Bank of Canada   Banque du Canada

Testing the Stability of the Canadian Phillips Curve Using Exact Methods

by

Lynda Khalaf and Maral Kichian
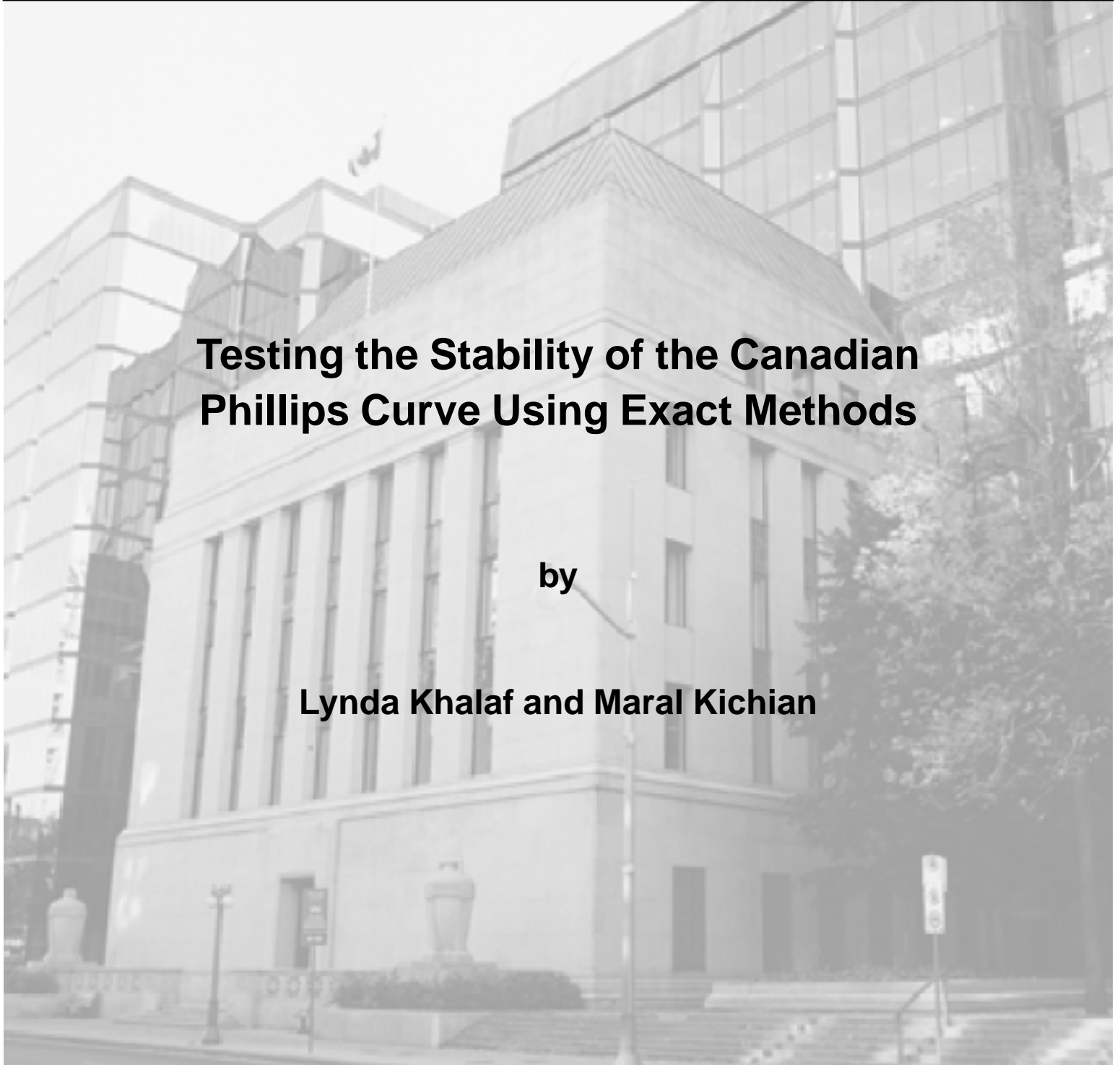
Testing the Stability of the Canadian Phillips Curve Using Exact Methods

by

Lynda Khalaf[1] and Maral Kichian[2]

[1]Département d'économique
Université Laval
Québec, Canada G1K 7P4

[2]Research Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
mkichian@bankofcanada.ca

Contents

Acknowledgements

Abstract

Postulating two different specifications for the Canadian Phillips curve (a purely backward-looking model, and a partly backward-, partly forward-looking model), the authors test for structural breaks in the parameters of the equation. In each case, they account for the possibilities that: (i) breaks can be discrete, or continuous, and (ii) available data samples may be too small to justify using asymptotically valid structural-change tests. Thus, the authors use recent testing procedures that are valid in finite samples, applying the Dufour-Kiviet (1996) methodology for discrete-type breaks, and the Dufour (2002) Maximized Monte Carlo test method for continuous-type shifts. The second test accounts for nuisance parameters that appear only under the alternative. The proposed alternative is a Kalman-filter-based time-varying-parameter specification, with coefficients that follow random walks. The authors find evidence for linear and non-linear breaks, the latter being characterized by continuous and unpredictable-type shifts in the inflation-dynamics coefficients.

JEL classification: E31, E37, C15, C52
Bank classification: Econometric and statistical methods

Résumé

Postulant deux formulations différentes de la courbe de Phillips au Canada (l'une purement rétrospective et l'autre reposant sur des composantes rétrospective et prospective), les auteurs cherchent à déceler la présence de ruptures structurelles dans les paramètres de l'équation. Dans les deux cas, ils tiennent comptent des possibilités que : i) celles-ci soient de type discret ou continu; ii) les échantillons disponibles soient trop petits pour justifier l'utilisation de tests de rupture structurelle valables asymptotiquement. Les auteurs ont donc recours à deux tests récents applicables aux échantillons finis, soit la méthode Dufour-Kiviet (1996) dans le cas des changements structurels de type discret et le test de Monte-Carlo maximisé de Dufour (2002) pour les changements de type continu. Le second test fait intervenir des paramètres de nuisance uniquement dans le modèle représenté par l'hypothèse alternative; fondé sur le filtre de Kalman, le modèle en question comporte des paramètres qui varient dans le temps selon une marche aléatoire. Les auteurs concluent à l'existence de ruptures linéaires et non linéaires, les secondes étant caractérisées par des changements continus et imprévisibles des coefficients de la dynamique d'inflation.

Classification JEL : E31, E37, C15, C52
Classification de la Banque : Méthodes économétriques et statistiques

1. Introduction

The effort, in recent years, to model the short-run dynamics of inflation starting from optimization principles culminated in the so-called New Keynesian Phillips curve (NKPC) relationship. This equation stipulates that inflation at time t is a function of expected future inflation and the current output gap. With its clearly elucidated theoretical foundations, the NKPC possesses a straightforward structural interpretation and therefore presents a strong theoretical advantage to the traditional reduced-form Phillips curve (which is justified only statistically).

The NKPC, however, did not perform very well when confronted with the data, This prompted two main modifications to the basic equation: (i) the use of aggregate marginal-cost measures instead of output-gap estimates, and (ii) the inclusion of lagged inflation terms in the estimation equation (a theoretical justification being that a proportion of firms use myopic price-setting behaviour instead of an intertemporal optimizing strategy).[1]

The resulting, so-called "hybrid," NKPC, was shown to have more empirical support, but important concerns remain. First, Rudd and Whelan (2001) suggest that the improved performance of the model comes from the essentially backward-looking nature of the estimated equation. That is, despite the theoretical model's forward-looking premise, the empirical counterpart manages to capture only backward-lookingness, and hence the good performance. Second, there are issues with respect to the estimation and testing of these models. Arellano, Hansen, and Sentana (1999) show that generalized method of moments (GMM) estimations of such models are subject to underidentification, leading to possibly spurious outcomes. Guay, Luger, and Zhu (forthcoming) show that the hybrid model is not supported by the data in Canada when they correct for the bias related to the number of GMM instruments and address the role of lag-length selection for the Newey-West standard errors. Third, Kurmann (2002) shows that the predicted inflation process obtained from estimated hybrid NKPC models and the actual inflation series diverge substantively.

