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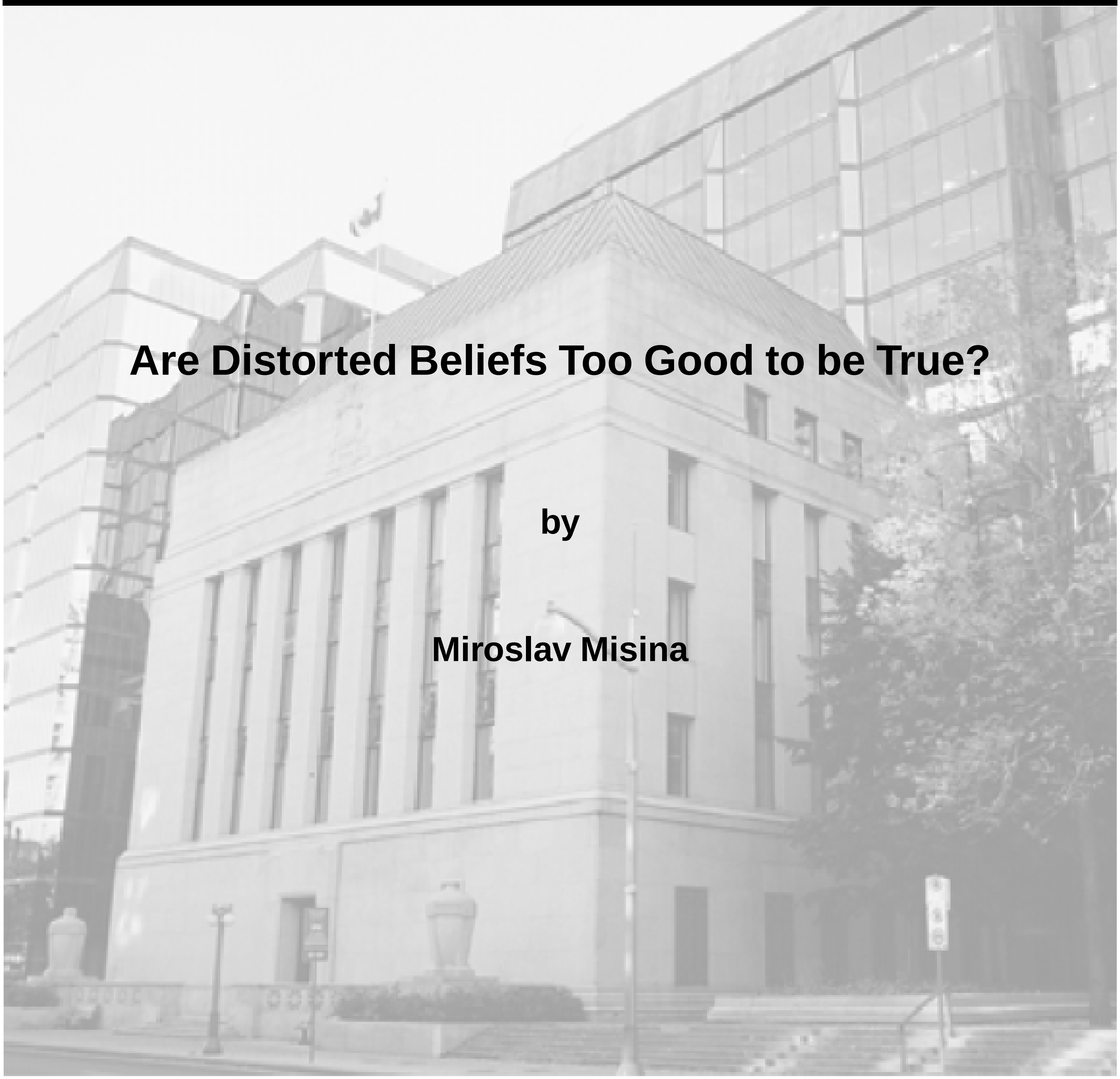

Are Distorted Beliefs Too Good to be True?

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Contents

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Abstract

In a recent attempt to account for the equity-premium puzzle within a representative-agent model, Cecchetti, Lam, and Mark (2000) relax the assumption of rational expectations and in its place use the assumption of distorted beliefs. The author shows that the explanatory power of the distorted beliefs model is due to an inconsistency in the model and that an attempt to remove this inconsistency removes the model's explanatory power. Using the theory of rational beliefs, the author constructs a model in which the inconsistency is not present, compares its performance with that of the distorted beliefs model, and gives a simple interpretation of the results obtained.

