time *T*,[15] and the elasticities in the transmission mechanism. The policy-makers' belief about any particular parameter at time *T* is identified with the model-consistent expectations of that parameter based on all the information available. Learning can occur, but it is assumed to be passive (see below). The policy-makers then take action that optimizes the objective function.

The solution to the optimization problem in full generality appears quite complex. We consider the case of data uncertainty and parameter uncertainty separately. The proofs are provided in Appendix C.

3.1 Parameter uncertainty[16]

Suppose that the state variables y_t, π_t, r_t, and X_t are observed at time *t*, and that learning is passive in the sense that actions taken at any time *T* or earlier are assumed not to affect the joint distribution of $\{\alpha_{t-1}, a_t, b_t, c_t, d_t, \Phi_t, \Psi_t, \Gamma_t, \varepsilon_t, \eta_t, \theta_t\}_{t \geq T+2}$, conditional on all information available at time $T + 1$. (This would follow automatically if the series $\{\alpha_{t-1}, a_t, b_t, c_t, d_t, \Phi_t, \Psi_t, \Gamma_t, \varepsilon_t, \eta_t, \theta_t\}_{t \geq T+2}$ is independently distributed over time.)

15. The uncertainty about r_t stems from the uncertainty about the equilibrium interest rate.
16. The implications of parameter uncertainty for policy were first studied by Brainard (1967), and a number of other authors since then (see Estrella and Mishkin 1998, Sack 1998, Svensson 1997, and Srour 1999).

Under these conditions, it can be shown that the optimum rule is to some extent of a Taylor form, except that the response coefficients may vary in time (see Chow 1975, ch. 10). The optimum rule has the form

$$r_t = A_t y_t + B_t \pi_t + C_t X_t + D_t, \tag{14}$$

where the response coefficients A_t and B_t depend on the data-generating process governing the parameters $\{\alpha_t, a_t, b_t, c_t, d_t\}$, conditional on information available as of time *t*; and the response coefficient C_t and the auxilliary scalar D_t depend on the data-generating process governing all of the model's parameters $\{\alpha_{t-1}, a_t, b_t, c_t, d_t, \Phi_t, \Psi_t, \Gamma_t, \varepsilon_t, \eta_t, \theta_t\}$, also conditional on information available as of time *t*.[17]

As a benchmark for the conduct of monetary policy, the above rule would be impractical unless the response coefficients on the most relevant state variables are relatively stable. The stability of the response coefficients in turn depends on the data-generating process governing the model's parameters. Since X_t is a vector of diverse autonomous variables—other than output and inflation—affecting the economy, one cannot expect the process governing X_t, or that governing $\{\Phi_t, \Psi_t, \Gamma_t\}$, and hence the response coefficient C_t, to be stable.

Except in times of structural transition, however, there are indications that the data-generating process governing at least the parameters $\{a_t, b_t, c_t, d_t\}$ may be stable. This is apparent from the consensus among central bankers regarding a number of key stylized facts about the transmission mechanism. Furthermore, it seems safe to assume that the weight α_t in the social loss function is exogenously given, i.i.d., and independent from all the other parameters of the model. In that case, one can without loss of generality replace α_t by its unconditional mean, and hence assume that the weight in the social loss function is constant.

Accordingly, suppose that the data-generating process governing the parameters $\{\alpha_t, a_t, b_t, c_t, d_t\}$ is stationary. One can then show that the data-generating process governing the response coefficients A_t and B_t in the optimal rule is also stationary (see Appendix C).[18] Nonetheless, A_t and B_t may vary in time, as the parameters $\{\alpha_t, a_t, b_t, c_t, d_t\}$ vary, and one

17. Thus, strictly speaking, the rule is not linear, since the response coefficients may be correlated with the state variables. For completeness, and because of some differences in model specification, we provide a proof in Appendix C.

18. If *all* of the model's parameters $\{\alpha_t, a_t, b_t, c_t, d_t, \Phi_t, \Psi_t, \Gamma_t, \varepsilon_t, \eta_t, \theta_t\}$ are stationary, then so are all of the response coefficients in the optimal rule.

learns more about their values over time.[19] However, if one learns about the changes in $\{\alpha_t, a_t, b_t, c_t, d_t\}$ slowly, i.e., if by the time one learns about the past changes in $\{\alpha_t, a_t, b_t, c_t, d_t\}$, the current new values are unrelated, then for all practical purposes, $\{\alpha_t, a_t, b_t, c_t, d_t\}$ can be considered to be i.i.d. In that case (in particular if $\{\alpha_t, a_t, b_t, c_t, d_t\}$ are constant), then A_t and B_t can be shown to be functions solely of the means and variances of a_t, b_t, c_t, d_t, and α_t, and hence constant. This is not surprising since, under the assumption of an i.i.d. distribution, the dynamic evolution of changes in inflation and output is unchanged regardless of the realizations of other variables or shocks in the model. Moreover, if the shocks ε_t, η_t, and θ_t are independent from the model's parameters $\{\alpha_t, a_t, b_t, c_t, d_t, \Phi_t, \Psi_t, \Gamma_t\}$, then $D_t = 0$.

One concludes from the above analysis that a certain amount of discretion, exhibited by a time-varying coefficient C_t and stemming from the diversity and unpredictability of exogenous shocks, is likely to be unavoidable. However, if there is an underlying core to the transmission mechanism that is relatively stable, e.g., if the data-generating process governing the parameters a_t, b_t, c_t, d_t, and α_t is exogenous to the shocks affecting the economy, and changes in these parameters are not very persistent relative to the speed with which one learns about the changes, then the coefficients A_t, B_t will be practically stable and will therefore provide benchmark responses to output and inflation shocks. Otherwise, a rule such as equation (15) cannot act as a benchmark for monetary policy, and monetary responses must be decided essentially on a case-by-case basis.

3.2 Numerical results

Srour (1999) ([S]) relied on strong simplifying assumptions, such as the restriction to two-period models and strict inflation targeting, to analyze the effects of parameter uncertainty on monetary policy. It was argued, for example, that uncertainty about the interest rate elasticity of demand, *c*, calls for more cautious policy responses to shocks, i.e., weaker response coefficients in the policy rule, than when no such uncertainty is present; whereas uncertainty about the degree, *a*, to which inflation feeds into future inflation calls for sharper responses. Whether the effect of simultaneous uncertainty about several parameters calls for more cautious or bolder policies than when no such uncertainty exists depends on the relative magnitude of uncertainty about the various parameters, and the correlations between the parameters, and is therefore an empirical issue. In this section,

19. For example, if the parameters $\{\alpha_t, a_t, b_t, c_t, d_t\}$ are observed at time *t*, and they follow an AR process where the innovations are independent from the model's other parameters, then one can show that A_t and B_t are functions of the realizations of $\{\alpha_t, a_t, b_t, c_t, d_t\}$ as of time *t*. If changes in $\{\alpha_t, a_t, b_t, c_t, d_t\}$ are persistent, then changes in A_t and B_t will be as well, in which case a benchmark rule would be impractical.

we examine numerically the effects of parameter uncertainty on the optimal rule under much more general conditions than in [S].

We assume that a_t, b_t, c_t, d_t, and α_t are i.i.d., uncorrelated with the shocks ε_t and η_t, with mean a, b, c, d, and α, and that there are no exogenous variables ($\Phi_t = \Psi_t = 0$). Under these conditions, the optimum rule has the same form as in the case without parameter uncertainty, e.g.,

$$r_t = Ay_t + B\pi_t,$$

but the response coefficients *A* and *B* are now complex functions of the variances of the model's parameters as well as their means, *a, b, c*, and *d*, and α and δ. *A* and *B* can be evaluated numerically.

Tables 1 and 2a, b, and c provide the optimal response coefficients under the specification

$$a = 1 \qquad b = 0.8 \qquad c = 1 \qquad d = 0.4$$

and a variance for the output gap 2.5 times larger than that of inflation. Four cases are considered: no uncertainty; *c* alone is random with standard deviation 0.5 (hence its *t*-statistic equals 2); *a* alone is random with standard deviation 0.5 (hence its *t*-statistic equals 2); and both *a* and *c* are random, uncorrelated, and with equal standard deviation 0.5. For each case, Table 2 provides the variances of output and inflation under the optimal rule as well as the variances (denoted $\text{var}_c(.)$) under the optimal rule that obtains if one ignores parameter uncertainty (i.e., the rule exhibited under the no-uncertainty case in Table 1).

For example, when $\alpha = 0.5$ and there is no uncertainty, the optimal rule is $r_t = 1.13y_t + 0.82\pi_t$, and the variances of inflation and output under this policy are 3.55 and 4.22, respectively. If, in fact, *a* and *c* are random with equal standard deviations 0.5, then the optimal rule is $r_t = 1.04y_t + 1.01\pi_t$, and the variances of inflation and output under this policy are 9.95 and 12.25, respectively. Hence, the overall loss is 11.1, whereas the variances of inflation and output under the previous rule, which ignores the uncertainty about the parameters, are 14.41 and 11.43, and the overall loss is 12.9.[20]

In general, Table 2 shows that, for $a = 1$, and for a given weight α, the response coefficients and the individual variances of output and inflation may differ significantly between the optimal rules

20. Not shown in the table, when both *a* and *c* are random with equal standard deviations 0.5, and perfectly positively correlated, the optimal rule is $r_t = 1.23y_t + 1.11\pi_t$, the variances of inflation and output under this policy are 13.35 and 18.58 respectively; and when *a* and *c* are perfectly negatively correlated, the optimal rule is $r_t = 0.95y_t + 0.97\pi_t$, and the variances of inflation and output are 7.82 and 9.85.