Until the above-noted practical difficulties with the NKPC are resolved, forecasters may find it more useful to resort to the statistical (reduced-form) Phillips curve. But the usefulness of the latter rests strongly on properly capturing changes in parameter values that may have occurred over time, and for which there seems to be ample heuristic evidence.

We assess this issue in this paper. Specifically, we posit two versions of a standard reduced-form Phillips curve – an entirely backward-looking model and another that includes forward-looking survey expectations – which we test for shifting parameters. This includes: (i) discrete breaks in mean or variance, and (ii) continuous and unpredictable shifts in parameters over time (a random-walk parameter specification). These are accom-

---

[1]See, for example, Galí and Gertler (1999) and Galí, Gertler, and Lopez-Salido (2001).

plished by performing two different stability tests: one against the alternative hypothesis of discrete breaks, and the other against the hypothesis of changing conditional variance.

The second test is a formal likelihood-ratio- (LR) type simulation-based procedure to test our dynamic regression model against one with random-walk coefficients. In this context, hypothesis testing is particularly challenging, since the tested constraints are nested at the boundary. Specifically, the test involves nuisance parameters that are not identified under the null hypothesis (the parameters describing the variances of the random-walk processes postulated for the regression coefficients). As a result: (i) the usual asymptotic theory breaks (i.e., the tests' limiting null distributions are highly non-standard and can even be nuisance-parameter-dependent), and (ii) even bootstraps may fail; see, for example, Bera and Ra (1995), Hansen (1996), and Andrews (2000, 2001). It is therefore important to use test procedures the statistical properties of which are reliable in finite samples. We use the maximized Monte Carlo (MMC) test (Dufour 2002). This simulation-based procedure yields a level-correct test for any sample size, if the null distribution of the test statistic is simulable conditional on a finite set of nuisance parameters. The fact that the relevant analytical (finite sample and/or asymptotic) distributions are quite complicated is not a problem in this context.[2]

This paper makes two main contributions to the literature. First, on empirical grounds, we provide answers to questions regarding whether some or all of the parameters of the Canadian Phillips curve changed over time; when breaks occurred, if any; and whether these were abrupt or continuous. In particular, we show how to use a general time-varying-parameter (TVP) econometric framework – with coefficients that evolve as state variables and that are estimated using Kalman filtering techniques – to address the issue of continuous breaks in the data. We find that, whether the inflation dynamics are defined as purely backward looking or partly forward looking, certain parameters of the Canadian statistical Phillips curve have changed over time. Furthermore, in the partly forward-looking case, we document a transfer of weight from the coefficient of lagged inflation to that of the forward-looking component, especially after 1990. Second, we indicate the nature of the shift in the data, showing evidence for both linear- and continuous-type breaks, the latter occurring only in the coefficients of the inflation dynamics.

Second, on theoretical grounds, our results illustrate the merits of the MMC test method in a highly non-standard context. Indeed, to the best of our knowledge, the TVP test problem has not been approached from a Monte Carlo (MC) test perspective. For further references on alternative MC tests when nuisance parameters are unidentified under the null hypothesis, see Dufour and Khalaf (2001), Dufour et al. (2001), and

---

[2] In nuisance-parameter-dependent test problems, the test level is *exactly* α if the largest rejection probability (over relevant nuisance parameters) is $\leq \alpha$. The MMC critical region corresponds to the largest simulated p-value (over the relevant nuisance parameter space), which controls the test level by construction. The only required condition is the possibility of simulating the relevant test statistic under the null hypothesis.

Saphores, Khalaf, and Pelletier (2002).[3] In the same vein, the results of the Dufour and Kiviet (1996) type tests are noteworthy. Such tests are exact yet conservative; since we detect break dates with plausible economic justifications, this provides an interesting example of the usefulness of these (very simple) generalized Chow tests.

This paper is organized as follows. In section 2, standard Phillips curve models are estimated for Canada and some diagnostic checks are run. Section 3 describes the application of discrete break tests to these models and documents the results. Section 4 proposes, estimates, and tests TVP versions of these models, and reports results. Section 5 concludes.

2. Model and Preliminary Diagnostics

The backward-looking statistical Phillips curve models inflation as a function of lagged inflation, the output gap, and aggregate relative price movements. The model is written as:

$$\pi_t = b_0 + b_1(L)\pi_t + b_3 g_{t-1} + b_4 \Delta g_{t-1} + b_5 \Delta m_{t-1} + \epsilon_t, \ t = 1, ..., T, \tag{1}$$

where π_t is the inflation rate, g_t is the output gap, Δg_t is the first difference of the output gap (included to account for the speeds of expansions and recessions), and Δm_{t-1} captures exogenous changes in relative prices. The last term in the equation is an identically and independently distributed (i.i.d.) innovation term and $b_j(L)$ represents a lag polynomial.

The data are quarterly and two versions of the model are estimated: one with a single lag of the dependent variable (called the AR(1) model), and another with two inflation lags (the AR(2) model). Our dependent variable is Canadian core inflation, which is defined as total CPI inflation excluding the prices of food and energy and the effect of changes in indirect taxes. For g_t we use the Bank of Canada Quarterly Projection Model gap measure.[4] We use the average change (over the past year) of U.S. import inflation relative to Canadian core inflation for our Δm_t variable,[5] because Canada is a small open

---

[3]The test statistics considered in Dufour and Khalaf (2001) and Dufour et al. (2001) are pivotal; i.e., they do not depend on (identified) nuisance parameters (besides those that are not identified). The tests described in Saphores, Khalaf, and Pelletier (2002) relate to the problem we study in this paper, in that, in addition to the unidentified nuisance parameters, further unknown (yet identifiable) nuisance parameters must be dealt with. Saphores, Khalaf, and Pelletier rely on the bounds MC technique, which bases the MC p-value on a pivotal bound; the MMC technique can be considered a numerical search for the tightest (optimal) bound, which provides (in general) a more powerful test. For further applications of the MMC test method (although no further unidentified parameters arise in these examples), see Dufour and Khalaf (2002c) and Beaulieu, Dufour, and Khalaf (2002).

[4]This measure is constructed using Hodrick-Prescott-filtered elements of various economic relationships. Moreover, results are qualitatively similar throughout the paper when a simple Hodrick-Prescott-filtered output gap is used.

[5]The average change over the past four quarters is used to account for the fact that local currency

3

Table 1 - OLS Estimates of Phillips Curves

	Estimated coefficients (p-values)	
	AR(1)	AR(2)
Constant	1.387* (0.000)	1.013* (0.003)
π_{t-1}	0.768* (0.000)	0.482* (0.000)
π_{t-2}	-	0.360 (0.000)
Δm_{t-1}	0.110 (0.084)	0.119* (0.040)
g_{t-1}	0.305* (0.000)	0.311* (0.000)
Δg_{t-1}	0.360 (0.179)	0.438 (0.085)
Adjusted R^2	0.71	0.75
No. of obs.	109	108
Q-stat(4)	24.1 (0.000)	13.1 (0.011)
ARCH(4)	24.0 (0.000)	17.0 (0.002)

Heteroscedasticity-consistent standard errors. The Q-stat(4) is the four-lag Ljung-Box statistic, while the ARCH(4) test is an LM statistic. * = significant at the 5 per cent level.

economy that has an 80 per cent trade share with the United States and is prone to shocks through changes in relative import prices. Imports are consumption goods that exclude food and energy, and are expressed in Canadian dollars. The models were estimated by ordinary least squares (OLS) on a sample extending from 1972Q4 to 1999Q4 and some diagnostic tests were run: a Ljung-Box test for the hypothesis of no autocorrelation in the residuals over four lags and an LM autoregressive conditional heteroscedasticity (ARCH) test for the hypothesis of no heteroscedasticity in the residuals over four periods. Standard errors are calculated using heteroscedasticity- and autocorrelation-consistent methods. Estimation and test results are reported in Table 1, which shows that the adjusted R^2 values are quite high for the two models (0.71 and 0.75) and that most of the parameters are significant at the 10 per cent level. The hypotheses of no-autocorrelation and no-ARCH effects are strongly rejected. In particular, the latter rejection could be the result of parameter instability and, as a first check, we conduct standard Breusch-Pagan tests against the alternative that the heteroscedasticity is related to specific regression variables.