JEL classification: D84, G12
Bank classification: Economic models; Financial markets

Résumé

Dans une étude récente qui tente d'expliquer l'énigme de la prime de risque rattachée aux actions dans le cadre d'un modèle à agent représentatif, Cecchetti, Lam et Mark (2000) écartent l'hypothèse des anticipations rationnelles pour utiliser à sa place l'hypothèse de la distorsion des croyances. L'auteur montre que le pouvoir explicatif du modèle de distorsion des croyances est dû à une incohérence intrinsèque et que tenter de l'éliminer revient à supprimer le pouvoir explicatif du modèle. Se fondant sur la théorie des croyances rationnelles, l'auteur construit un modèle qui, lui, est cohérent, compare sa tenue avec celle du modèle de distorsion des croyances et donne une interprétation simple des résultats obtenus.

Classification JEL : D84, G12
Classification de la Banque : Modèles économiques; Marchés financiers

1. Introduction

It is well known that a simple Lucas-type representative-agent model cannot account for the historically observed differences in returns on equity and bonds. The problem, originally identified by Mehra and Prescott (1985),[1] has generated much work in attempting to resolve what has become known as the "equity-premium puzzle." As Kocherlakota (1996) emphasizes in his survey of attempts to solve the problem, the relationship between the returns on stocks and bonds found in the data is in line with the predictions of the asset-pricing theory. The puzzle refers to a failure of a particular type of model to match the quantitative properties of the data.

Three main attempts have been made to resolve the equity-premium puzzle within the Lucas-type representative-agent model:[2]

- modifying the utility function,
- relaxing the assumption of market completeness, and
- introducing transactions costs.

After an extensive survey of these efforts, Kocherlakota concludes that the literature has offered only two plausible explanations of the equity-premium puzzle:

(i) unusually risk-averse investors, and

(ii) the presence of transactions costs.

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1 Hereafter referred to as MP.

2 See Kocherlakota (1996) for references.

The problem with the first explanation is that most researchers are unwilling to assume the degrees of risk aversion necessary to explain the puzzle. Regarding the second explanation, the presence of transactions costs is not in itself sufficient to explain the puzzle: it turns out that the assumption of transactions cost asymmetries across markets is necessary. In particular, one must assume that it is much more costly to participate in the stock market than in the bond market. Since the evidence does not support the cost differentials needed to explain the puzzle, Kocherlakota concludes that the equity premium is still a puzzle.

A common characteristic of all attempts surveyed by Kocherlakota is that the assumption of rational expectations (RE) is maintained. Since the properties of prices depend, among other things, on the probability structure, it is natural to try to relax the assumption that agents know the true data-generating process and that they use it in their computations. Cecchetti, Lam, and Mark (2000)[3] relax the assumption of RE and introduce a degree of persistent irrationality into the model. In particular, they assume that the agent's probability beliefs systematically deviate from the true beliefs. CLM show that the introduction of this form of irrationality, termed "distorted beliefs," with all other features of the original MP model remaining intact, can explain the equity-premium puzzle.

This type of explanation is potentially problematic**.** Substitution of the assumption of RE by distorted beliefs in the manner of CLM is based on an interpretation of RE in behavioural terms. Although RE can be interpreted in this way, the emphasis on behavioural interpretation tends to obscure the role of this assumption as a consistency condition in the model. Relaxing the assumption of RE will, in general, result in incon-

[3] Hereafter referred to as CLM.

sistent models. To relax the assumption of RE is not simply a matter of replacing it with an alternative behavioural assumption—an alternative equilibrium concept is required. An alternative to RE is the theory of rational beliefs and the notion of a rational beliefs equilibrium, introduced by Kurz (1994). Rationality of beliefs can be interpreted as a weak consistency condition: although heterogeneity of beliefs is allowed, a priori restrictions are imposed on the set of admissible beliefs in a way that model consistency is preserved.

In this paper, we demonstrate the consequences of using an inconsistent model to explain the equity-premium puzzle. Inconsistently specified models are shown to result in additional degrees of freedom, which increases their explanatory power to an arbitrary degree. After the nature of the problem is demonstrated, the equity-premium puzzle is examined in a representative-agent model in which rationality of beliefs is used as a consistency condition. Rational beliefs are compared and contrasted with distorted beliefs and some conclusions are drawn.