Table 1: Optimal response coefficients (*a* = 1)

	no uncertainty		*c* random with 0.5 standard deviation		*a* random with 0.5 standard deviation		*a* and *c* random with 0.5 standard deviation	
α	π	y	π	y	π	y	π	y
0.00	2.50	1.80	1.50	1.24	2.50	1.80	1.50	1.24
0.05	1.98	1.59	1.31	1.16	2.20	1.68	1.42	1.21
0.10	1.70	1.48	1.17	1.11	2.01	1.60	1.35	1.18
0.15	1.50	1.40	1.07	1.07	1.86	1.55	1.29	1.16
0.20	1.35	1.34	0.98	1.03	1.75	1.50	1.24	1.13
0.25	1.23	1.29	0.91	1.00	1.65	1.46	1.19	1.12
0.30	1.13	1.25	0.84	0.98	1.57	1.43	1.15	1.10
0.35	1.04	1.22	0.79	0.95	1.50	1.40	1.11	1.08
0.40	0.96	1.18	0.73	0.93	1.43	1.37	1.07	1.07
0.45	0.89	1.16	0.68	0.91	1.37	1.35	1.04	1.06
0.50	0.82	1.13	0.63	0.89	1.31	1.33	1.01	1.04
0.55	0.76	1.10	0.59	0.88	1.26	1.30	0.97	1.03
0.60	0.69	1.08	0.54	0.86	1.21	1.28	0.94	1.02
0.65	0.63	1.05	0.50	0.84	1.16	1.27	0.91	1.01
0.70	0.57	1.03	0.46	0.82	1.12	1.25	0.89	0.99
0.75	0.51	1.01	0.41	0.80	1.07	1.23	0.86	0.98
0.80	0.45	0.98	0.36	0.79	1.03	1.21	0.83	0.97
0.85	0.39	0.95	0.31	0.76	0.98	1.19	0.80	0.96
0.90	0.31	0.92	0.25	0.74	0.93	1.17	0.77	0.95
0.95	0.22	0.89	0.18	0.71	0.88	1.15	0.73	0.93
0.99	0.10	0.84	0.08	0.67	0.84	1.14	0.71	0.92

Table 2a: Variance (*a* = 1)

	no uncertainty			*c* random with 0.5 standard deviation					
α	**var(π)**	**var(y)**	**loss**	**var(π)**	**var(y)**	**loss**	**var$_c$(π)**	**var$_c$(y)**	**loss**
0.00	2.40	11.25	2.40	3.28	10.26	3.28	29.00	343.75	29.00
0.05	2.46	8.25	2.75	3.32	8.57	3.58	4.13	24.81	5.16
0.10	2.56	7.00	3.00	3.40	7.60	3.82	3.55	15.40	4.74
0.15	2.66	6.26	3.20	3.49	6.94	4.01	3.42	11.91	4.69
0.20	2.77	5.75	3.37	3.59	6.46	4.16	3.40	10.02	4.72
0.25	2.88	5.36	3.50	3.70	6.07	4.29	3.44	8.80	4.78
0.30	3.00	5.06	3.62	3.82	5.75	4.40	3.51	7.93	4.84
0.35	3.12	4.80	3.71	3.95	5.48	4.49	3.60	7.26	4.88
0.40	3.25	4.58	3.78	4.10	5.25	4.56	3.70	6.73	4.91
0.45	3.40	4.39	3.85	4.25	5.04	4.61	3.83	6.29	4.94
0.50	3.55	4.22	3.89	4.42	4.85	4.64	3.97	5.92	4.95
0.55	3.73	4.06	3.91	4.62	4.67	4.65	4.13	5.59	4.93
0.60	3.93	3.91	3.92	4.85	4.50	4.64	4.33	5.30	4.91
0.65	4.16	3.77	3.91	5.11	4.34	4.61	4.56	5.03	4.87
0.70	4.44	3.64	3.88	5.43	4.19	4.56	4.84	4.78	4.80
0.75	4.79	3.50	3.82	5.83	4.04	4.49	5.20	4.55	4.71
0.80	5.25	3.37	3.75	6.37	3.88	4.38	5.67	4.32	4.59
0.85	5.91	3.23	3.63	7.13	3.72	4.23	6.35	4.08	4.42
0.90	6.99	3.08	3.47	8.38	3.55	4.03	7.47	3.84	4.20
0.95	9.36	2.90	3.22	11.15	3.34	3.73	9.96	3.56	3.88
0.99	19.04	2.68	2.84	22.51	3.07	3.26	20.16	3.23	3.40

Table 2b: Variances (*a* = 1)

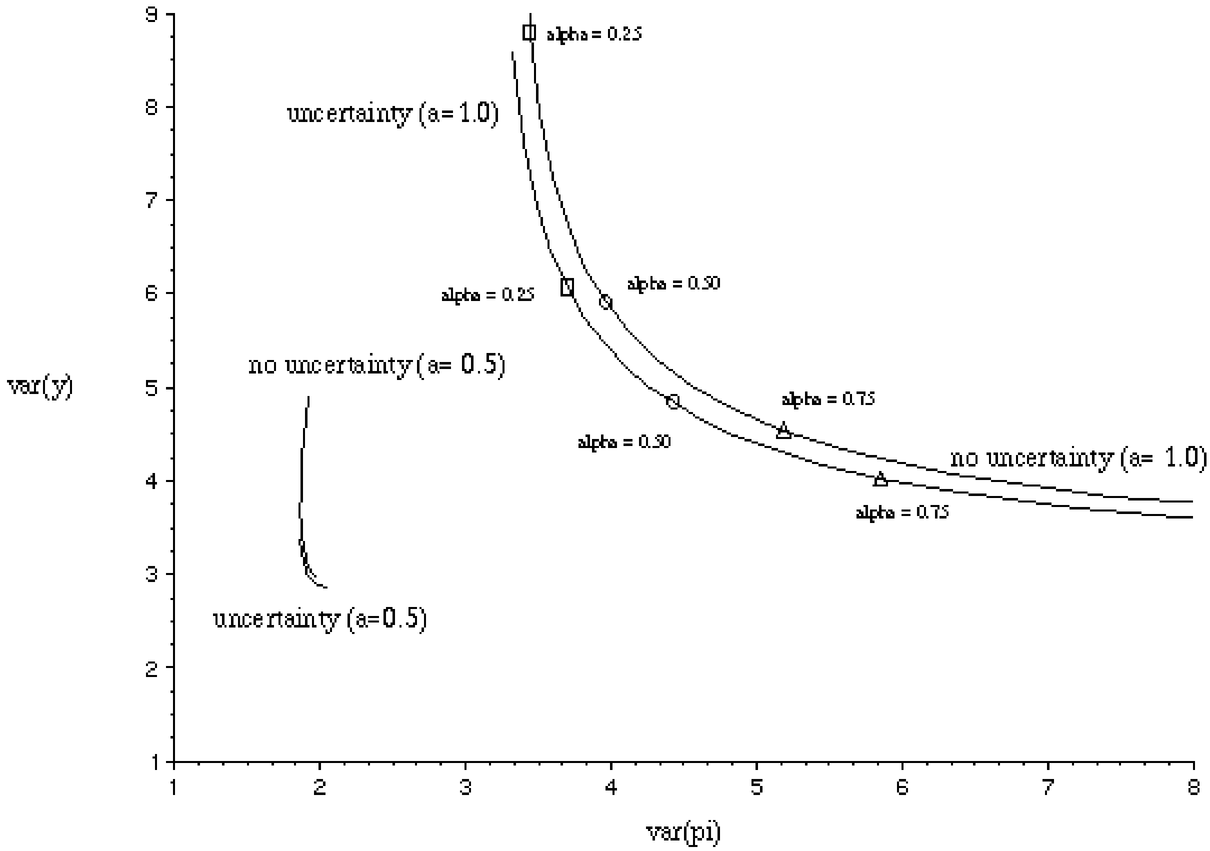
		***a* random with 0.5 standard deviation**				
α	**var(π)**	**var(y)**	**loss**	**var$_c$(π)**	**var$_c$(y)**	**loss**
0.00	4.80	18.75	4.80	4.80	18.75	4.80
0.05	4.88	15.39	5.41	5.04	13.42	5.46
0.10	5.01	13.63	5.87	5.43	11.37	6.02
0.15	5.17	12.51	6.27	5.89	10.21	6.54
0.20	5.34	11.70	6.61	6.40	9.46	7.01
0.25	5.52	11.08	6.91	6.97	8.93	7.46
0.30	5.71	10.59	7.17	7.64	8.54	7.91
0.35	5.90	10.18	7.40	8.42	8.26	8.36
0.40	6.11	9.84	7.60	9.37	8.06	8.85
0.45	6.33	9.54	7.77	10.55	7.95	9.38
0.50	6.56	9.28	7.92	12.08	7.92	10.00
0.55	6.82	9.05	8.05	14.19	8.00	10.79
0.60	7.10	8.84	8.14	17.30	8.27	11.88
0.65	7.40	8.66	8.22	22.44	8.86	13.61
0.70	7.75	8.50	8.28	32.80	10.29	17.04
0.75	8.14	8.35	8.30	65.71	15.28	27.89
0.80	8.59	8.22	8.29			
0.85	9.12	8.10	8.25			
0.90	9.78	8.01	8.19			
0.95	10.61	7.94	8.07			
0.99	11.48	7.92	7.96			

Table 2c: Variances (*a* = 1)

	a and *c* random with 0.5 standard deviation					
α	**var(π)**	**var(y)**	**loss**	**var$_c$(π)**	**var$_c$(y)**	**loss**
0.00	8.19	19.52	8.19			
0.05	8.23	17.76	8.71	13.49	63.07	15.97
0.10	8.33	16.51	9.15	9.44	30.65	11.56
0.15	8.46	15.57	9.53	8.81	22.14	10.81
0.20	8.62	14.83	9.86	8.86	18.12	10.71
0.25	8.80	14.22	10.16	9.20	15.76	10.84
0.30	8.99	13.71	10.41	9.75	14.20	11.09
0.35	9.20	13.27	10.62	10.49	13.11	11.41
0.40	9.43	12.89	10.81	11.46	12.33	11.81
0.45	9.68	12.55	10.97	12.72	11.78	12.30
0.50	9.95	12.25	11.10	14.41	11.43	12.92
0.55	10.25	11.98	11.20	16.78	11.29	13.76
0.60	10.58	11.73	11.27	20.35	11.44	15.00
0.65	10.95	11.51	11.31	26.38	12.07	17.08
0.70	11.36	11.31	11.33	38.92	13.91	21.41
0.75	11.83	11.14	11.31	82.06	21.28	36.48
0.80	12.38	10.98	11.26			
0.85	13.02	10.84	11.17			
0.90	13.80	10.73	11.04			
0.95	14.77	10.65	10.86			
0.99	15.77	10.62	10.67			

that take parameter uncertainty into consideration and those that ignore it. However, for the middle range of values for α, the rules that ignore parameter uncertainty remain close to the efficiency frontier. This is apparent in Figure 2, which plots the efficiency frontier in the case where *c* is random with standard deviation equal to 0.5, together with the frontier associated with the optimal rules that ignore the uncertainty about *c*. (The efficiency frontiers associated with the other cases have similar qualitative properties.) Thus, ignoring parameter uncertainty seems to lead more to a lateral displacement of the efficiency frontier rather than to a level shift. An intuitive explanation is that taking parameter uncertainty into consideration amounts to some extent to rescaling the relative weight on price and output stability.[21] Consequently, the additional overall loss implied by the optimal rules that ignore parameter uncertainty is not substantial.

Figure 2: Efficiency frontiers *a* = 1.0 and *a* = 0.5



21. For instance, taking uncertainty about the interest rate elasticity of demand, *c*, into consideration amounts to placing more weight on output variability.