Results documented in Table 2 show that, for all examined models, the null hypothesis of stable coefficients is rejected against the alternative that lagged inflation terms affect the residual variance. In the AR(1) case, the null hypothesis of stable coefficients is also rejected against the alternative that the heteroscedasticity is related to the gap variables.

Of course, care must be exercised in interpreting the above diagnostics, since autocorrelation and heteroscedasticity tests are not robust to structural breaks in the estimated

prices of imports are fairly sticky.

Table 2 - Results of Standard Breusch-Pagan Tests

Alternative hypothesis	χ^2-statistic (p-value)	
	AR(1)	AR(2)
$\hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 \pi_{t-1}$	9.8* (0.002)	-
$\hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-2}$	-	13.4* (0.001)
$\hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 g_{t-1} + \alpha_2 \Delta g_{t-1}$	6.1* (0.047)	2.9 (0.229)
$\hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 \Delta m_{t-1}$	0.7 (0.410)	0.9 (0.355)

* = significant at the 5 per cent level.

coefficients. We therefore consider the above-detected specification problem as a motivation for formal testing against specific hypotheses.

3. Discrete Break Tests

The first alternative hypothesis considered is the case of discrete breaks in the regression coefficients. The standard Chow test is not valid in our dynamic setting, since this F-distributed test strictly requires fixed regressors. We therefore apply an extension of this test to dynamic regressions along the lines proposed by Dufour and Kiviet (1996). We describe the test as it applies to a first-order specification; an extension to a second-order model is straightforward.

Consider the following maintained model (H_0):

$$Y_t = \lambda Y_{t-1} + X_t'\beta + u_t, \ t = 1, ..., T,$$

where X_t denotes the k-dimensional vector of observations on the exogenous variables at time t. The tests are exact if the error terms, u_t, are $\overset{ind}{\sim} N(0, \sigma^2)$ and Y_0 is either fixed or independent of the u_ts. The alternative model (H_1) allows for breaks in all parameters, λ, β, and even σ^2, after a specific date, say T_1.

For a given value of λ, say λ_0, a predictive F-statistic may be obtained as follows:

$$PC(\lambda_0) = \frac{T_1 - k}{T_2} \left\{ \frac{S_0(\lambda_0) - S_1(\lambda_0)}{S_1(\lambda_0)} \right\},$$

where $T_2 = T - T_1$, $S_0(\lambda_0)$ refers to the (full-sample) OLS-based residual sum of squares (RSS) associated with

$$y_t(\lambda_0) = X_t'\beta + u_t, \ t = 1, ..., T$$
$$y_t(\lambda_0) = Y_t - \lambda_0 Y_{t-1},$$

and $S_1(\lambda_0)$ is the (first subsample) OLS-based RSS associated with

$$y_t(\lambda_0) = X_t'\beta^{(1)} + u_t, \ t = 1, ..., T_1.$$

If λ_0 is known, then $PC(\lambda_0) \sim F(T_2, T_1 - k)$. To account for an unknown λ, Dufour and Kiviet propose the following bound test: reject stability if $PC_{\min}$ is significant, accept stability if $PC_{\max}$ is not significant, where

$$PC_{\min} = \min_{\lambda_0 \in \Lambda_1} PC(\lambda_0)$$
$$PC_{\max} = \max_{\lambda_0 \in \Lambda_1} PC(\lambda_0),$$

and Λ_1 is a set of plausible values for the lag parameter over the first sample. Of course, if the smallest $PC(\lambda_0)$ has exceeded the cut-off point, then we are sure that $PC(\lambda_0)$ would exceed the cut-off point for any λ_0. Conversely, if the largest $PC(\lambda_0)$ could not exceed the cut-off point, then we are sure that all $PC(\lambda_0)$ are in the non-rejection region. The question is how to obtain Λ_1.

If a sample-based confidence set is considered, we must account for its estimation by correcting the cut-off point level. Formally, if Λ_1 is a $(1 - \alpha_1)$ confidence set, then to obtain an overall test of level α, $PC_{\min}$ should be referred to $F(T_2, T_1 - k; \alpha - \alpha_1)$ and $PC_{\max}$ to $F(T_2, T_1 - k; \alpha + \alpha_1)$. If Λ_1 is not estimated (i.e., if we sweep over the full relevant parameter space), then it is possible to use $F(.; \alpha)$-based cut-off points.

Dufour and Kiviet show that this test is equivalent to assessing the joint significance of the dummy variables in the augmented regression

$$y_t(\lambda_0) = X_t'\beta + \sum_{s=T_1+1}^{T} D_{ts}\gamma_s + u_t, \ t = 1, ..., T,$$

where
$$D_{ts} \begin{matrix} = 1, t = s \\ = 0, t \neq s \end{matrix} , \ s = T_1 + 1, ..., T.$$

To identify break points, the student t-test associated with each dummy variable can be used. Formally, let

$$t_s(\lambda_0) = \frac{y_s(\lambda_0) - X_s'\widehat{\beta}^{(1)}}{s_1(\lambda_0)\left[1 + X_s'\left(Z_{(1)}'Z_{(1)}\right)^{-1}X_s\right]^{1/2}}$$

$$s_1(\lambda_0)^2 = \frac{S_1(\lambda_0)}{T_1 - k},$$

where $Z_{(1)}$ refers to the matrix of regressors over the first sample. If λ_0 is known, then $(t_s(\lambda_0))^2 \sim F(1, T_1 - k)$. To account for an unknown λ, Dufour and Kiviet propose, in

6

Table 3 - Dufour-Kiviet Test on Dummies

| | First subperiod | 1973Q2 - 1982Q4 | 1985Q1 - 1990Q4 |
	Second subperiod	1983Q1 - 1990Q4	1991Q1 - 1999Q4
AR(1) break points		1984Q1 (0.070)	1991Q2 (0.014) 1993Q2 (0.082) 1997Q3 (0.081)
AR(2) break points		1984Q1 (0.076)	1991Q2 (0.042)

p-values are in parentheses.

line with the joint F-test, the following bound test: reject stability if $F_{\min}^s$ is significant, accept stability if $F_{\max}^s$ is not significant, where

$$
\begin{aligned}
F_{\min}^s &= \min_{\lambda_0 \in \Lambda_1} \left(t_s(\lambda_0) \right)^2, \\
F_{\max}^s &= \max_{\lambda_0 \in \Lambda_1} \left(t_s(\lambda_0) \right)^2.
\end{aligned}
$$

We apply the above student *t*-test to our AR(1) and AR(2) Phillips curve models and report the results in Table 3. We first consider the subsample 1973Q2-1982Q4 versus 1983Q1-1990Q4, which roughly correspond to the high- and medium-inflation periods in Canada. In each case, tests were conducted sweeping over a space composed of the OLS estimate ± 3 standard errors for each of the dynamic coefficients.

The results (columns 1 and 2) indicate evidence of a break in the last quarter of 1984 at the 10 per cent level, regardless of model specification.[6] Interestingly, this break point falls in the period after Canada came out of recession, having experienced a fair amount of restructuring over the previous two years or so.

Having determined that there was a break in 1984Q1, we consider two other subperiods: 1985Q1-1990Q4 and 1991Q1-1999Q4. These, more or less, represent the medium- and low-inflation periods in Canada. Again, with either model, we find evidence of a discrete break. This time, the evidence is at the 5 per cent level, and the break occurs in the second quarter of 1991 (see the last column of Table 3). This break point coincides with the start of the inflation-targeting period in Canada and comes shortly after Canada's adoption of the free-trade agreement with the United States.