In the following presentation, CLM's model is used as an illustration of this type of problem, although the issues raised may be relevant to the whole class of heterogeneous beliefs models. Our results suggest that one should be wary of any model in which the expectational assumptions are motivated by behavioural concerns, without an explicit demonstration of the consistency of the resulting model.

This paper is organized as follows.

In section 2, we introduce the ideas of microspace and macrospace and use these to describe model consistency. These concepts are then applied to models with distorted beliefs and an example is given that illustrates the sources of these models' explanatory power. Ideas illustrated in this example are developed fully in the rest of the paper.

In section 3, three types of models are described: (i) the basic RE model used by MP, (ii) a simplified version of the distorted beliefs model used by CLM, and (iii) a representative-agent model with rational beliefs. The properties of these models are analyzed, and their relationship to each other is explained. As will be shown, both the distorted beliefs and rational beliefs models are derived by expanding the state space. The differences between them lie in the types of restrictions imposed on the agent's beliefs.

Section 4 contains the simulation results. First, we reproduce the equity-premium puzzle results in all three models. Some of CLM's results are then replicated and the sources of their model's explanatory power are examined. This is followed by an investigation of the relationship between distorted and rational beliefs and an investigation of the equity-premium puzzle in a representative-agent model with rational beliefs. Section 5 concludes.

2. Model Consistency

The notion of model consistency that is introduced here is based on a simple observation: since the properties of any model follow from the specification of the characteristics of agents, the properties of the model must be related to those characteristics. Our focus on the role of expectations in models translates into the analysis of the relationship between the stochastic properties of the model and those of the agents. To formalize these ideas, the economy is viewed as a stochastic dynamical system. A model is consistent if the stochastic properties of the system depend on the stochastic properties of agents.

The following definitions formalize the above intuition:

Microspace: A space in which individual k operates, characterized by observable variables and private signals. Denote this space by $V_k, \forall k.$

Microsystem: A stochastic dynamical system defined on the microspace; i.e.,

$$\left(V_k^{\infty}, \mathcal{B}\left(V_k^{\infty}\right), Q_k, T\right),$$

where, Q_k is the individual probability measure and T is the shift operator.

Macrospace (aggregate space): Product space $X = \times_k V^k$.

Economy: A stochastic dynamical system defined on the product space; i.e.,

$$\left(X^{\infty}, B\left(X^{\infty}\right), \Pi, T\right).$$

In the special case of the representative-agent models, the microspace and macrospace coincide.

The question of model consistency is a question of the relationship between the individual probability measure, Q_k, and the probability measure, Π, defined on the macrospace.[4] Since the macrospace is a product space, it is clear that the properties of Π must in some ways be related to the properties of Q_k. One form of consistency that can be used is to assume that $Q_k = \Pi$. This is known as the assumption of rational expectations.

Definition *A model is said to be inconsistent if Q_k and Π are independent.*

A model is inconsistent if it is specified in such a way that the properties of the true probability measure, Π, are unrelated to the properties of individual probability mea-

[4] Π will be referred to as the true probability measure associated with the macrospace.

sures. In representative-agent economies the problem is particularly apparent. Since there is, in effect, only one agent there, the independence of Q and Π leaves us without satisfactory answers to the following questions:

- Assuming that Q_k is given, what is the origin of Π? The use of Q_k by agents will not result in Π.
- If Π is taken as a primitive, what is the origin of Q_k?

If one interprets Π as summarizing the properties of the data, the above problems can be restated as follows: a model is inconsistent if the properties of the data, represented by Π, are not implied by, or are not related to, the properties of Q_k.

Models with distorted beliefs as well as models with rational beliefs relax the assumption of strong-form consistency. Models with rational beliefs impose the rationality-of-beliefs condition, which can be thought of as a weak-form consistency condition. Models with distorted beliefs do not impose any consistency condition relating Π to Q_k.

2.1 Rationality of beliefs as a weak consistency condition

The idea behind the rationality-of-beliefs condition can be illustrated as follows.[5] The economy is represented as $(X^\infty, B(X^\infty), \Pi, T)$. An agent has available some data and forms a belief about Π, denoted by Q. The agent's view of the economy is summarized by $(X^\infty, B(X^\infty), Q, T)$.

[5] See Kurz (1997), Introduction, for an excellent summary of the main ideas.