Since smaller degrees of persistence of inflation tend to collapse the efficiency frontier, it is not surprising to find that for values of the degree of persistence, a, smaller than 0.5, one can safely ignore parameter uncertainty of the order of magnitude considered above. This is also apparent in Tables 3 and 4, which provide the optimal response coefficients and overall loss under the rules that take parameter uncertainty into consideration and those that ignore it, when $a = 0.5$ and c is random with standard deviation equal to 0.5. A more formal explanation is that the smaller the degree of persistence of inflation, the less current volatility feeds into the future, and the quicker the state variables can be brought back to equilibrium, hence the less parameter uncertainty ought to matter for policy.

Our numerical results confirm that whether parameter uncertainty calls for more or less cautious responses depends on the relative magnitude of uncertainty about the various parameters.[22] Here, "more cautious" means smaller responses to shocks. Alternatively, caution can be defined as inertia, i.e., acting in a manner that deviates relatively little from past policies. Sack (1999) showed that, under this alternative interpretation, if the uncertainty about the parameters stems from ordinary least squares (OLS) estimation, then in a sense it always calls for more caution. That is, taking into consideration the statistical uncertainty attached to the parameters when estimated by OLS over the past, leads to an optimal rule that is closer to the past behaviour of monetary policy. This proposition is not inconsistent with our results, since parameter uncertainty can be inherent in the monetary transmission mechanism and not necessarily due to econometric estimation, and past policies can be either too weak or too strong to begin with, relative to the optimal policy without uncertainty. However, it raises the issue of how to evaluate parameter uncertainty. It does not appear easy to disentangle the uncertainty inherent in the transmission mechanism from the statistical uncertainty arising from econometric estimation.

3.3 Taylor rules versus forecast-based rules

In Taylor-type rules, the instrument of policy is set as a function of *contemporaneous* variables. Alternatively, one can consider forecast-based rules whereby the policy instrument is set as a function of *forecasted* variables. As a matter of fact, since the objective function is a function solely of expected (squared deviations of) inflation and output from equilibrium, the optimal rule can always be stated as a function solely of forecasted future deviations of inflation and output from equilibrium. In the context of the present model of the transmission mechanism, since forecasts can always be expressed in terms of contemporaneous variables, Taylor-type rules and forecast-based rules are completely interchangeable, except that in Taylor form, the optimal type

22. See Craine (1979) for an early discussion of the effects of parameter uncertainty.

Table 3: Optimal response coefficients (*a* = 0.5)

	no uncertainty		*c* random with 0.5 standard deviation	
α	π	**y**	π	**y**
0.05	0.48	1.18	0.32	0.90
0.10	0.38	1.11	0.27	0.85
0.15	0.32	1.05	0.22	0.82
0.20	0.27	1.01	0.19	0.79
0.25	0.23	0.98	0.16	0.77
0.30	0.19	0.96	0.14	0.75
0.35	0.17	0.93	0.12	0.74
0.40	0.14	0.91	0.11	0.72
0.45	0.12	0.90	0.09	0.71
0.50	0.11	0.88	0.08	0.70
0.55	0.09	0.87	0.07	0.69
0.60	0.08	0.86	0.06	0.68
0.65	0.06	0.85	0.05	0.68
0.70	0.05	0.84	0.04	0.67
0.75	0.04	0.83	0.03	0.66
0.80	0.03	0.82	0.02	0.66
0.85	0.02	0.82	0.02	0.65
0.90	0.01	0.81	0.01	0.65
0.95	0.01	0.81	0.01	0.64
0.99	0.48	1.18	0.32	0.90

Table 4: Variances (*a* = 0.5)

			***c* random with 0.5 standard deviation**			
alpha	**var(π)**	**var(y)**	**loss**	**var$_c$(π)**	**var$_c$(y)**	**loss**
0.05	1.86	3.38	1.93	1.91	4.91	2.06
0.10	1.87	3.22	2.00	1.88	4.24	2.11
0.15	1.88	3.12	2.07	1.87	3.88	2.17
0.20	1.90	3.06	2.13	1.87	3.66	2.22
0.25	1.91	3.01	2.19	1.87	3.50	2.28
0.30	1.92	2.97	2.24	1.87	3.39	2.33
0.35	1.94	2.95	2.29	1.88	3.31	2.38
0.40	1.95	2.93	2.34	1.89	3.24	2.43
0.45	1.96	2.91	2.39	1.90	3.19	2.48
0.50	1.97	2.90	2.44	1.903	3.15	2.53
0.55	1.98	2.89	2.48	1.91	3.12	2.58
0.60	1.99	2.88	2.53	1.92	3.09	2.62
0.65	2.00	2.88	2.57	1.93	3.07	2.67
0.70	2.01	2.88	2.61	1.93	3.05	2.71
0.75	2.02	2.87	2.66	1.94	3.03	2.76
0.80	2.02	2.87	2.70	1.94	3.02	2.80
0.85	2.03	2.87	2.74	1.95	3.00	2.85
0.90	2.04	2.87	2.78	1.96	2.99	2.89
0.95	2.04	2.87	2.83	1.96	2.98	2.93

rule must involve all of the contemporaneous variables, including the exogenous variables, X_t, since they help forecast future deviations in inflation and output from equilibrium. The forecast-based version of the optimal rule, on the other hand, does not have to refer explicitly to the exogenous variables X_t.

In fact, the forecast-based rule needs to refer to forecasts of inflation and output only up to the horizon over which the changes in the exogenous variables are expected to persist, since forecasts of inflation and output up to that horizon are sufficient to forecast inflation and output at all horizons. Thus, for a country such as Canada, where the exogenous variables are mainly variables that describe commodity prices or the state of the economy in the United States, and that are expected to return to equilibrium within a two- to three-year period, the forecast-based optimal rule needs to refer only to forecasted deviations of inflation and output up to a two- to three-year horizon.

The forecast-based version can therefore provide a more parsimonious representation of the optimal rule than the Taylor-type version, and furthermore, should be more efficient than simple Taylor-type rules that ignore changes in exogenous variables.[23] However, this apparent advantage of forecast-based rules must be strongly qualified by the fact that, in order to implement these rules, one must use the full model anyway to evaluate the forecasts, and that requires taking into consideration changes in all the variables in the model, including the exogenous ones.

3.4 Model uncertainty

Suppose that there are several possible models of the transmission mechanism, only one of which is true. (The alternative models are all backward-looking, but differ with respect to the elasticities attached to the variables.) What then is the optimum rule for monetary policy?

One can tackle this question within the framework described above if one approaches the problem from a Bayesian perspective. That is, one first assigns a probability to each model of being the true model, and then one applies the analysis above to derive the optimum rule under the assigned parameter uncertainty.[24]

There is an important caveat to this approach, however. Under the formulation of the objective function used in the previous sections, what matters for welfare are the ex ante expected

23. Indeed, Armour, Fung, and Maclean (2002) find that in the Quarterly Projection Model used at the Bank of Canada, inflation-forecast-based rules do significantly better than Taylor-type rules where the policy instrument responds to deviations of output and inflation from equilibrium.

24. Note that it also depends on subjective beliefs.

deviations in the state variables. A rationale is that the uncertainty about the outcome results in a welfare cost. However, one can argue that what matters for welfare are the deviations that take place, i.e., the deviations ex post. In that case, any given monetary policy must be evaluated separately in each model. The true welfare function is likely to combine both ex ante expectations, arising from, for instance, risk aversion, and ex post deviations resulting from the direct effects of disequilibrium on welfare.[25]

3.5 Data uncertainty

Suppose now that the parameters $a_t, b_t, c_t, d_t, \Phi_t, \Psi_t, \Gamma_t$, and α_t are known and constant,[26] but the state variables y_t, π_t, r_t, and X_t are not observed at time t: let μ_t, ν_t, υ_t, and τ_t denote their forecast errors respectively (e.g., $y_t = E_t(y_t) + \mu_t$). It is well known that as an immediate consequence of the certainty-equivalence property of the model, the optimum rule under these conditions has the form

$$r_t = AE_t(y_t) + BE_t(\pi_t) + CE_t(X_t), \tag{15}$$

where coefficients A, B, and C are identical to those obtained when there is no data uncertainty. Thus, data uncertainty has no bearing on the optimum monetary policy rule. Of course, the larger the uncertainty about the data, the less reliable are the forecasts, $E_t(y_t)$, $E_t(\pi_t)$, and $E_t(X_t)$, and the greater the potential for error, but the optimal response to the forecasts remains the same.

The same conclusion, however, may not be true if policies are restricted to a particular form (e.g., Taylor-type rules with a restricted number of variables), or if policies are formulated in terms of some measure of the state variables that does not use all relevant information. In that case, data uncertainty may require either stronger or weaker responses to shocks than in the certainty case, depending on the type of policy.

To illustrate, suppose that the true value of π_t is observed, but not the output gap, and that the monetary rules pursued by the central bank are of the form

25. It is not immediately clear how the two approaches relate to each other. For example, it is not clear whether a rule that is found to be efficient in every model separately would be efficient with respect to minimizing ex ante variability, and vice versa. One attractive feature of the approach based on parameter uncertainty is that it provides an already established framework within which to derive an efficient rule under model uncertainty. Moreover, because of the quadratic form of the loss function, such a rule would automatically put more weight on avoiding worst case scenarios, as intuition would suggest.

26. In fact, we need only the parameters a_t, b_t, c_t, d_t to be known and constant.

$$r_t = \hat{A}\pi_t + \hat{B}y_t^m,$$

where y_t^m is a forecast of the output gap that does not use all available information. Then, a priori, it is unclear how the optimal coefficients $\hat{A}$ and $\hat{B}$ would relate to the coefficients A and B that would be optimal if the output gap is in fact observed. For instance, if y_t^m is based on a measure of potential output that relies mostly on past observations of output, as is the case with many filter-based measures, on the one hand, the larger the uncertainty about the output gap, the larger the weight $\hat{A}$ one may want to place on contemporaneous inflation, on the basis that contemporaneous inflation becomes a relatively more reliable indicator for the output gap. On the other hand, contemporaneous inflation may not be a good indicator of the contemporaneous output gap because of lagged effects on prices, in which case one must exercise caution in reacting too strongly to inflation. Consequently, the overall effect of uncertainty about the output gap on $\hat{A}$ is ambiguous.

Likewise, the implications for the response coefficient $\hat{B}$ are not clear a priori. Suppose, for instance, that in a given period, potential output is suspected to have increased and, accordingly, higher output is observed in the economy. The true output gap is therefore negative, and large or small depending on how quickly actual output catches up with the increase in potential. If actual output adjusts quickly to changes in potential, then the true output gap is likely to be relatively small, while the measured output gap, y_t^m, is likely to be positive (because the measure of potential is based on past observations) and relatively large. Under these circumstances, a more cautious policy reaction to the measured output gap, i.e., a smaller response coefficient $\hat{B}$ (than the one without uncertainty), would be warranted. But, if actual output adjusts very slowly to the change in potential (as might be the case in the wake of industrial restructuring or a large technological innovation), then the true output gap is likely to be relatively large (and negative), while the measured output gap, y_t^m, is likely to be found negative but smaller than the true output gap. A stronger reaction would be called for to help close the gap more quickly.