---

[6]Results are similar when the second subsample is extended to 1999Q4.

4. Continuous Break Tests and the TVP Model

The second alternative hypothesis we consider is that of continuous and unpredictable shifts in the model parameters over time. These can loosely be interpreted as reflecting gradual changes in the underlying structural model parameters, such as those affecting policy credibility. A convenient way of capturing such changes is to consider coefficients that follow random-walk processes (see Kichian 2001). In this case, the backward-looking model (say, in the case of an AR(2)) is given by:

$$\pi_t = \beta_{0t} + \beta_{1t}\pi_{t-1} + \beta_{2t}\pi_{t-2} + \beta_{3t}g_{t-1} + \beta_{4t}\Delta g_{t-1} + \beta_{5t}\Delta m_{t-1} + \epsilon_t$$
$$\beta_{it} = \beta_{it-1} + \eta_{it}, \ i = 0, ..., 5, \ t = 1, ..., T, \tag{2}$$

where the β_{it} coefficients follow random-walk processes, and where the η_{it}'s are assumed to be i.i.d. with variances $\sigma_{\eta_i}^2$, respectively. The model is written in state-space form and is estimated using Kalman filtering and maximum-likelihood methods (see Appendix B for a detailed example).

Kim and Nelson (1989) consider a specific Breusch-Pagan homoscedasticity test for detecting shifts of the above nature in the parameters. For each AR model, the test statistic is obtained by regressing the ratio of the equation (1) residuals squared to their variance on $t * X_t^2$, where t is the number of observations and X_t is the vector of k regressors. Then, under the null hypothesis of stable coefficients, the resulting explained sum of squares divided by 2 is shown to be distributed as a $\chi^2(k)$.

Table 4 - Breusch-Pagan Tests against Random-Walk Coefficient Alternatives

Alternative hypothesis	χ^2-statistic (p-value)	
	AR(1)	AR(2)
All coefficients are random walks	14.3* (0.014)	15.9* (0.014)
Inflation coefficients are random walks	12.5* (0.002)	14.8* (0.002)
Gap coefficients are random walks	4.3 (0.112)	2.7 (0.254)
Relative price coefficient is random walk	2.1 (0.145)	2.5 (0.111)

* = significant at the 5 per cent level.

Table 4 documents the results of these Kim-Nelson tests. Again, for all examined models, we see that the null hypothesis of stable coefficients is rejected when the intercept and the lagged inflation terms are jointly tested. Yet the Kim-Nelson tests are asymptotically valid and our span of 25 years may not be sufficient to justify the outcomes. In other words, these results need to be qualified within small sample sizes.

We therefore apply a formal LR-type simulation-based test procedure that has the correct cut-off point irrespective of the sample size. Formally, we test the dynamic regression

8

model (1) (the null hypothesis, imposing i.i.d. normal errors) against the random-walk-TVP model (2) (the alternative hypothesis), using the quasi-likelihood ratio statistic.

Before proceeding with the testing, we introduce a second specification for the inflation dynamics of our Phillips curve. As section 1 indicated, and as the optimization models of the Phillips curve show, forward-looking inflation expectations likely play an important role in determining inflation dynamics and therefore must be accounted for. In particular, since inflation expectations are forward looking and continuously updated, it is important to ascertain that detected continuous breaks in the backward-looking specifications are not due to such an omitted term in the equation.

Accordingly, we add a survey-based inflation-expectations variable to both our AR(1) and AR(2) Phillips curve equations. In our reduced-form context, the inflation dynamics are given by a weighted average of forward- and backward-looking inflation terms. The time t value of the expectation variable is the average annual total CPI inflation that is expected for the next calendar year (see Figure 2 for a graph of this series).[7] Since the forecasts pertain to total CPI inflation, our partly forward-looking models feature total CPI inflation for the π_t variable.

Thus, in the AR(2) case, the model under the null hypothesis is given by:

$$
\begin{aligned}
\pi_t &= \lambda_1 \pi_{t-1} + \lambda_2 \pi_{t-2} + (1 - \lambda_1 - \lambda_2)\pi_t^e + b_3 g_{t-1} + b_4 \Delta g_{t-1} \\
&\quad + b_5 \Delta m_{t-1} + \epsilon_t, \ t = 1, ..., T,
\end{aligned} \tag{3}
$$

where π_t is total CPI inflation, π_t^e is this quarter's CPI inflation expectations for the next year, and λ_1 and λ_2 are the weights on the first and second lags of inflation, respectively.

This is tested against an alternative model that allows for continuous and unpredictable shifts over time, as characterized by its random-walk coefficients. In other words, the alternative model is described by the system:

$$
\begin{aligned}
\pi_t &= \lambda_{1t} \pi_{t-1} + \lambda_{2t} \pi_{t-2} + (1 - \lambda_{1t} - \lambda_{2t})\pi_t^e + \beta_{3t} g_{t-1} + \beta_{4t} \Delta g_{t-1} \\
&\quad + \beta_{5t} \Delta m_{t-1} + \epsilon_t, \ t = 1, ..., T \\
\lambda_{it} &= \lambda_{it-1} + \eta_{it}^*, \ i = 1, 2 \\
\beta_{jt} &= \beta_{jt-1} + \eta_{jt}, \ j = 3, ..., 5,
\end{aligned} \tag{4}
$$

where the λ_{it} and the β_{jt} coefficients follow random-walk processes, and where the η_{jt}'s and the η_{it}^*'s are assumed to be i.i.d. with variances $\sigma_{\eta_j}^2$ and $\sigma_{\eta_i^*}^2$, respectively.

Regarding the MC LR tests on these models, Appendix A provides a formal exposition of the MC test method. Below, we summarize the technique as it applies to our testing

---

[7] These expectations series were obtained from Canada's Conference Board Survey, where, each quarter, participants are asked to forecast the average total CPI inflation in Canada for both the current and next calendar years. We consider the next-year forecasts in our models to avoid simultaneity with the lagged inflation terms in our equations.

problem for the models. For this case, the nuisance-parameter vector, denoted by ω, is composed of the regression coefficients of the null model (1) and the variance of the regression error.

First, we calculate the likelihood-ratio statistic using the likelihood values of the TVP model (the alternative model) against its equivalent constant-coefficient model (the null model). This value is denoted as LR_0. As emphasized earlier, referring LR_0 to a standard (e.g., χ^2) cut-off point will lead to invalid inference. Indeed, the results of Andrews (2000, 2001) imply that the limiting null hypothesis distribution of this statistic is not χ^2, since

$$(1) \sqsubseteq (2) \text{ when } \sigma_{\eta_i}^2 \to 0;$$

i.e., the models are nested at parameter space boundaries.

Second, we generate data from the model under the null hypothesis drawing from the normal distribution and setting ω to its estimated quasi-maximum-likelihood values. With this data, we re-estimate the null and the alternative models and calculate the LR statistic based on the obtained likelihood values. This data generation and subsequent LR-value calculation is repeated in 199 replications. Thus, we obtain a 199-point distribution of generated-data LR values. The actual data LR value (LR_0) is then compared with distribution, and its p-value is calculated based on the rank of LR_0 relative to its simulated counterparts; see equations (5), (6), and (9) in Appendix A. This leads to a bootstrap-type p-value, which we report in Table 5 (and which we denote as the MC p-value). Unfortunately, the results of Andrews (2000, 2001) and Dufour (2002) imply that such p-values may still be invalid. Ideally, the size of a test based on these p-values will converge to its nominal size (e.g., 5 per cent), if the sample size (and the number of MC replications) $\to \infty$. In this case, however, the regularity conditions underlying the latter convergence result are not necessarily verified. We thus apply a size-correction technique, the MMC technique of Dufour (2002).