A stochastic dynamical system is stable if the frequencies of finite-dimensional events converge. The assumption of stability implies the existence of an empirical distribution function. Kurz (1994) proves the following proposition:

Proposition 1 *(Kurz 1994)* $(X^{\infty}, \mathcal{B}(X^{\infty}), \Pi, T)$ *is stable iff it is weakly asymptotically mean stationary.*

Under the assumption of stability, it can be shown that there are unique stationary measures, m^{Π} and m^{Q}, associated with each of the above systems. The rationality-of-beliefs condition requires that m^{Π} and m^{Q} coincide.

Stability is a weaker requirement than stationarity.[6] If the system is stationary m^{Π} and Π will coincide, but more general possibilities are allowed. In particular, non-stationarity is allowed. This implies that if the data are generated by a stable system, agents, in learning the empirical distribution, will end up learning the stationary distribution of the system. The rationality-of-beliefs condition requires that this distribution coincide with m^{Π}. By contrast, the assumption of rational expectations requires that $Q = \Pi$.

2.2 Model consistency and explanatory power

Regarding the explanatory power of inconsistently specified models, it is clear that when Q_k and Π are independent an additional degree of freedom exists in the model that can be used to improve its explanatory power. Indeed, as the following example illustrates, almost anything can be "explained" using these models.

[6] The assumption of stability imposes restrictions on the time-heterogeneity of the process. Some restrictions on time-heterogeneity of the stochastic process are necessary to make an operational model that can be used to draw inferences about the data.

Example 1 *In the representative-agent economy analyzed by MP, the individual optimization implies the following expressions for stock and bond prices*[7]*:*

$$p_i^s = \beta \sum_j \pi_{ij} \left(g_j\right)^{1-\gamma} \left(p_j^s + 1\right),$$

for stocks, and

$$p_i^b = \beta \sum_j \pi_{ij} \left(g_j\right)^{-\gamma},$$

for bonds. β *is the subjective discount rate,* γ *is the risk-aversion parameter,* π_{ij} *is the transition probability to state* j *given state* i*, and* g_j *is the growth rate of endowments in state* j*. Setting* $\gamma = 1$ *results in the following expressions*[8]*:*

$$p_i^s = \beta \sum_j \pi_{ij} \left(p_j^s + 1\right), \forall i,$$

for stocks, and

$$p_i^s = \beta \sum_j \pi_{ij} \left(g_j\right)^{-1}, \forall i,$$

for bonds. Using MP's parameterization, with

$$\begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 1.054 \\ 0.982 \end{bmatrix}, \quad \Phi = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} 0.43 & 0.57 \\ 0.57 & 0.43 \end{bmatrix},$$

and setting $\beta = 0.95$*, results in the following values for prices:*

$$p_1^b = 0.939, \; p_2^b = 0.930, \; p_i^s = 19, i = 1, 2.$$

The expected returns are computed using

$$\rho^s = \sum_{i=1}^n \bar{\pi}_i R_i^s, \quad \rho^b = \sum_{i=1}^n \bar{\pi}_i R_i^b,$$

where R_i^s *and* R_i^b *are state-*i *returns on stocks and bonds, respectively, and* $\bar{\pi}_i$ *is the unconditional probability of state* i*. Under the assumption of rational expectations, the prices are computed using* Φ *and* $\bar{\pi}_i$ *is the unconditional state* i *probability implied by* Φ*. In this example, the expected rate of return on stocks is approximately 7.16 per cent*

[7] The details of the model are described in section 3.

[8] The choice of $\gamma = 1$ is for convenience only. The points made herein do not depend on the choice of γ. Section 4 contains the results for different values of γ.

and on bonds it is 7.03 per cent. This results in an equity premium of 0.13 per cent, as opposed to 6-7 per cent observed in the data. This is the equity-premium puzzle.
Suppose that the agents' belief is represented by Φ' and that $\Phi' \neq \Phi$. For concreteness,[9] *let*

$$\Phi' = \begin{bmatrix} 0.1 & 0.9 \\ 0.1 & 0.9 \end{bmatrix}.$$

Under this specification, leaving the other parameters unchanged, the following values are obtained for the prices of stocks and bonds:

$$p_i^b = 0.9608,\ p_i^s = 19,\ i = 1, 2.$$

These prices were computed using Φ'. Computing the expected returns using Φ' gives $\rho^b = 0.0408$ and $\rho^s = 0.0413$, so that the equity premium is 0.05 per cent. The puzzle remains.
Suppose that the expected returns are computed using Φ while the prices are still computed using Φ'. In this case, $\rho^b = 0.0408$ and $\rho^s = 0.0716$, which gives an equity premium of 3.1 per cent, much closer to the one observed in the data.