In any event, one is bound to make a forecast, perhaps based on judgment, about the true state variables to infer the right monetary policy response under data uncertainty.[27]

4. The Case of a Small Open Economy

The analysis so far has been restricted to a closed economy. In a small open economy like Canada, an important role in the transmission mechanism must be assigned to the exchange rate and exogenous variables such as real commodity prices. Consider, therefore, the extended model[28]

$$\pi_{t+1} = a\pi_t + dy_t - f(e_t - e_{t-1}) + \Psi X_t + \eta_{t+1} \tag{16}$$

$$y_{t+1} = by_t - cr_t - ge_t + \Phi X_t + \varepsilon_{t+1} \tag{17}$$

$$e_t = hr_t + \Omega X_t + \nu_t, \tag{18}$$

where e_t is the percentage deviation of the real exchange rate from its equilibrium (assumed for now to be constant)—a greater *e* means appreciation of the domestic currency; X_t is a vector of exogenous variables observed at the beginning of period *t*, before any monetary action is taken; $X, \varepsilon, \eta, \nu$ are assumed to be white noise; and $h \geq 1$. For now, all of the model's parameters are assumed to be known and constant.

This is essentially the baseline closed-economy model of section 2, with the exchange rate and the exogenous variables X_t added as new explanatory variables. The exchange rate affects demand through foreign trade, and the change in the exchange rate affects inflation through import prices because, for instance, foreign firms desire constant real prices in their home currencies. Exchange

27. Orphanides (1998) argues that since implementation of the rule $r_t = A\pi_t^m + By_t^m$ would add noise to the true optimal rule because of the measurement errors, one ought to respond cautiously to changes in measured state variables. (Orphanides uses this argument to explain why historical monetary reactions to shocks appear to be more cautious than model-based optimal reactions derived ex post from the revised historical series of output and inflation.) However, this claim depends on the assumption that the measurement errors are indeed noise, i.e., uncorrelated or at least positively correlated with the true variables so that the variances of the measured variables are greater than those of the true variables. But it is not clear that this is the case for the measures used in practice. Of course, in the case where the measured variables are the best forecasts, the measurement errors are negatively correlated with the true state variables. We suspect that is the case for the majority of measures used, although the variances of the measures may still be greater than the variances of the true variables. In fact, of eight different real-time measures of the output gap listed in Orphanides and van Norden (1999, Table 1), four have a smaller variance than the revised values of the output gap (which can be considered an approximation of the true output gap). (See Swanson (2000) for an interpretation of Orphanides' argument as a signal extraction problem.)

28. Except for the exogenous variables X_t, this is the model used by Ball (1997).

rate fluctuations are assumed to affect domestic demand and prices with a one-period lag. The rationale for equation (17) linking the interest rate and exogenous variables to the exchange rate is that a rise in the interest rate or such variables as commodity prices increases the demand for domestic currency. The shock ν captures other influences on the exchange rate, such as shifts in expectations and investor confidence.

4.1 Optimal rules

If the direct effect of the exchange rate on inflation is ignored, i.e., $f = 0$, then the present Phillips curve and demand equation in equations (15) and (16) are identical to those described earlier for the closed economy (with exogenous variables added) once the term $\frac{cr_t + ge_t}{c}$ rather than r_t is thought of as the policy instrument. In that case, it follows that the optimal rule in the small open economy has the form

$$cr_t + ge_t = Ay_t + B\pi_t + CX_t,$$

where the response coefficients A, B, and C equal those obtained in the closed-economy case up to the constant *c*.[29] Thus, when the direct effect of the exchange rate on inflation is ignored, the expression of the optimal rule does not require knowledge of the exchange rate-interest rate relationship embodied in equation (17). This is particularly useful, since the exchange rate-interest rate relationship is known to be difficult to evaluate.

Dividing by $c + g$, the above rule can be written in the form

$$wr_t + (1 - w)e_t = Ay_t + B\pi_t + CX_t,$$

where the weights on the real interest rate and the real exchange rate, w and $1 - w$, are proportional to the coefficients c and g in the demand equation. $wr_t + (1 - w)e_t$ can be thought of as a monetary conditions index (MCI).

If $f \neq 0$, then the optimal rule (18) must be adjusted to take into consideration the direct effect of the exchange rate on inflation through import prices. In this case, one can show that the optimal rule takes the form[30]

29. As noted in the previous section, *A* and *B* are independent of the coefficients on the exogenous variables. Moreover, if $\Psi = 0$ then C equals Φ. To see this clearly, think of the term $cr_t + ge_t - \Phi X_t$ as the instrument of policy and identify the model with the closed-economy model.

30. See Ball (1997) or Srour (1999).

$$w'r_t + (1 - w')e_t = A'y_t + B'\left(\pi_t + \frac{f}{a}e_{t-1}\right) + C'X_t, \tag{19}$$

where the coefficients w', A', and B' also now depend on the coefficients f and h in the interest rate-exchange rate relationship.[31] Because the direct effect of the exchange rate on inflation is transitory, the optimal rule calls for a response to the deviation in inflation that excludes the previous period's effect of the exchange rate, i.e., it calls for a response to the term $\pi_t + \frac{f}{a}e_{t-1}$.

The relative weight on the exchange rate, $\frac{1-w'}{w'}$, in the case $f \neq 0$, ought always to be larger than the relative weight, $\frac{1-w}{w}$, in the case $f = 0$, since in the former case an increase in the exchange rate is presumed to have a direct dampening effect on inflation. However, the comparison between the optimal response coefficients in the two cases is a priori ambiguous. On the one hand, the response coefficients A', B', and C' ought to be smaller when $f \neq 0$, to the extent that the direct effect of the exchange rate on inflation reinforces the desired effect of monetary action following an inflationary shock. On the other hand, A', B', and C' ought to be larger, to the extent that the direct effect can be used to control inflation more quickly and this requires a larger increase in the exchange rate than would be needed if inflation were affected through demand only.

In any case, the fact that the direct effect of a change in the exchange rate on inflation is relatively small and transitory implies that the cumulative effect of a change in the exchange rate on inflation is also relatively small. It follows that an attempt to control inflation through this channel will require large fluctuations in the exchange rate, hence large fluctuations in the interest rate and output. One would therefore expect that, except in the case where the policy-maker places a relatively large weight, $1 - \alpha$, on inflation in the loss function, the optimal response coefficients ought to be fairly close to the response coefficients obtained when one ignores the direct effects of the exchange rate on inflation. In particular, the relative weights on the interest rate and the exchange rate in the MCI introduced above should be approximately proportional to the effects that these variables have on demand.

Table 5 confirms these claims. It provides the ratio $\frac{w'}{1-w'}$ and the optimal response coefficients A' and B' under the specification

$$a = 1, b = 0.8, c = 0.6, g = 0.2, d = 0.4, f = 0.2, h = 2,$$

31. w', A', and B', however, continue to be independent of the coefficients Φ, Ψ, Ω on the vector of exogenous variables X_t. Also, C' does not depend on Ω: the effect of the exogenous variables on the exchange rate is subsumed in the level of the exchange rate.

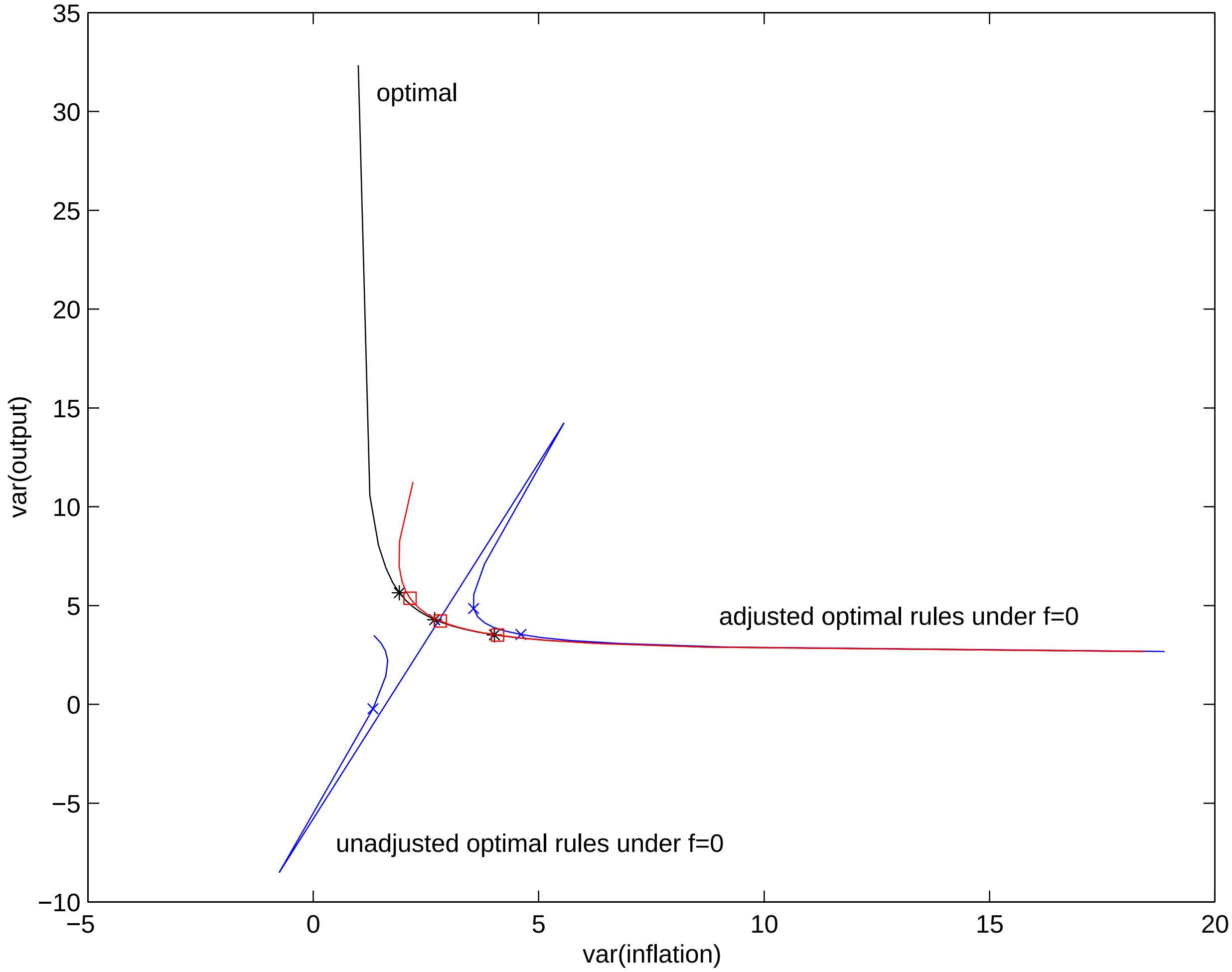
Table 5: Optimal coefficients, open-economy case