Third, the MMC technique involves repeating the second step, sweeping over combinations of admissible values of ω.[8] Thus, we obtain an MC p-value for each such combination. The maximized MC p-value is then the highest obtained MC p-value amongst these; since the maximized p-value function is a non-differentiable step function, we use simulated annealing (a global non-gradient-based algorithm) to obtain the latter maxima.[9] The MMC test is significant at level α if the MMC p-value $\leq \alpha$. Of course, if the MC p-value

---

[8]Specifically, we sweep over the space spanning the OLS estimated values ± 5 standard deviations for the mean parameters, while we adopt the interval [1,5] for the variance parameter (OLS estimates of the variance falling about the centre of this interval).

[9]The MMC procedure is computationally involved, particularly because the underlying statistic is based on an iterative Kalman-filter based quasi-maximum-likelihood estimation (QMLE). Since no other reliable test procedure is available, however, one objective of this paper is to emphasize that computational costs, particularly with current computer facilities, should not deter practitioners from applying a method that provably leads to non-spurious rejections.

obtained in the second step has already exceeded α (e.g., 5 per cent), there is no need to proceed with the maximization; this saves execution time.

The above tests were applied to both the purely backward-looking, and the partly backward-, partly forward-looking, specifications of the Phillips curve equations. For each, we tested the general case where all parameters follow random walks, as well as cases where only a subset of the model parameters are time-varying. Three such subsets are considered: (i) the constant and coefficients of inflation lags, (ii) the coefficient of the gap, and (iii) the coefficient of the relative price variable.

The alternative models, the MC p-value and corresponding MMC p-values, are reported in Table 5. In general, the results indicate that it would have been dangerous to rely on MC test outcomes alone: there are two cases where the MMC test reverses the decision at the 5 per cent level (cases A4 and D3), and more where the decision is reversed at the 10 per cent level (B4, C3, and C4). Thus, the outcomes demonstrate empirically what the theoretical arguments had already established. We will therefore rely on the MMC test outcomes for our economic decisions.

First, we shall examine the case where the regression coefficients are assumed to follow random walks (models A1, B1, C1, and D1 in Table 5). In all cases, the tests reject the hypothesis of stable regression coefficients in favour of continuous and unpredictable shifts in the parameters. Thus, whether the inflation dynamics are purely backward looking (models A1 and B1) or partly forward looking (models C1 and D1), there is very strong evidence of continuous breaks in the equation coefficients.

For the cases where only a subset of model parameters are assumed to be time-varying, the MMC test decisively shows that the null hypothesis of stable coefficients is rejected against only one alternative: the case where the inflation dynamics coefficients are time-varying (models A2, B2, C2, and D2). Again, regardless of how these dynamics are defined – purely backward looking, or partly forward looking, an AR(1) specification, or an AR(2) – there is strong evidence that the coefficients of the inflation dynamics underwent continuous shifts over time. In contrast, it is not possible to reject the stability of the gap coefficients or of the coefficient of the relative price-shock variable.

Having determined that the continuous-type parameter instability in the inflation process comes from the inflation dynamics of the equation, we can ask exactly how these coefficients have evolved over time. In particular, if the instability is related to underlying policy changes and their gradual effects on the inflation process, an examination of the time-path of the continuously evolving coefficients may provide some insights to that effect. An advantage of our state-space modelling strategy is that it is possible to obtain such time paths.[10]

---

[10]There is a small drawback. Since state variables are assumed to follow normal distributions – an assumption required in the context of the optimality of the Kalman filter – parameters invariably take negative values at certain times. Therefore, rather than look at the numerical values, one should focus on the overall form of such graphs to determine whether they are increasing or decreasing.

Table 5 - Estimation and Test Results of TVP Phillips Curves

Model description (most general case)	Time-varying coefficients	Model name	MC p-value	MMC p-value	LR
$\pi_t = \beta_{0t} + \beta_{1t}\pi_{t-1} + \beta_{3t}g_{t-1} +$ $\beta_{4t}\Delta g_{t-1} + \beta_{5t}\Delta m_{t-1} + \epsilon_t,$ $\beta_{it} = \beta_{it-1} + \eta_{it},$ $i = 0, 1, 3, 4, 5$	$\beta_{0t}, \beta_{1t}, \beta_{3t}, \beta_{4t}, \beta_{5t}$	A1	0.01	**0.01**	60.71
	β_{0t}, β_{1t}	A2	0.01	**0.01**	86.03
	β_{3t}, β_{4t}	A3	0.23	-	11.41
	β_{5t}	A4	0.04	0.17	18.34
$\pi_t = \beta_{0t} + \beta_{1t}\pi_{t-1} + \beta_{2t}\pi_{t-2} +$ $\beta_{3t}g_{t-1} + \beta_{4t}\Delta g_{t-1} +$ $\beta_{5t}\Delta m_{t-1} + \epsilon_t,$ $\beta_{it} = \beta_{it-1} + \eta_{it},$ $i = 0, ..., 5$	$\beta_{0t}, \beta_{1t}, \beta_{2t},$ $\beta_{3t}, \beta_{4t}, \beta_{5t}$	B1	0.01	**0.01**	50.24
	$\beta_{0t}, \beta_{1t}, \beta_{2t}$	B2	0.01	**0.01**	85.88
	β_{3t}, β_{4t}	B3	0.125	-	54.95
	β_{5t}	B4	0.065	0.25	59.67
$\pi_t = \lambda_t\pi_{t-1} + (1 - \lambda_t)\pi_t^e +$ $\beta_{3t}g_{t-1} + \beta_{4t}\Delta g_{t-1} +$ $\beta_{5t}\Delta m_{t-1} + \epsilon_t,$ $\lambda_t = \lambda_{t-1} + \eta_t^*,$ $\beta_{it} = \beta_{it-1} + \eta_{it},$ $i = 3, ..., 5$	$\lambda_t, \beta_{3t}, \beta_{4t}, \beta_{5t}$	C1	0.015	**0.02**	23.76
	λ_t	C2	0.015	**0.02**	35.30
	β_{3t}, β_{4t}	C3	0.06	0.12	23.80
	β_{5t}	C4	0.06	0.145	26.51
$\pi_t = \lambda_{1t}\pi_{t-1} + \lambda_{2t}\pi_{t-2} +$ $(1 - \lambda_{1t} - \lambda_{2t})\pi_t^e + \beta_{3t}g_{t-1} +$ $\beta_{4t}\Delta g_{t-1} + \beta_{5t}\Delta m_{t-1} + \epsilon_t,$ $\lambda_{it} = \lambda_{it-1} + \eta_{it}^*,$ $\beta_{jt} = \beta_{jt-1} + \eta_{jt},$ $i = 1, 2, j = 3, 4, 5$	$\lambda_{1t}, \lambda_{2t}, \beta_{3t}, \beta_{4t}, \beta_{5t}$	D1	0.01	**0.01**	36.73
	$\lambda_{1t}, \lambda_{2t}$	D2	0.01	**0.01**	46.80
	β_{3t}, β_{4t}	D3	0.015	0.135	30.82
	β_{5t}	D4	0.105	0.415	31.92

Notes: The number of replications is 199 in each of the MMC tests. Examples of estimated variances are: for model A1: $\sigma_\epsilon^2 = 0.16, \sigma_{\eta_0}^2 = 0.65, \sigma_{\eta_1}^2 = 0.24, \sigma_{\eta_3}^2 = 0.01, \sigma_{\eta_4}^2 = 0.23, \sigma_{\eta_5}^2 = 10^{-4}$, for model C1: $\sigma_\epsilon^2 = 1.67, \sigma_{\eta_*}^2 = 0.07, \sigma_{\eta_3}^2 = 2\text{x}10^{-7}, \sigma_{\eta_4}^2 = 4\text{x}10^{-4}, \sigma_{\eta_5}^2 = 0.02$.

We provide one such example in Figure 1. This figure depicts the evolution of the estimated lamda parameter (the weight on the lagged inflation coefficient) of model C1, where the gap measure used is the Hodrick-Prescott-filtered output gap. The graph reveals an important decline in the value of lamda that starts around mid-1982 and continues till 1990–91. After 1991, the parameter value seems to settle around the 0.1 level, followed by a further small decline after 1996. Remembering that as λ_t declines, $1-\lambda_t$ increases, we see that there has been a transfer of weight from the backward-looking to the forward-looking component of inflation dynamics over time.