The preceding example illustrates the way in which an inconsistently specified model can be used to explain the equity-premium puzzle. By properly choosing γ and Φ', while computing the expected values using Φ one can obtain any result.

3. Models

In this section, three types of models are presented:

(i) Model 1—rational expectations (RE),

(ii) Model 2—distorted beliefs (DB), and

(iii) Model 3—rational beliefs (RB).

[9] Again, the numbers chosen are for convenience only. Section 4 contains the results for several choices that differ from the one here.

The first model is used by MP; the second is used by CLM. The only difference between these models is in the assumptions imposed on the agent's beliefs.

Let $\Theta(m)$, $m = RE, RB, DB$, denote the solution sets associated with these models. One can then write

$$\Theta(RE) \subset \Theta(RB) \subset \Theta(DB).$$

This relationship follows from the assumption about the agent's beliefs that is used. The assumption of RE is the most restrictive, since it amounts to assuming, as noted in section 2, that the agent knows and uses the true probability distribution in computing the expected values. In the RB model, beliefs are restricted in such a way that the long-run properties of the data are reproduced. This assumption is less restrictive than the assumption of RE, since it does not rely on the knowledge of the true data-generating process but only on some of its characteristics. RE models are special cases of RB models when RB are the same as RE.

The assumption of DB is the least restrictive, since none of the characteristics of the true data-generating process are used to restrict the beliefs that the agent can hold. Even if agents can learn the characteristics of the true data-generating process, they ignore them. That is why they are called "irrational."

There is another way to view the models introduced above. As shown in sections 3.2 and 3.3, both DB and RB models are obtained from the RE model by the appropriate expansion of the state space. Although the expansion of the state space is used in both DB and RB models, the key difference is that in DB models no consistency condition is specified, whereas in RB models it is present.

3.1 Representative-agent economy with rational expectations

The model used in this section corresponds to the models used by MP and CLM. It is a representative-agent model of an exchange economy with a single consumption good and endowment uncertainty. There are two assets in the model: a unit of stock, with price p^s, and a bond, with price p^b. Both pay in units of the single consumption good. The payoff of the stock varies across states, whereas bonds pay one unit in each state. Since there is only one consumption good in the model, and bonds pay a fixed amount in units of this consumption good in each state, it follows that the bond is a riskless asset.

The structure of the model implies that the solution will take a rather special form: since there is in effect only one agent in the model, there will be no trade in equilibrium, so that it is optimal for the agents to consume the available endowment.

The evolution of the endowment is modelled by a Markov chain, specifying the transition probabilities at each point. In MP, it is assumed that the endowments are

$$D_j = g_j D_i.$$

The growth rate of endowments, g_j, follows a two-state Markov chain with the transition matrix

$$\Gamma_m \equiv \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} \phi_1 & 1-\phi_2 \\ 1-\phi_1 & \phi_2 \end{bmatrix}, \tag{1}$$

with $\phi_1 = \phi_2$. CLM use the same specification, except that $\phi_1 \neq \phi_2$.[10]

[10] The differences in the specification of the endowment growth process are discussed in section 4.1.1.

The utility function used is the constant relative risk-aversion type:

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}.$$

From the first-order conditions, we get the following expressions for stock and bond prices in each state i:

$$p_i^s = \beta \sum_j \pi_{ij} (g_j)^{1-\gamma} \left(p_j^s + 1\right), \tag{2}$$

and

$$p_i^b = \beta \sum_j \pi_{ij} (g_j)^{-\gamma}. \tag{3}$$

Since there is no other uncertainty, and under the assumption that Φ is used in computing asset prices, there will be two possible states of the world, h and l, with bond and stock prices defined in each state. For convenience, assume that the state at each point is selected by a signal $S \in \{1, 0\}$, where the evolution of S is described by (1). The link between this formulation and the one in MP is established by defining

$$g_j = \begin{cases} g^h & \text{if } S = 1 \\ g^l & \text{if } S = 0 \end{cases}.$$

The state space looks as follows:

State	**S**
1	1
2	0

The price mapping is given by $P : S \to R$. The consequence of the fact that agents know and use Φ is that $\pi_{ij} = \phi, i = j$, and $\pi_{ij} = 1 - \phi, i \neq j$. The knowledge of (d, ϕ) makes it possible to compute the prices in different states.