α	f = 0.2			f = 0.0		
	ratio	π	**y**	**ratio**	π	**y**
0.00	0.00	4.99	2.00	3.00	3.13	2.25
0.05	0.67	3.05	1.75	3.00	2.48	1.99
0.10	1.03	2.46	1.66	3.00	2.12	1.85
0.15	1.29	2.11	1.60	3.00	1.88	1.75
0.20	1.51	1.86	1.55	3.00	1.69	1.68
0.25	1.69	1.67	1.51	3.00	1.54	1.62
0.30	1.84	1.51	1.47	3.00	1.41	1.57
0.35	1.98	1.38	1.44	3.00	1.30	1.52
0.40	2.10	1.26	1.41	3.00	1.20	1.48
0.45	2.21	1.16	1.38	3.00	1.11	1.44
0.50	2.31	1.07	1.36	3.00	1.02	1.41
0.55	2.40	0.98	1.33	3.00	0.94	1.38
0.60	2.48	0.89	1.31	3.00	0.87	1.35
0.65	2.56	0.81	1.28	3.00	0.79	1.32
0.70	2.63	0.73	1.26	3.00	0.72	1.29
0.75	2.70	0.65	1.23	3.00	0.64	1.26
0.80	2.77	0.57	1.21	3.00	0.57	1.23
0.85	2.83	0.49	1.18	3.00	0.48	1.19
0.90	2.89	0.39	1.15	3.00	0.39	1.16
0.95	2.94	0.27	1.11	3.00	0.27	1.11
0.99	2.99	0.12	1.05	3.00	0.12	1.05

as well as the optimal response coefficients A and B when f is assumed to equal 0. Figure 3 plots the efficiency frontier, the frontier associated with the optimal rules derived under the assumption $f = 0$, and the frontier associated with the optimal rules derived under the assumption $f = 0$ and adjusted to exclude the transitory effects of the exchange rate on inflation.

It is apparent that, in the middle range of values for α, the rules derived under the assumption $f = 0$ and that respond only to inflation that excludes transitory effects of the exchange rate rather than overall inflation are nearly as efficient as the fully optimal rules. In contrast, rules that respond to overall inflation, and hence to transitory effects of the exchange rate, can lead to unstable outcomes.

Figure 3: Efficiency frontiers



Note: The straight diagonal segment in the graph above is a fluke of the program, and should be ignored. It corresponds to cases that do not, in fact, admit finite variances.

For example, when an equal weight is placed on output and price stability ($\alpha = 0.5$), the optimum rule under the specifications above is

$$0.7r_t + 0.3e_t = 1.36y_t + 1.07(\pi_t + 0.2e_{t-1}),$$

and the variances of output and inflation are respectively 4.29 and 2.69. If the indirect effect of the exchange rate on inflation is ignored, i.e., $f = 0$, then the adjusted optimal rule would be

$$0.75r_t + 0.25e_t = 1.41y_t + 1.02(\pi_t + 0.2e_{t-1}),$$

and the variances of output and inflation would be respectively 4.22 and 2.82, whereas the unadjusted optimal rule would be

$$0.75r_t + 0.25e_t = 1.41y_t + 1.02\pi_t,$$

and the variances of output and inflation would be 4.85 and 3.56, respectively.

Static changes in α, δ, and the model's parameters can be shown to have similar effects on the response coefficients as in the closed-economy case (see section 3), whereas the effect on the relative weight $\frac{w'}{1-w'}$ can be ambiguous. It is worth mentioning, however, that a decrease in the degree of persistence of inflation, *a*, leads the policy-maker to respond less sharply to inflationary shocks, hence to put less weight on autonomous exchange rate shocks, i.e., to choose a larger ratio $\frac{w'}{1-w'}$.

4.2 Supply shocks

So far, potential output, the equilibrium interest rate, and the equilibrium real exchange rate have been assumed to be constant (or at least growing at a steady rate in the case of potential output). Suppose now that these variables vary in time—denote them y^*_t, r^*_t, and e^*_t respectively. Potential output may vary because of productivity shocks, changes in the supply of factors of production, or structural changes in the economy. The equilibrium interest rate and equilibrium real exchange rate may vary as a result of changes in potential output (e.g., changes in productivity) or because of autonomous factors, such as changes in the risk premium due to domestic or external shocks or changes in commodity prices. The three variables are therefore closely, but not perfectly, correlated.

If the economy adjusts symmetrically to changes in supply, e.g., potential output and demand, and to changes in the actual and equilibrium interest rates and exchange rates, then what matters for the conduct of monetary policy are the deviations of the state variables from equilibrium. Whether the deviations are driven by a demand shock or a supply shock is immaterial. Thus, an increase in potential output, which translates into a drop in the output gap, would call for a similar ease in monetary conditions *relative* to equilibrium as would a decrease in demand. However, whether this implies an increase or a decrease of the *level* of the MCI depends on whether the equilibrium level of the MCI has risen in relation to the change in potential output, and by how much.

An autonomous change in the equilibrium level of the monetary conditions index that is unrelated to a change in contemporaneous potential output, on the other hand, ought to be accommodated by a similar change in the actual level of monetary conditions. For example, an autonomous drop in the exchange rate, resulting from portfolio rebalancing and accompanied by an ensuing increase in equilibrium (e.g., long-run) interest rates, ought to be accommodated by a corresponding increase in (short-term) interest rates.[32]

4.3 Parameter uncertainty

Consider now the case where the coefficients c and g on the interest rate and the exchange rate in the demand equation are uncertain. As in the closed-economy case with c random, such uncertainty calls for more cautious monetary responses to shocks, i.e., smaller response coefficients, A', B', and C', than in the case where all parameters are known (see [S]). The direction in which the weights w' and $1-w'$ ought to be adjusted, however, depends on the relative degrees of uncertainty on c and g: a larger degree of uncertainty on the coefficient g of the exchange rate requires a larger response to autonomous changes in the exchange rate, hence a larger weight $1-w'$ on the exchange rate in the policy rule, in order to reduce the uncertainty of the effects of the exchange rate on demand.

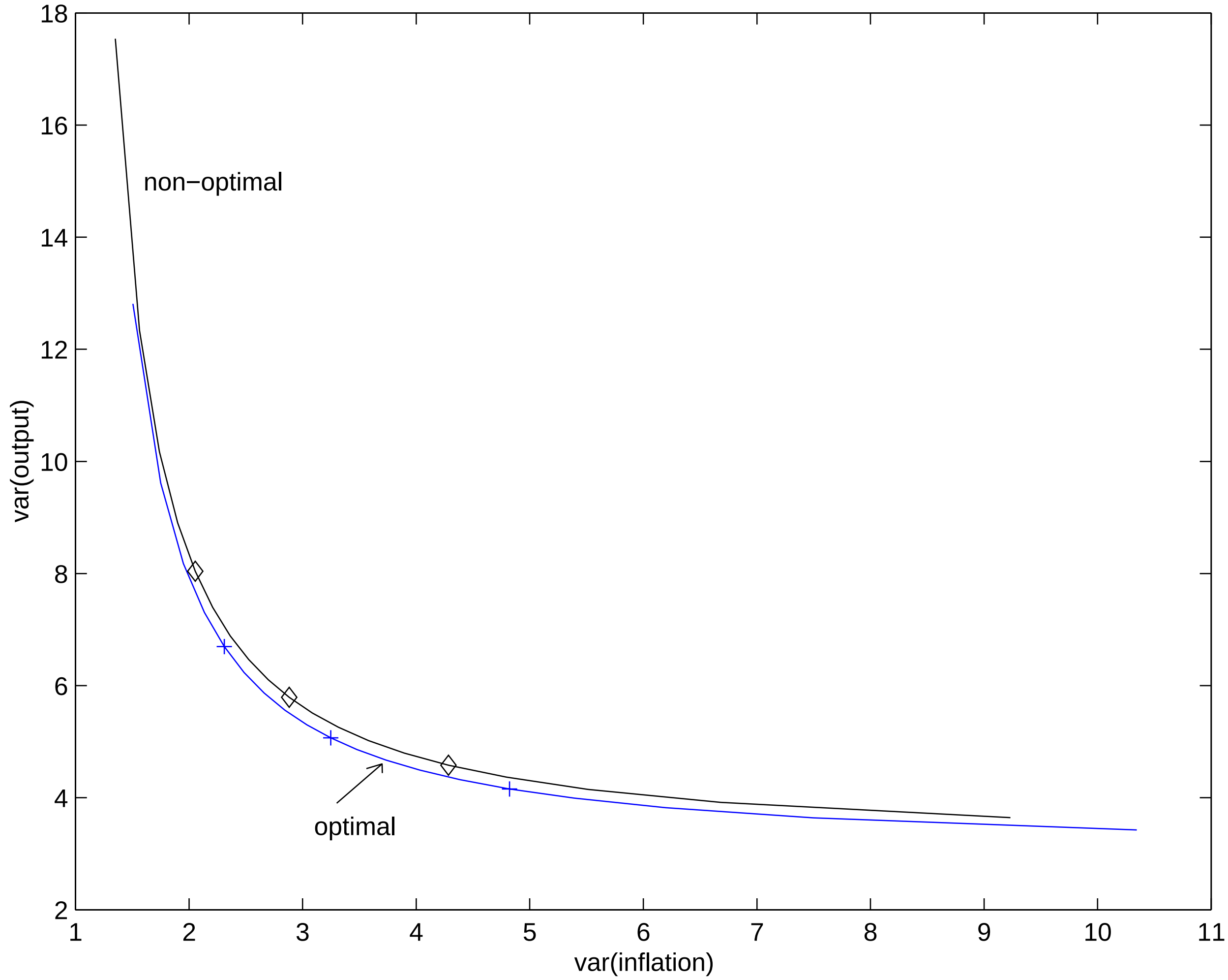
Table 6 provides the optimal coefficients and weights under alternative assumptions about the magnitude of the parameter uncertainty and different choices of the relative weight α in the loss function. The table also provides the variances of output and inflation under the optimal rule as well as under the optimal rule that ignores uncertainty. Figure 4 plots the efficiency frontier in the case where the standard deviations on c and g are 0.3 and 0.2 respectively, as well as the frontier associated with the optimal rules that ignore uncertainty. As in the closed-economy case, ignoring uncertainty leads more to a lateral displacement of the efficiency frontier than to a level shift.