The graph of inflation expectations in Figure 2 shows a similar pattern. Johnson (2002) has shown that inflation targeting played an important role in bringing down the level of this variable over time. Given that the value of λ more or less settled after 1991, and that inflation targeting in Canada officially started in February 1991, one might well ask whether there is also a link between policy change and the evolution of lamda. Indeed, Clifton, Leon, and Wong (2001), using a smooth transition model for the OECD countries, show empirically that inflation expectations are mostly backward looking in the pre-targeting period, and that they are partly backward and partly forward looking after the adoption of targets. Thus, one could make the interpretive argument that, as expectations in Canada became more and more anchored, policy credibility increased, and agents began to assign a higher weight to expectations and a lower weight to past inflation.

5. Conclusion

Evidence has suggested that certain parameters of the Canadian statistical (reduced-form) Phillips curve may have changed in value because of underlying structural breaks in the economy. Parameters that may have shifted over time include the coefficients of the autoregressive lags of inflation, those of the output gap, and coefficients of the relative price-shock variables. The question is whether some or all of these parameters have changed; when breaks have occurred, if any; and whether these have been abrupt or continuous.

These uncertainties exist because: (i) the validity of structural-break tests (as with tests based on GMM, regime-switching, and traditional Kalman-filtering estimations) relies on having a very large data sample size, (ii) most of these tests are applicable to a univariate series rather than its components (as described by a model), leaving the source of instability undefined, and (iii) it is methodologically difficult, even asymptotically, to test for breaks in one parameter while accounting for possible breaks in other parameters of the same model.

In this study, we have addressed the above issues using recent testing procedures that are valid in small data samples. These methods are applicable to specific parameters of

a model describing a series and they detect breaks in one parameter while allowing other parameters to change. We have therefore tested whether the different parameters of the Canadian statistical Phillips curve changed over time, taking into account the smaller variance of inflation during the nineties and given our small sample sizes. We have also investigated whether parameter changes are abrupt or continuous.

In addition to asymptotic Breusch-Pagan tests (a test recommended by Kim and Nelson 1989 against a random-walk-coefficients alternative), we have performed two different stability tests: (i) a generalized Chow-type predictive test for abrupt breaks, based on Dufour and Kiviet (1996), and (ii) an LR-type simulation-based maximized Monte Carlo test (see Dufour 2002), to address the random-walk-coefficients alternative. Both of these tests account for the presence of nuisance parameters that appear under the alternative model and not under the null hypothesis. We have shown that the Monte Carlo test procedure circumvents such intractable null distribution problems.

We have found two dates where a linear break seems to have occurred in the Canadian Phillips curve parameters. These are the first quarter of 1984 and the second quarter of 1991. In addition, regardless of whether the Phillips curve is purely backward looking or partly backward and partly forward looking, we have found evidence for non-linear breaks. That is, we have documented continuous and unpredictable shifts in the inflation dynamics parameters over time. In particular, in the partly forward looking case, we have documented a transfer of weight from the lagged inflation terms in the equation to the forward-looking variable.

14

Bibliography

Andrews, D.W.K. 2000. "Inconsistency of the Bootstrap When a Parameter is on the Boundary of the Parameter Space." *Econometrica* 68: 399-405.

——. 2001. "Testing When a Parameter is on the Boundary of the Maintained Hypothesis." *Econometrica* 69: 683-734.

Arellano, M., L.P. Hansen, and E. Sentana. 1999. "Underidentification?" CEMFI. Mimeo.

Beaulieu, M.-C., J.-M. Dufour, and L. Khalaf. 2002. "Testing Black's CAPM in Possibly Non-Gaussian Contexts: An Exact Simulation-Based Approach." Working Paper, CIREQ, Université de Montréal and Université Laval.

Bera, A.K. and S. Ra. 1995. "A Test for the Presence of Conditional Heteroskedasticity within ARCH-M Framework." *Econometric Reviews* 14: 473-85.

Breusch, T.S. and A.R. Pagan. 1979. "A Simple Test for Heteroskedasticity and Random Coefficient Variation." *Econometrica* 47: 1287-94.

Clifton, E., H. Leon, and C-H. Wong. 2001. "Inflation Targeting and the Unemployment-Inflation Trade-Off." IMF Working Paper No. WP/01/166.

Dufour, J.-M. 2002. "Monte Carlo Tests in the Presence of Nuisance Parameters with Econometric Applications." Working Paper, Centre de recherche et développement en économique (C.R.D.E.) and Département de sciences économiques, Université de Montréal.

Dufour, J.-M., A. Farhat, L. Gardiol, and L. Khalaf. 1998. "Simulation-Based Finite Sample Normality Tests in Linear Regressions." *The Econometrics Journal* 1: 154-73.

Dufour, J.-M. and L. Khalaf. 2001. "Monte Carlo Tests Methods in Econometrics." In *Companion to Theoretical Econometrics*, edited by B. Baltagi, Blackwell.

——. 2002a. "Exact Tests for Contemporaneous Correlation of Disturbances in Seemingly Unrelated Regressions." *Journal of Econometrics* 106: 143-70.

——. 2002b. "Simulation-Based Finite and Large Sample Tests in Multivariate Regressions." *Journal of Econometrics* forthcoming.

——. 2002c. "Finite Sample Simulation-Based Tests in Seemingly Unrelated Regressions." In *Computer-Aided Econometrics*, edited by D.E.A. Giles, Marcel Dekker, forthcoming.

Dufour, J.-M., L. Khalaf, J.-T. Bernard, and I. Genest. 2001. "Simulation-Based Finite-Sample Tests for Heteroskedasticity and ARCH Effects." *Journal of Econometrics* forthcoming.

Dufour, J.-M. and J. Kiviet. 1996. "Exact Tests for Structural Change in First-Order Dynamic Models." *Journal of Econometrics* 70: 39-68.

Galí, J. and M. Gertler. 1999. "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics* 44: 195-222.

Galí, J., M. Gertler, and D. Lopez-Salido. 2001. "European Inflation Dynamics." *European Economic Review* 45: 1237-70.

Guay, A., R. Luger, and Z. Zhu. 2002. "The New Phillips Curve in Canada." Bank of Canada conference proceedings. Forthcoming.

Hansen, B.E. 1996. "Inference When a Nuisance Parameter is not Identified Under the Null Hypothesis." *Econometrica* 64: 413-30.

Johnson, D.R. 2002. "The Effect of Inflation Targeting on the Behavior of Expected Inflation: Evidence from an 11 Country Panel." *Journal of Monetary Economics* 49: 1521-38.

Khalaf, L. and M. Kichian. 2002. "Simulation-Based Tests of Pricing-to-Market." In *Computational Methods in Decision-Making, Economics and Finance*, edited by E. Kontoghiorghes, B. Rustem, and S. Siokos, 583-603. The Kluwer Applied Optimization Series, Chapter 29.

Kichian, M. 2001. "On the Nature and the Stability of the Canadian Phillips Curve." Bank of Canada Working Paper No. 2001-4.

Kim, C-J., and C.R. Nelson. 1989. "The Time-Varying-Parameter Model for Modeling Changing Conditional Variance: The Case of the Lucas Hypothesis." *Journal of Business and Economic Statistics* 7(4): 433-40.

——. 1999. *State-Space Models with Regime-Switching*. MIT Press, U.S.A.

Kurmann, A. 2002. "Quantifying the Uncertainty about a Forward-Looking New Keynesian Phillips Curve." University of Virginia. Mimeo.

Rudd, J. and C. Whelan. 2001. "New Tests of the New-Keynesian Phillips Curve." Federal Reserve Board discussion paper.