The returns on equity in this model are given by the returns matrix

$$\left[\begin{array}{cc} r_{11}^s & r_{12}^s \\ r_{21}^s & r_{22}^s \end{array}\right],$$

where

$$r_{ij}^s = \frac{p_j^s + d_j^s}{p_i^s} = g_j \left(\frac{p_j + 1}{p_i}\right).$$

The expected return on equity if the current state is i is

$$R_i^s = \sum_{j=1}^{n} \phi_{ij} r_{ij}^s,\ \forall i,$$

where ϕ_{ij} is the ijth element of Γ_m.

The return matrix for bonds is

$$\left[\begin{array}{cc} r_{11}^b & r_{12}^b \\ r_{21}^b & r_{22}^b \end{array}\right],$$

where

$$r_{ij} = \frac{d_j^b}{p_i^b}.$$

Since, in the model, $d_j = 1, \forall j$,

$$r_{ij} = \frac{1}{p_i^b}.$$

The return on bonds if the current state is i is

$$R_i^b = \sum_{j=1}^{n} \phi_{ij} r_{ij}^b,\ \forall i.$$

The expected returns on equity and bonds are

$$\rho^s = \sum_{i=1}^{n} \bar{\pi}_i R_i^s, \quad \rho^b = \sum_{i=1}^{n} \bar{\pi}_i R_i^b,$$

where $\bar{\pi}_i$ is the unconditional probability of state $i,$ which exists under the assumption of ergodicity and is represented by the normalized eigenvector associated with the unit eigenvalue of matrix $\Gamma_m.$

3.2 Representative-agent economy with distorted beliefs

The model used by CLM is identical in all respects to the model described in section 3.1, except in the specification of the sources and the nature of the uncertainty. The key to CLM's approach is to relax the assumption that agents know and use Φ. The endowment process (growth rates) will evolve according to the Markov process, which takes the form

$$\Phi \equiv \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} \phi_1 & 1-\phi_2 \\ 1-\phi_1 & \phi_2 \end{bmatrix},$$

with $\phi_1 \neq \phi_2$.[11] This is assumed to describe the true process that generated the data.

Agents do not believe that this is the true process, and they form their own beliefs about the transition probabilities and the endowment growth rates. Their decisions depend on pseudo-signals, which will in turn affect prices. Prices will depend on the state of these signals as well as on the state of the observables in the economy. Thus, $\mathcal{S} = D \times S$ in a representative-agent economy and the resulting prices are $P : \mathcal{S} \to R.$

CLM assume that there are, apart from the signal $S \in \{0, 1\}$ indicating the current state of the economy, two types of pseudo-signals in the model: $S_e \in \{1, -1\}$, which influences agents' behaviour in expansions, and $S_c \in \{1, -1\}$, which influences agents' behaviour if the economy is in recession. Each of these signals is assumed to follow a

[11] The implications of the difference in specification of the endowment growth process are discussed in section 4.1.1.

two-state Markov chain with transition matrices of the form:

$$\Gamma_{S_e} = \left[\begin{array}{cc} \tilde{\phi}_e & \tilde{\phi}_e^* \\ \tilde{\phi}_e^* & \tilde{\phi}_e \end{array} \right],$$

and

$$\Gamma_{S_c} = \left[\begin{array}{cc} \tilde{\phi}_c & \tilde{\phi}_c^* \\ \tilde{\phi}_c^* & \tilde{\phi}_c \end{array} \right],$$

where $\tilde{\phi}_i^* = 1 - \tilde{\phi}_i,\ i = s, c.$

The joint transition matrix of the pseudo-signals is then

$$\Gamma_{ec} = \Gamma_{S_e} . * \Gamma_{S_c} = \left[\begin{array}{cc} \tilde{\phi}_e \Gamma_{S_c} & \tilde{\phi}_e^* \Gamma_{S_c} \\ \tilde{\phi}_e^* \Gamma_{S_c} & \tilde{\phi}_e \Gamma_{S_c} \end{array} \right].$$

The state space now looks as follows:

State	$(\mathbf{S}, \mathbf{S}_e, \mathbf{S}_c)$
1	$(1, 1, 1)$
2	$(1, 1, -1)$
3	$(1, -1, 1)$
4	$(1, -1, -1)$.
5	$(0, 1, 1)$
6	$(0, 1, -1)$
7	$(0, -1, 1)$
8	$(0, -1, -1)$

The representative agent's belief about the true transition probabilities can be described as follows[12]:

$$\tilde{\pi}(S_e) = \tilde{\mu}_{\tilde{\pi}_e} + \tilde{\delta}_e S_e,$$

and

$$\tilde{\pi}(S_c) = \tilde{\mu}_{\tilde{\pi}_c} - \tilde{\delta}_c S_c.$$

12 Similar expressions are used to describe the beliefs about the endowment growth rates in different states.

This is used to define the following:

$$p_0 = \tilde{\mu}_{\tilde{\pi}_e} + \tilde{\delta}_e, \;\; p_p = \tilde{\mu}_{\tilde{\pi}_e} - \tilde{\delta}_e,$$

$$q_0 = \tilde{\mu}_{\tilde{\pi}_c} - \tilde{\delta}_c, \; q_q = \tilde{\mu}_{\tilde{\pi}_c} - \tilde{\delta}_c.$$

The following two transition matrices characterize the agent's subjective belief:

$$\tilde{F}_1 = \left[\begin{array}{cc} p_0 & 1-p_0 \\ p_p & 1-p_p \end{array} \right],$$

if the current state is expansionary, and

$$\tilde{F}_2 = \left[\begin{array}{cc} 1-q_0 & q_0 \\ 1-q_p & q_p \end{array} \right],$$

if the current state is contractionary. These matrices, together with the selection criterion for the matrix $\tilde{P}$, specify the conditional probabilities used in the computations:

$$\tilde{P} \equiv [p_j] = \left\{ \begin{array}{lll} \tilde{F}_1[1,.].*\Gamma_{ec}[i,.] & \text{if} & S=1, \; S_e=1, \\ \tilde{F}_1[2,.].*\Gamma_{ec}[i,.] & \text{if} & S=1, \; S_e=-1, \\ \tilde{F}_2[1,.].*\Gamma_{ec}[i,.] & \text{if} & S=0, \; S_c=1, \\ \tilde{F}_2[2,.].*\Gamma_{ec}[i,.] & \text{if} & S=0, S_c=-1, \end{array} \right. , \tag{4}$$

where $i = 1, ..., 4, \; j = 1, ..., 8.$

In the ensuing discussion, two simplifying assumptions are made:

(i) there is only one pseudo-signal governing behaviour in both the contractionary and expansionary states, and

(ii) the agent's beliefs about the endowment growth rates are not distorted.

Use of these assumptions makes the similarities (and differences) between the models with distorted beliefs and those with rational beliefs much more obvious without affecting any of the findings related to the first moment of the equity-premium puzzle.[13] Indeed, CLM's success in explaining the first moments of the puzzle does not depend on the presence of asymmetries arising from two different signals, S_e and S_c. We will first replicate the results of CLM and then investigate the sources of successes in this model.

Due to the fact that there is only one pseudo-signal in the model, the dimension of the state space is reduced from 8 to 4. The state space looks as follows:

State	$(\mathbf{S}, \mathbf{S}_{ec})$
1	$(1, 1)$
2	$(1, -1)$
3	$(0, 1)$
4	$(0, -1)$

.

The transition matrices used by agents are still $\tilde{F}_1$ and $\tilde{F}_2$. The pseudo-signal is, as before, assumed to follow a two-state Markov process with the transition matrix

$$\Gamma_{S_{ec}} = \begin{bmatrix} \tilde{\phi} & \tilde{\phi}^* \\ \tilde{\phi}^* & \tilde{\phi} \end{bmatrix},$$

where $\tilde{\phi}^* = 1 - \tilde{\phi}$.

The selection criteria for probabilities used in computations of prices are the same as in (4), once $\Gamma_{S_{ec}}$ is substituted for Γ_{ec}. Note that if $\tilde{\delta}_i = 0$, $i = e, c$, $\tilde{\mu}_{\tilde{\pi}_e} = \phi_1$, and $\tilde{\mu}_{\tilde{\pi}_c} = \phi_2$, these matrices coincide with the true transition matrix, which is called the stationary transition matrix Γ (described in section 3.3).