32. One small qualification to the previous argument is that in the Phillips curve, inflation ought to depend on the change in the level of the exchange rate, which is not the same as the change in the deviation of the exchange rate from equilibrium if the equilibrium exchange rate is not constant. As argued in the previous section, however, this should have little consequence for the optimal rule, given the small magnitude and transitional nature of the direct effects of the exchange rate on inflation.

Table 6: Response coefficients and variances

	α	ratio	π	y	var(π)	var(y)	var$_c$(π)	var$_c$(y)
$\sigma_c = 0.0$, $\sigma_g = 0.0$	0.25	1.69	1.67	1.51	1.91	5.65		
	0.50	2.31	1.07	1.36	2.69	4.29		
	0.75	2.70	0.65	1.23	4.02	3.52		
$\sigma_c = 0.3$, $\sigma_g = 0.0$	0.25	2.04	1.50	1.38	2.04	6.01	1.95	6.32
	0.50	2.71	0.96	1.23	2.87	4.55	2.75	4.72
	0.75	3.13	0.59	1.11	4.29	3.74	4.10	3.83
$\sigma_c = 0.0$, $\sigma_g = 0.1$	0.25	1.63	1.62	1.48	1.96	5.82	1.93	5.93
	0.50	2.16	1.04	1.33	2.77	4.42	2.72	4.47
	0.75	2.49	0.64	1.21	4.13	3.63	4.05	3.66
$\sigma_c = 0.0$, $\sigma_g = 0.2$	0.25	1.47	1.49	1.39	2.14	6.32	1.99	6.96
	0.50	1.81	0.96	1.24	3.02	4.79	2.80	5.14
	0.75	2.01	0.60	1.13	4.50	3.94	4.18	4.14
$\sigma_c = 0.3$, $\sigma_g = 0.1$	0.25	1.96	1.46	1.35	2.10	6.18	1.97	6.68
	0.50	2.53	0.93	1.20	2.96	4.68	2.78	4.95
	0.75	2.88	0.58	1.09	4.41	3.84	4.14	3.99
$\sigma_c = 0.3$, $\sigma_g = 0.2$	0.25	1.75	1.34	1.27	2.31	6.70	2.05	8.04
	0.50	2.11	0.87	1.13	3.25	5.07	2.88	5.79
	0.75	2.32	0.54	1.03	4.82	4.16	4.28	4.58

Figure 4: Efficiency frontiers $\sigma_c = 0.3, \sigma_g = 0.2$



5. Estimated VARs

The previous results relied on calibrated models. In this section, we estimate the transmission mechanism in Canada by means of a vector autoregression (VAR), and derive the optimal rules associated with this model. The VAR involves two blocks of variables. One consists of the United States growth in real GDP, the percentage change in a real non-energy commodity price index, the percentage change in the price of oil, a measure of the United States CPI inflation rate, and the United States real federal funds rate. This block of variables is assumed to be exogenous, reflecting the fact that Canada is a small economy. The second block consists of the Canadian output gap, inflation, the percentage change in the Canada-U.S. real exchange rate, and the real yield spread, that is, the difference between the 90-day prime corporate paper and the 10-year-and-over Government of Canada bond yield average deflated by the CPI inflation rate, excluding food and energy and the effect of changes in indirect taxes. We view the real yield spread as a proxy for the deviation of the short-term real interest rate, or instrument of monetary policy, from its equilibrium.

The data cover the period from 1961 to 1999, at annual frequency. The measure of real GDP in the United States is the U.S. Department of Commerce chain volume real GDP measure, in 1996 prices, seasonally adjusted at annual rates. The inflation measure is the log difference of the CPI, excluding food and energy. The measure of output in Canada is the real GDP in 1992 prices,[33] seasonally adjusted at annual rates, and the measure of inflation is the log difference of the CPI, excluding food, energy, and the effect of changes in indirect taxes. Real crude oil prices and the real non-energy commodity price index are both in U.S. dollars deflated by the U.S. consumer price index (excluding food and energy). The real exchange rate is defined as the U.S.-Canada nominal exchange rate (e.g., the price of a unit of domestic currency in terms of the U.S. currency) multiplied by the ratio of the Canadian GDP deflator to the U.S. GDP deflator.

The VAR is identified via standard Choleski decompositions, where the variables are ordered in the manner they are listed above. The monetary policy instrument is placed last to capture the idea that monetary policy may adjust to current events but its effects on output and prices occur with a lag. A single lag on each variable is used, and the coefficient on lagged inflation in the output equation is constrained to equal 0.

Table 7 provides the estimated reduced-form Canadian demand equation and Phillips curve. One apparent difference with the stylized model used earlier is that while the coefficient on the lagged real yield spread in the Phillips curve is smaller than in the output equation, it is nevertheless significant.

In deriving the optimal rules, a weight of 0.05 on changes in the real yield spread was incorporated in the loss function.[34] Incorporating such a weight has little effect on the loss due to output and inflation variability under the optimal rule, but it reduces substantially the variability of interest rates. The latter is implausibly large without such a constraint. The presence of such a weight also implies that the optimal reaction function will involve a certain degree of persistence (represented by a coefficient on lagged interest rates) in addition to responses to deviations of inflation, output, and the exchange rate from equilibrium.

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33. Our measures do not incorporate the changes in the measures of GDP in the national accounts since May 2001. For real GDP at market prices, these changes involve a move to chain volume measures, while those for real GDP at factor cost involve a move to real GDP at basic prices.

34. In other words, the loss function now has the form $L(\pi, y) \equiv \alpha(y - y^*)^2 + (1 - \alpha)(\pi - \pi^*)^2 + 0.05(r_t - r_{t-1})^2$.

Table 7: Reduced-form estimated equations

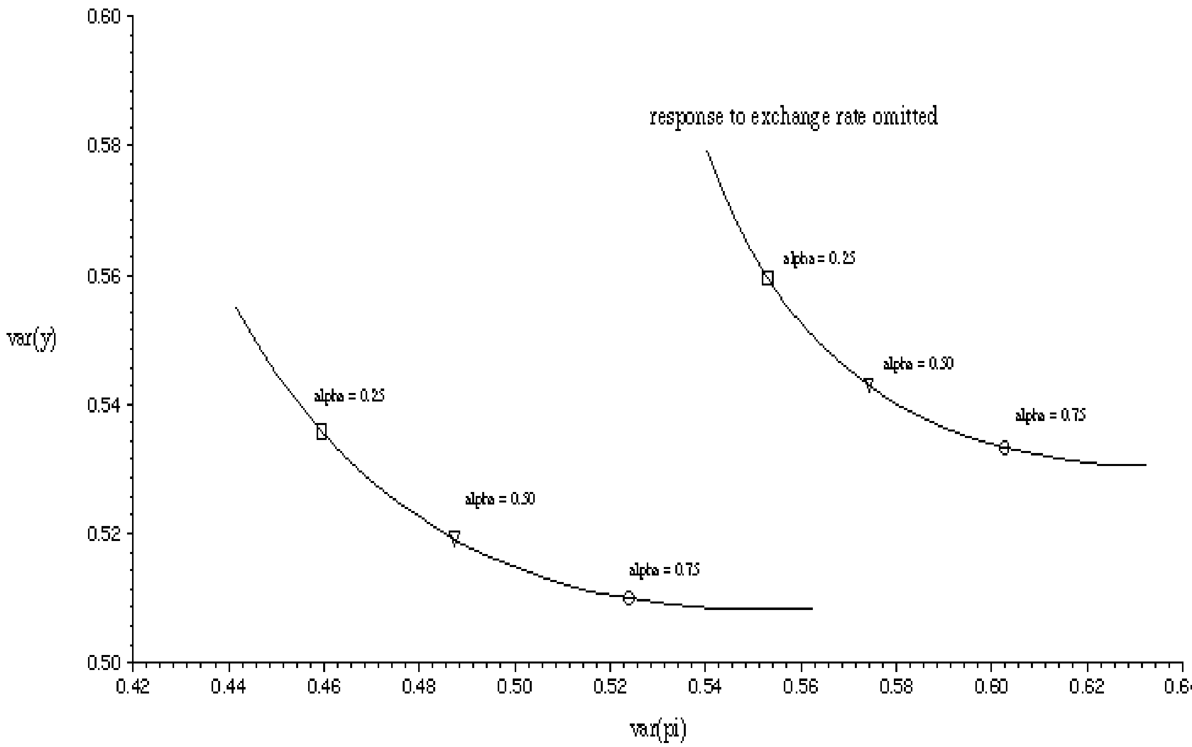
Variable	**output equation**	**inflation equation**
Constant	–0.22 (0.83)	0.40 (0.50)
U.S. gap	0.43 (0.07)	0.11 (0.42)
Oil	–0.01 (0.48)	0.01 (0.40)
Commodity prices	0.05 (0.09)	0.04 (0.03)
U.S. inflation	–0.24 (0.20)	0.30 (0.02)
RFF	–0.19 (0.11)	0.01 (0.83)
Core inflation (Canada)		0.47 (0.01)
Canada gap	0.35 (0.26)	0.46 (0.02)
Real exchange rate	–0.05 (0.48)	–0.09 (0.02)
Real yield spread	–0.49 (0.02)	–0.28 (0.08)

Table 8 shows the optimal response coefficients, and Figure 5 plots the efficiency frontier when the economy is subject only to domestic shocks. For example, when an equal weight is placed on output and inflation variability, the optimal rule is

Table 8: Response coefficients based on estimated VAR

α	π	$\boldsymbol{y}$	$\Delta \boldsymbol{e}$	$\boldsymbol{r}(-\mathbf{1})$
0.05	0.56	0.62	–0.15	0.16
0.10	0.52	0.61	–0.15	0.15
0.15	0.49	0.60	–0.14	0.15
0.20	0.43	0.59	–0.14	0.15
0.25	0.43	0.58	–0.13	0.15
0.30	0.40	0.58	–0.13	0.15
0.35	0.37	0.57	–0.12	0.10
0.40	0.34	0.56	–0.12	0.15
0.45	0.31	0.55	–0.12	0.14
0.50	0.28	0.55	–0.11	0.14
0.55	0.25	0.54	–0.11	0.14
0.60	0.22	0.53	–0.10	0.14
0.65	0.19	0.53	–0.10	0.14
0.70	0.16	0.52	–0.09	0.14
0.75	0.14	0.51	–0.09	0.14
0.80	0.11	0.51	–0.09	0.14
0.85	0.08	0.50	–0.08	0.14
0.90	0.05	0.49	–0.08	0.14
0.95	0.02	0.48	–0.07	0.14

Figure 5: Efficiency frontier based on the estimated VAR



$$r_t = 0.55y_t + 0.28\pi_t - 0.11\Delta e_t + 0.14r_{t-1}.$$

The long-run responses to output and inflation are therefore 0.64 and 0.32, respectively. Rules with standard Taylor coefficients and without interest persistence, i.e.,

$$r_t = 0.5y_t + 0.5\pi_t - 0.11\Delta e_t,$$

or

$$r_t = y_t + 0.5\pi_t - 0.11\Delta e_t,$$

achieve almost equal total loss with respect to output and inflation variability alone compared with the optimal rule—the variances of output and inflation are 0.53 and 0.47 for the former rule, and 0.54 and 0.44 for the latter—but the variability of the interest rate is almost twice as large. Interestingly, omitting the response to the change in the exchange rate from the optimal rules, shifts the efficiency frontier significantly to the right (see Figure 5).