Saphores, J.-D., L. Khalaf, and D. Pelletier. 2002. "On Jumps and ARCH Effects in Natural Resource Prices. An Application to Stumpage Prices from Pacific Northwest National Forests." *American Journal of Agriculture Economics* 84(2): 387-400.
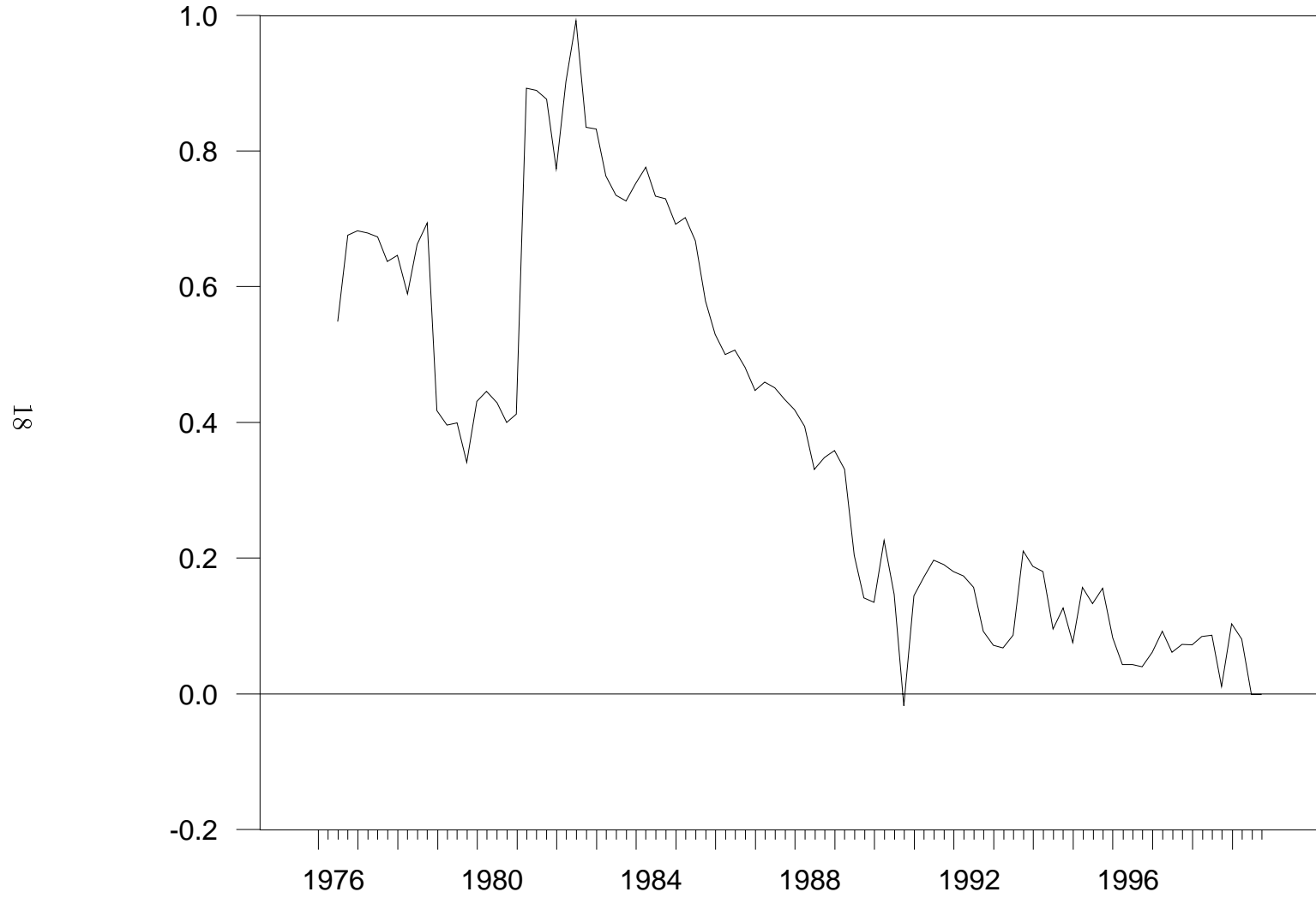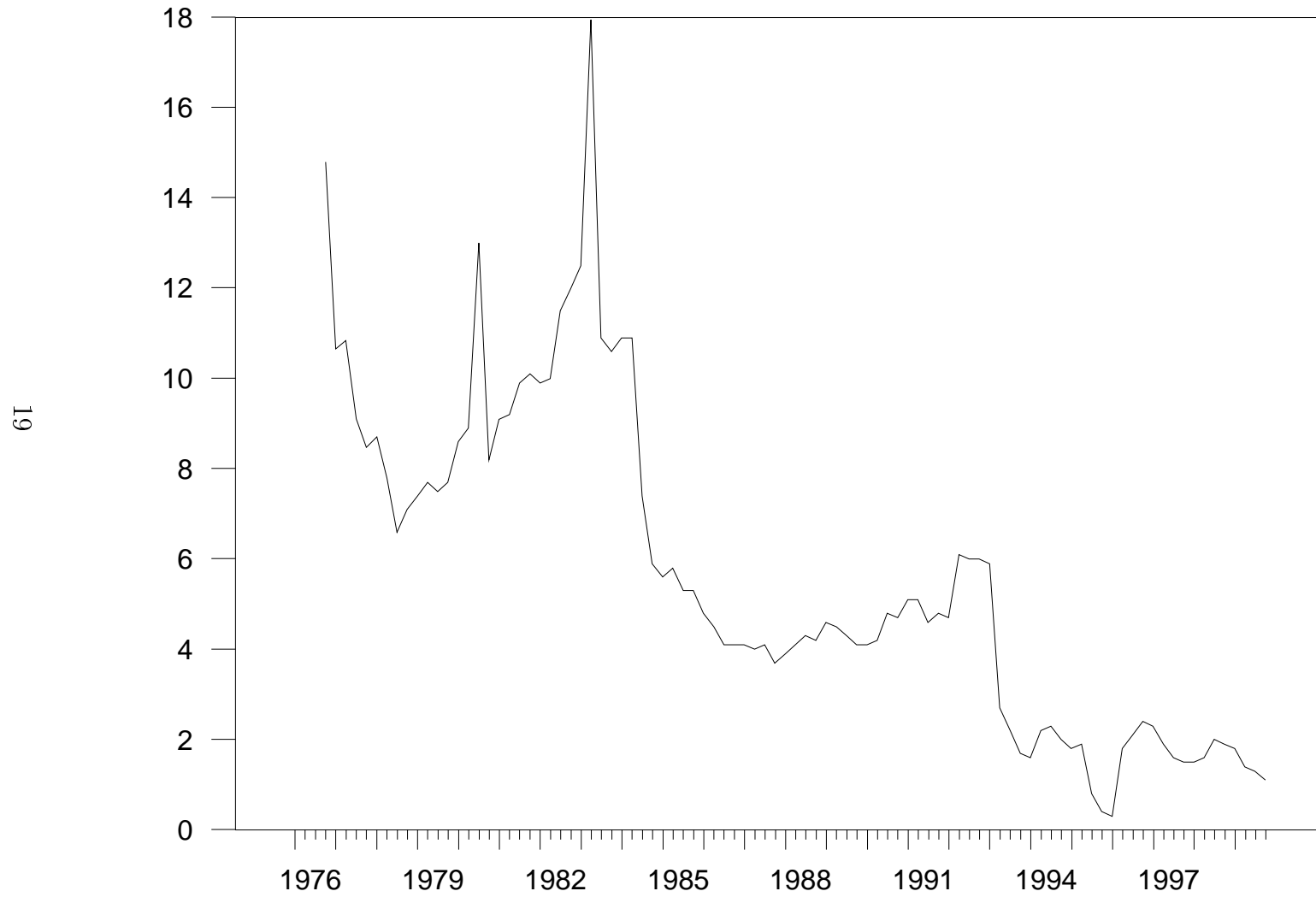
Figure 1: Model C1, Evolution of the Lamda Parameter