In so far as the outcomes of the optimal rules for values of the weight α in the middle range are fairly close to each other, the results based on the estimated VAR are consistent with those obtained earlier with calibrated models when the degree of persistence in inflation, a, is relatively small.

6. Conclusion

This paper explores the role that Taylor-type rules may play in the conduct of monetary policy. In contrast to the literature that examines efficient rules in the context of large macroeconometric models, this paper uses small, stylized, calibrated, backward-looking models. Although these models are simplistic, and subject to the Lucas critique, they can still act as a very useful benchmark for more complex models, and they are easily amenable to investigating the robustness of rules and the effects of uncertainty.

We showed that if the degree of persistence of inflation in the Phillips curve is not high (below 0.5, for example), then a simple rule that mostly offsets demand shocks, and puts a relatively small weight on inflation shocks, is efficient. Furthermore, such a rule appears to be robust to alternative preferences regarding the relative weight on inflation and output stability, the presence of diverse exogenous variables and, as long as the model's key relationships are relatively stable, parameter uncertainty.

We would therefore submit that a Taylor-type rule such as the above, for example, one that puts a weight 0.75 or 1 on the output gap, a weight 0.5 on deviations of inflation from target, a small degree of persistence in policy and, in the case of Canada, perhaps a small negative response to changes in the exchange rate, can act as a reasonable benchmark for monetary policy responses to domestic shocks. Inevitably, of course, uncertainty and the diversity of shocks affecting the economy require policy-makers to adjust their policy responses to particular events as appropriate on a case-by-case basis.

This type of rule differs from the type usually found in the literature in the context of established macroeconometric models. However, these studies assume commitment to the rule, whereas the optimal rules derived in this paper are better compared with time-consistent rules. We have argued that time-consistent rules are more reasonable choices for public policy. Nonetheless, since the models we used are backward-looking, it would be most useful (and relatively easy) to examine how the introduction of forward-looking elements in the model might affect efficient rules. And finally, although we provide some results in the open-economy case, our main focus has been on domestic shocks. Further study is needed to analyze responses to foreign shocks.

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Appendix A. Comparative Statics

Elementary algebraic manipulation shows that *k* is a constant between 0 and *a* which increases with α and *a*, and decreases with d and δ; *k* is, however, independent from the parameters *b* and *c* in the IS curve. The interpretation is as follows.

The coefficient *k* reflects the trade-off between inflation and output as witnessed by the Phillips curve: bringing inflation down requires a temporary output contraction; the greater the output contraction, the greater the drop in inflation.[1] If no weight is placed on output stability ($\alpha = 0$), then the policy-maker will attempt to achieve the inflation target as quickly as possible ($k = 0$), but at the cost of large fluctuations in output. If a positive weight is placed on output stability ($\alpha > 0$), then following a shock the policy-maker will bring inflation back to its initial target more slowly ($k > 0$) so as to reduce the fluctuations in output—the greater is the weight α on output stability, the larger is the coefficient *k* and the more gradual is the adjustment of inflation. At the other extreme where the policy-maker is targeting only output (i.e., $\alpha = 1$), inflation returns to the target at the speed implied by the degree of mean reversion in the Phillips curve: $k = a$.

A larger coefficient *a* means that inflation is more persistent,[2] and would therefore return more slowly to the target level, ceteris paribus, hence *k* will be larger; whereas a larger *d* means that inflation responds more strongly to changes in the output gap, hence inflation will return more quickly to target, ceteris paribus, and *k* will be smaller. A larger δ means that a smaller weight is placed on short-run variations. To the extent that variations in output are needed early to stabilize inflation, a larger δ implies that a smaller weight is placed on output variability, and therefore leads to a greater speed of adjustment, i.e., a smaller *k*.[3]

Similarly, one can verify that the response coefficients *A* and *B* in the optimal instrument rule (6) increase with *a* and δ, and decrease with α. The interpretation is as follows. A larger *a* implies a more persistent inflation, and therefore requires sharper monetary policy responses to reduce deviations in inflation. A larger δ means that a smaller weight is placed on short-run variations

1. Note that from equations (2) and (6), under the optimal policy rule, the dynamics governing the movement of the output gap are summarized by

$$y_{t+1} - y^* = -(a-k)(y_t - y^*) - \frac{a(a-k)}{d}(\pi_t - \pi^*) + \eta_{t+1}.$$

A positive output gap at time *t* or an inflation rate that is higher than the target level leads to a negative output gap next period.

2. It may be more persistent because the central bank's policy is less credible or because agents are more backward-looking.
3. We thank Pierre Duguay for that observation.

and therefore induces stronger immediate reaction to shocks. It is sometimes mistakenly thought that a greater weight, α, on output stability calls for a greater output-response coefficient, *A*. This is not true in the present case because, following an inflationary shock, the actions taken by the monetary authorities are intended to achieve a temporary output contraction. Greater concern about output stability therefore implies that the output contraction should not be as sharp, hence it requires a smaller coefficient *A*; similarly for *B*.

A increases with *d*, whereas *B* decreases with *d*: the larger is the coefficient *d* on output in the Phillips curve, i.e., the larger is the effect of an output gap on future prices, the larger ought to be the output response coefficient *A*, in order to offset the greater expected deviation in inflation following a demand shock. But the smaller ought to be the inflation response coefficient *B*, since a smaller change in output is needed to achieve the desired effect on future inflation.

A increases with *b*, whereas *B* is independent from *b*: the larger is the coefficient *b* on output in the demand equation, i.e., the more persistent are changes in demand, the greater ought to be the response to a demand shock. In other words, the larger ought to be the output response coefficient *A* in order to offset the greater expected future output gap.

Finally, for obvious reasons, the optimal response coefficients *A* and *B* are inversely proportional to the real interest rate demand elasticity, *c*.

It must be emphasized that the above comparative results depend heavily on the specification of the loss function and the dynamic structure of the model, in particular the frequency of the data. However, similar types of interpretation ought to apply in general. For example, it may turn out that in a more complex model, say with quarterly data or perhaps a concern for interest rate variability incorporated in the objective function, the optimal rule calls for a smaller positive, rather than an immediately negative, output gap in the quarter following a positive demand shock. In that case, a greater weight, α, on output stability is likely to call for a greater, not smaller, response coefficient on contemporaneous output in order to further close the output gap in the quarter following a shock.

Appendix B. Commitment Versus Discretion: An Example

Consider the following model[4]

$$\begin{aligned} \pi_t &= \pi_{t+1|t} + y_{t|t-1} + \varepsilon_t \\ y_t &= -r_t \end{aligned}, \tag{B1}$$

and suppose that, at time *t*, the loss function is the expected discounted sum of squared deviations of inflation from a target (assumed equal to 0), e.g.,

$$E_t \sum_{i=0}^{\infty} \delta^i \pi_{t+i}^2 . \tag{B2}$$

Clearly, if in every period of time, policy-makers take bygones as bygones and seek to minimize the loss function described above, then, at time *t* forward, unless there is a new shock, ε_t, they would set output equal to 0 so as to achieve 0 inflation. Under the time-consistent optimal rule, the public would therefore expect future output and inflation to equal 0, hence $\pi_t = \varepsilon_t$, in which case the unconditional variance of inflation would equal the variance of ε_t, say normalized to equal 1.

As an alternative policy, suppose now that policy-makers commit to the rule

$$r_t = \pi_{t-1} . \tag{B3}$$

One can then show that inflation follows the process

$$\pi_t = a\pi_{t-1} - a\varepsilon_t , \tag{B4}$$

where $a = \frac{1-\sqrt{5}}{2}$, in which case the unconditional variance of inflation would equal $\frac{a^2}{1-a^2}$, which is smaller than 1. Thus, the time-consistent optimal rule does not perform as well as the alternative rule under commitment.

4. The notation is standard.

Appendix C. Solution to the Linear-Quadratic Problem Under Uncertainty

Suppose that one seeks to minimize the loss function

$$E_t \sum_{i=0}^{\infty} \delta^i (Z'_{t+i} R_{t+i} Z_{t+i} + u'_{t+i} Q_{t+i} u_{t+i}), \tag{C1}$$

subject to

$$Z_{t+1} = A_{t+1} Z_t + B_{t+1} u_t + e_{t+1}, \tag{C2}$$

where R_t and Q_t are negative semi-definite symmetric matrices of random weights, Z_t is a vector of state variables, u_t is a vector of control variables, e_t is a vector of random shocks, and A_t and B_t are matrices of random parameters. Z_t is not necessarily observed at time *t*: let μ_t be the forecast error, $Z_t = E_t(Z_t) + \mu_t$.

We assume that learning is passive. That is, in every period *T*, the policy-makers assume that actions taken at time *T* or earlier do not to affect the joint distribution of $\{R_t, Q_t, e_{t+1}, \mu_{t+1}, A_{t+1}, B_{t+1}\}_{t \geq T+1}$ conditional on all information available at time $T + 1$. Note that this assumption is fully rational if the series of parameters $\{R_t, Q_t, e_{t+1}, \mu_{t+1}, A_{t+1}, B_{t+1}\}$ is independently distributed over time. Let V_t denote the value function at time *t*.

Case A. Suppose that Z_t is observed at time *t*, before actions are taken.

We conjecture that V_t has the form

$$V_t = Z_t' P_t Z_t + 2 Z_t' W_t + d_t,$$

where P_t is a positive semi-definite symmetric matrix, W_t is a vector, and d_t is a scalar. P_t, W_t, and d_t are assumed to be unaffected by past choices of the control variable.

From the Bellman equation,

$$\begin{aligned} V_t &= min_u E_t\{Z'_t R_t Z_t + u'_t Q_t u_t + \delta V_{t+1}\} \\ &= min_u E_t\{Z'_t R_t Z_t + u'_t Q_t u_t + \delta Z'_{t+1} P_{t+1} Z_{t+1} + 2\delta Z'_{t+1} W_{t+1} + \delta d_{t+1}\}, \end{aligned}$$

one derives the first-order equation,

$$E_t[\delta B'_{t+1}P_{t+1}Z_{t+1} + \delta B'_{t+1}W_{t+1} + Q_t u_t] = 0\,.$$

Substituting the expression of Z_{t+1} in terms of Z_t and expanding, one deduces the optimal rule

$$u_t = F_t Z_t + G_t\,,$$

where

$$F_t = -\delta E_t[Q_t + \delta B'_{t+1}P_{t+1}B_{t+1}]^{-1}E_t[B'_{t+1}P_{t+1}A_{t+1}]$$

$$G_t = -\delta E_t[Q_t + \delta B'_{t+1}P_{t+1}B_{t+1}]^{-1}E_t[B'_{t+1}P_{t+1}e_{t+1} + B'_{t+1}W_{t+1}]\,.$$

Substituting back the expression of u_t into the Bellman equation and expanding Z_{t+1} in terms of Z_t, one obtains

$$\begin{aligned}
V_t &= E_t\{Z'_t R_t Z_t + u'_t Q_t u_t + \delta Z'_{t+1}P_{t+1}Z_{t+1} + 2\delta Z'_{t+1}W_{t+1} + \delta d_{t+1}\} \\
&= E_t\{Z'_t R_t Z_t\} + \delta E_t\{Z'_t A'_{t+1}P_{t+1}A_{t+1}Z_t\} + E_t\{u'_t[Q_t + \delta B'_{t+1}P_{t+1}B_{t+1}]u_t\} \\
&\quad + 2\delta E_t\{u'_t B'_{t+1}P_{t+1}A_{t+1}Z_t + u'_t B'_{t+1}P_{t+1}e_{t+1} + Z'_t A'_{t+1}P_{t+1}e_{t+1}\} \\
&\quad + \delta E_t\{2Z'_t A'_{t+1}W_{t+1} + 2u'_t B'_{t+1}W_{t+1} + 2e'_{t+1}W_{t+1} + e'_{t+1}P_{t+1}e_{t+1} + d_{t+1}\} \\
&= E_t\{Z'_t R_t Z_t\} + \delta E_t\{Z'_t A'_{t+1}P_{t+1}A_{t+1}Z_t\} \\
&\quad (-\delta E_t\{Z'_t A'_{t+1}P_{t+1}B_{t+1} + e'_{t+1}P_{t+1}B_{t+1} + W'_{t+1}B_{t+1}\}u_t) \\
&\quad + 2\delta E_t\{u'_t B'_{t+1}P_{t+1}A_{t+1}Z_t + u'_t B'_{t+1}P_{t+1}e_{t+1} + Z'_t A'_{t+1}P_{t+1}e_{t+1}\} \\
&\quad + \delta E_t\{2Z'_t A'_{t+1}W_{t+1} + 2u'_t B'_{t+1}W_{t+1} + 2e'_{t+1}W_{t+1} + e'_{t+1}P_{t+1}e_{t+1} + d_{t+1} \\
&= E_t\{Z'_t R_t Z_t\} + \delta E_t\{Z'_t A'_{t+1}P_{t+1}A_{t+1}Z_t + Z'_t A'_{t+1}P_{t+1}B_{t+1}u_t + e'_{t+1}P_{t+1}B_{t+1}u_t\} \\
&\quad + \delta E_t\{2Z'_t A'_{t+1}W_{t+1} + u'_t B'_{t+1}W_{t+1} + 2Z'_t A'_{t+1}P_{t+1}e_{t+1}\} \\
&\quad + \delta E_t\{e'_{t+1}P_{t+1}e_{t+1} + 2e'_{t+1}W_{t+1} + d_{t+1}\} \\
&= E_t\{Z'_t R_t Z_t\} + \delta E_t\{Z'_t A'_{t+1}P_{t+1}A_{t+1}Z_t + Z'_t A'_{t+1}P_{t+1}B_{t+1}F_t Z_t\} \\
&\quad + \delta E_t\{2Z'_t A'_{t+1}W_{t+1} + Z'_t F'_t B'_{t+1}W_{t+1} + Z'_t A'_{t+1}P_{t+1}B_{t+1}G_t + e'_{t+1}P_{t+1}B_{t+1}F_t Z_t \\
&\quad + \delta E_t\{2Z'_t A'_{t+1}P_{t+1}e_{t+1}\} \\
&\quad + \delta E_t\{G_t' B'_{t+1}W_{t+1} + 2e'_{t+1}W_{t+1} + e'_{t+1}P_{t+1}e_{t+1} + e'_{t+1}P_{t+1}B_{t+1}G_t + d_{t+1}\}
\end{aligned}$$

Comparing the latter expression with that of V_t, it follows that

$$P_t = E_t[R_t] + \delta E_t[A'_{t+1}P_{t+1}A_{t+1}]$$
$$-\delta^2 E_t[A'_{t+1}P_{t+1}B_{t+1}]E_t[Q_t + \delta B'_{t+1}P_{t+1}B_{t+1}]^{-1}E_t[B'_{t+1}P_{t+1}A_{t+1}]$$

$$W_t = \delta E_t[A'_{t+1}W_{t+1}] + \delta E_t\{A'_{t+1}P_{t+1}e_{t+1}\}$$
$$(-\delta^2 E_t[A'_{t+1}P_{t+1}B_{t+1}]E_t[Q_t + \delta B'_{t+1}P_{t+1}B_{t+1}]^{-1}E_t[B'_{t+1}P_{t+1}e_{t+1} + B'_{t+1}W$$

$$d_t = \delta E_t[trace(P_{t+1}e_{t+1}e'_{t+1})] + \delta E_t[d_{t+1}] + 2\delta E_t[e'_{t+1}W_{t+1}]$$
$$(-\delta^2 E_t[e'_{t+1}P_{t+1}B_{t+1}]E_t[Q_t + \delta B'_{t+1}P_{t+1}B_{t+1}]^{-1}E_t[B'_{t+1}P_{t+1}e_{t+1}])$$
$$(-2\delta^2 E_t[e'_{t+1}P_{t+1}B_{t+1}]E_t[Q_t + \delta B'_{t+1}P_{t+1}B_{t+1}]^{-1}E_t[B'_{t+1}W_{t+1}]),$$

assuming that these implicit equations do have solutions. Given the data-generating process of the model's parameters, $\{R_t, Q_t, e_t, A_t, B_t\}$, one can solve numerically for the matrices P_t and W_t, and hence for the coefficients, F_t and G_t, in the optimal rule.

Note that P_t and therefore the optimal response vector F_t in the optimal rule, depend solely on the distributions of A_{t+1}, B_{t+1}, R_t, and Q_t, conditional on all information available as of time *t*. W_t, and hence the optimal response vector G_t in the optimal rule, depend on the distributions of all the model's parameters, A_{t+1}, B_{t+1}, R_t, Q_t, and e_{t+1}, conditional on all information available as of time *t*.

In general, F_t and G_t will vary with the information acquired over time. If the data-generating process governing $\{R_t, Q_t, e_t, A_t, B_t\}$ is stationary, then so will be F_t and G_t.

If we suppose that R_t and Q_t are exogenously given, so that $E_t[R_t] = E[R_t]$ and $E_t[Q_t] = E[Q_t]$, and the parameters $\{A_t, B_t\}$ are independently and identically distributed over time, then P_t, and therefore the optimal response vector F_t, are constant.

If, moreover, e_t is white noise (specifically, $E_t[e_{t+1}] = 0$ at all *t*) and independent from $\{A_t, B_t\}$, then $E_t[A'_{t+1}P_{t+1}e_{t+1}] = 0$ and $E_t[B'_{t+1}P_{t+1}e_{t+1}] = 0$, and the vector 0 is a trivial solution for W_t. In that case, $G_t = 0$, and the optimal rule reduces to the usual form $u_t = FZ_t$.[5]

More generally, if $e_t = D_tX_t + f_t$ and $X_{t+1} = C_{t+1}X_t + g_{t+1}$, where X_t is an autonomous vector observed at time *t* and unaffected by the control variable, u_t, and f_t and g_t are white noise shocks that are independent from A_t, B_t, R_t, Q_t, C_t, and D_t, jointly, then $E_t[A'_{t+1}P_{t+1}e_{t+1}] = E_t[A'_{t+1}P_{t+1}D_{t+1}C_{t+1}]X_t$, and

5. This of course applies in particular to the case where all of the model's parameters are constant.

$E_t[B'_{t+1}P_{t+1}e_{t+1}] = E_t[B'_{t+1}P_{t+1}D_{t+1}C_{t+1}]X_t$. It follows that $W_t = K_tX_t$ for some vector K_t, which depends on the distributions of A_t, B_t, R_t, Q_t, C_t, and D_t, conditional on information available as of time *t*. In that case, the optimal rule takes the form $u_t = FZ_t + H_tX_t$ for some vector of coefficients H_t, which depends on the distributions of A_t, B_t, R_t, Q_t, C_t, and D_t, conditional on information available as of time *t*. If $\{A_t, B_t, C_t, D_t\}$ are independently and identically distributed over time, then H_t is constant. Then, one can use the identity $E[Z_tZ_t'] = E[Z_{t+1}Z_{t+1}']$ and equation (2) to evaluate numerically the covariance matrix $E[Z_tZ_t']$ under the optimal rule.

Case B (data uncertainty). Suppose that the parameters $\{R_t, Q_t, A_{t+1}, B_{t+1}\}$ are known and constant, but Z_t is not observed at time *t*. Then, the optimal rule is identical to the optimal rule found in the previous case, where there is no uncertainty about the data, except that it is expressed in terms of the forecasts of Z_t, i.e., it has the form

$$u_t = FE_t(Z_t) + G_t ,$$

where F and G_t are as described above. To see this formally, define the new state variables $Y_t \equiv E_t(Z_t)$. (Recall $Z_t = E_t(Z_t) + \mu_t$.) Note that the objective function can be written

$$\begin{aligned} & E_t\sum_{i=0}^{\infty}\delta^i(Z'_{t+i}RZ_{t+i} + u'_{t+i}Qu_{t+i}) \\ &= E_t\sum_{i=0}^{\infty}\delta^i(Y'_{t+i}RY_{t+i} + 2Y'_{t+i}R\mu_{t+i} + \mu'_{t+i}R\mu_{t+i} + u'_{t+i}Qu_{t+i}). \end{aligned}$$

Since $E_t(\mu_t) = 0$, and learning is assumed to be passive, and the policy instrument is assumed not to affect the distribution of the forecast error, μ_t, minimizing this objective function is equivalent to minimizing the function

$$E_t\sum_{i=0}^{\infty}\delta^i(Y'_{t+i}RY_{t+i} + u'_{t+i}Qu_{t+i}) .$$

Then, apply the results of the previous section to the model

$$Y_{t+1} = AY_t + Bu_t + \varepsilon_{t+1} ,$$

where $\varepsilon_{t+1} \equiv e_{t+1} + A\mu_t - \mu_{t+1}$. (Note that ε_t is white noise if e_t is.)

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