Figure 2: Next-Year Inflation Expectations

Appendix A

Maximized Monte Carlo

Monte Carlo tests have recently been generalized to the nuisance-parameter-dependent case by Dufour (2002). These tests are based on the following fundamental distributional result. Let S_0 denote the value of a continuous test statistic computed from the observed data and obtain N i.i.d. random draws from the statistic's null distribution, denoted S_j, $j = 1$, $\dots$, N. Then calculate

$$\widehat{p}_N(S_0) = \frac{N\widehat{G}_N(S_0) + 1}{N + 1}, \tag{5}$$

$$\widehat{G}_N(S_0) = \frac{1}{N}\sum_{i=1}^{N} I_{[0,\infty]}(S_i - S_0), \tag{6}$$

$$I_A(x) = \begin{cases} 1, & if\ x \in A, \\ 0, & if\ x \notin A. \end{cases}$$

If no nuisance parameters were involved in the drawing of S_j, $j = 1$, $\dots$, N,

$$P_{(H_0)}[\widehat{p}_N(S_0) \leq \alpha] = \alpha, \tag{7}$$

for all $0 < \alpha < 1$ where N is such that $\alpha(N + 1)$ is an integer and $P_{(H_0)}$ refers to the distribution under the null hypothesis.

Note that $N\widehat{G}_N(S_0)$ is the number of simulated criteria $\geq S_0$. The formula for $\widehat{p}_N(S_0)$ gives an *empirical p-value*, so the MC test's critical region of size α may be defined as:

$$\widehat{p}_N(S_0) \leq \alpha.$$

The fact that the latter critical region has size α exactly obtains from equation (7); see Dufour (2002) for more details and formal proofs.

Let us suppose that the null distribution of S depends on a nuisance parameters that we denote $\omega \in \Omega$, where Ω_0 refers to the nuisance-parameter subspace compatible with the null hypothesis, H_0, under test. In this case, the simulation algorithm underlying equation (5) may be applied conditional on ω. We denote the p-value so obtained $\widehat{p}_N(S_0|\omega)$ to emphasize conditioning on ω. The MC test technique generally requires that $\widehat{p}_N(S_0|\omega)$ be maximized with respect to $\omega \in \Omega_0$. Specifically, Dufour (2002) demonstrates that the test (denoted the *maximized* Monte Carlo (MMC) test), based on the critical region

$$\sup_{\omega \in \Omega_0} [\widehat{p}_N(S_0|\omega)] \leq \alpha, \tag{8}$$

is exact at level α; i.e.,

$$P_{(H_0)} \left\{ \sup_{\omega \in \Omega_0} [\widehat{p}_N(S_0|\omega)] \leq \alpha \right\} \leq \alpha.$$

Now consider the (parametric) bootstrap-type critical region

$$\widehat{p}_N(S_0|\widehat{\omega}) > \alpha, \tag{9}$$

where $\widehat{\omega}$ is any consistent estimate of ω that satisfies H_0. Obviously,

$$\widehat{p}_N(S_0|\widehat{\omega}) > \alpha \Rightarrow \sup_{\omega \in \Omega_0} [\widehat{p}_N(S_0|\omega)] \leq \alpha.$$

In other words, if the bootstrap-type test is not significant, then we can be sure that the exact MMC test is not significant at level α. It is thus a good strategy to start the MMC sup p-value step using a common (e.g., a constrained QMLE) estimate of ω.

Appendix B

Kalman Filtering and the TVP Model

This appendix draws heavily on Kim and Nelson (1999), chapter 3. Consider the AR(2)-TVP model:

$$
\begin{aligned}
\pi_t &= \beta_{0t} + \beta_{1t}\pi_{t-1} + \beta_{2t}\pi_{t-2} + b_3 g_{t-1} + b_4 \Delta g_{t-1} + b_5 \Delta m_t + \epsilon_t \\
\beta_{it} &= \beta_{it-1} + \eta_{it}, \ t = 1, ..., T \\
\epsilon_t &\sim \ i.i.d.N\left(0, \sigma_\epsilon^2\right) \\
\eta_{it} &\sim \ i.i.d.N\left(0, \sigma_{\eta_i}^2\right), \ i = 0, ..., 2.
\end{aligned}
\tag{10}
$$

In matrix notation, this is given by

$$
\begin{aligned}
y_t &= H_t \beta_t + A z_t + \epsilon_t \\
\beta_t &= F\beta_{t-1} + \eta_t, \ t = 1, ..., T, \\
\epsilon_t &\sim \ i.i.d.N\left(0, R\right) \\
\eta_t &\sim \ i.i.d.N\left(0, Q\right).
\end{aligned}
\tag{11}
$$

Thus, we have that

$$
\begin{aligned}
\begin{bmatrix} \pi_t \end{bmatrix} &= \begin{bmatrix} 1 & \pi_{t-1} & \pi_{t-2} \end{bmatrix} \begin{bmatrix} \beta_{0t} \\ \beta_{1t} \\ \beta_{2t} \end{bmatrix} + \begin{bmatrix} b_3 & b_4 & b_5 \end{bmatrix} \begin{bmatrix} g_{t-1} \\ \Delta g_{t-1} \\ \Delta m_t \end{bmatrix} + \epsilon_t \\
\begin{bmatrix} \beta_{0t} \\ \beta_{1t} \\ \beta_{2t} \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_{0t-1} \\ \beta_{1t-1} \\ \beta_{2t-1} \end{bmatrix} + \begin{bmatrix} \eta_{0t} \\ \eta_{1t} \\ \eta_{2t} \end{bmatrix},
\end{aligned}
\tag{12}
$$

with

$$
Q = \begin{bmatrix} \sigma_{\eta_0}^2 & 0 & 0 \\ 0 & \sigma_{\eta_1}^2 & 0 \\ 0 & 0 & \sigma_{\eta_2}^2 \end{bmatrix}.
\tag{13}
$$

The prediction equations in the Kalman filter algorithm are given by:

$$
\begin{aligned}
\beta_{t|t-1} &= F\beta_{t-1|t-1}, \\
\Sigma_{t|t-1} &= F\Sigma_{t-1|t-1}F' + Q,
\end{aligned}
\tag{14}
$$

where $\beta_{t|t-1}$ is the forecast value of β_t on the basis of information available through date $t-1$, and $\Sigma_{t|t-1}$ is its conditional variance. Then, the conditional forecast error and its conditional variance can be obtained as:

$$
\begin{aligned}
e_{t|t-1} &= y_t - H_t \beta_{t|t-1} - A z_t, \\
f_{t|t-1} &= H_t \Sigma_{t|t-1} H_t' + R.
\end{aligned}
\tag{15}
$$

These expressions can be used in the updating equations of the algorithm according to

$$
\begin{aligned}
\beta_{t|t} &= \beta_{t|t-1} + K_t e_{t|t-1}, \\
\Sigma_{t|t} &= \Sigma_{t|t-1} - K_t H_t \Sigma_{t|t-1},
\end{aligned}
\tag{16}
$$

where the Kalman gain term is $K_t = \Sigma_{t|t-1} H_t' f_{t|t-1}^{-1}$.

If, in addition to the error terms ϵ_t and η_{it}, the initial value of β is also Gaussian, then the distribution of y_t conditional on information available through time $t-1$ is also Gaussian, and its log-likelihood function is:

$$
lnL = -(1/2) \sum_{t=1}^{T} ln(2\pi f_{t|t-1}) - (1/2) \sum_{t=1}^{T} e_{t|t-1}' f_{t|t-1} e_{t|t-1}.
\tag{17}
$$

Therefore, given initial values for model parameters and state variables, the log-likelihood function can be maximized over the sample to yield maximum-likelihood parameter estimates. See Kim and Nelson (1999) for additional details.

Bank of Canada Working Papers
Documents de travail de la Banque du Canada

Working papers are generally published in the language of the author, with an abstract in both official languages. *Les documents de travail sont publiés généralement dans la langue utilisée par les auteurs; ils sont cependant précédés d'un résumé bilingue.*

2003

2002