[13] The intuition behind the results is summarized in Example 1 of section 2.2.

The prices in the model are computed from (2) and (3) using the distorted probabilities specified above. The expected values are computed as

$$\rho^s = \sum_{i=1}^{n} \bar{\pi}_i R_i^s, \quad \rho^b = \sum_{i=1}^{n} \bar{\pi}_i R_i^b,$$

where $\bar{\pi}_i$ represents the unconditional state probabilities associated with the **true** transition matrix that describes the behaviour of the economy. These results will be used in the simulation work in section 3.3.

3.3 Representative-agent economy with rational beliefs

We follow the same strategy as above, expanding the state space by allowing the agents to hold beliefs that may systematically differ from the beliefs implied by the data, while imposing the rationality-of-beliefs condition.

To keep things simple, assume that at any time the agent can use one of two transition matrices, F^1 and F^2, to compute prices for the next period. The agent's belief is then an infinite sequence, $Q = \left\{F_t^k\right\}$, $k = 1, 2$. The elements of the sequence are selected by a signal, $s_t \in S_p = \{0, 1\}$, with $\Pr\{s_t = 1\} = \alpha$. Assume that a large number of observations on variables of interest are available to the agent. Also assume that the agent has learned the stationary measure associated with the, possibly non-stationary, system $(X, B(X), \Pi, T)$ that generated the data. Denote this stationary measure as m^Π. The rationality criterion then requires that the agent's belief be consistent with this stationary measure, so that $m^Q = m^\Pi$.

The state space is $\mathcal{S} = D \times S_p$, where S_p is the space of signals that select the transition matrix to be used at any point in time. Since it was assumed that there are only two matrices to select from, our state space will have dimension 4. It looks as follows:

State	(S, S_p)
1	$(1,1)$
2	$(1,0)$
3	$(0,1)$
4	$(0,0)$

.

Prices will depend on the state of dividends as well as the state of the private signals. The stationary transition matrix in this case is given by[14]

$$\Gamma = \Phi .* A,$$

where

$$A = \begin{bmatrix} a_1\alpha & 1 - a_1\alpha \\ a_2\alpha & 1 - a_2\alpha \end{bmatrix}.$$

Here, $\alpha = \Pr(s = 1)$, $s \in S = \{1, 0\}$, and $a = (a_1, a_2)$ is a vector of parameters that allow for the possibility of asymmetries across states.

The rationality-of-beliefs condition requires that the selected matrices reproduce the stationary measure. This condition can be written as

$$\sum_k \alpha_k F^k = \Gamma, \quad \sum_k \alpha_k = 1. \tag{5}$$

Matrix F^1 takes the following form[15]:

$$\begin{bmatrix} \phi\lambda_1 a_1\alpha & \phi\lambda_1(1 - a_1\alpha) & (1 - \lambda_1\phi)\, a_1\alpha & (1 - \lambda_1\phi)(1 - a_1\alpha) \\ \phi\lambda_2 a_2\alpha & \phi\lambda_2(1 - a_2\alpha) & (1 - \lambda_2\phi)\, a_2\alpha & (1 - \lambda_2\phi)(1 - a_2\alpha) \\ (1 - \phi)\,\lambda_3 a_1\alpha & (1 - \phi)\,\lambda_3(1 - a_1\alpha) & (1 - (1 - \phi))\,\lambda_3 a_1\alpha & (1 - (1 - \phi))\,\lambda_3(1 - a_1\alpha) \\ (1 - \phi)\,\lambda_4 a_2\alpha & (1 - \phi)\,\lambda_4(1 - a_2\alpha) & (1 - (1 - \phi))\,\lambda_4 a_2\alpha & (1 - (1 - \phi))\,\lambda_4(1 - a_2\alpha) \end{bmatrix}.$$

[14] The formulation $\Gamma = \Gamma_c .* A$ will be used when we want to compare the performance of the models with distorted beliefs and with rational beliefs.

[15] Matrix F^2 is calculated using (5).

Here, $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ are parameters. These parameters represent proportional revisions of state probabilities relative to the stationary measure represented by Γ.[16] For example, $\lambda > 1$ implies that the agent assigns greater probability to states 1 and 2 than is assigned by the stationary measure Γ.

Conditional probabilities used in computations are selected according to the following rule:

$$\pi_{ij} = \begin{cases} F^1_{(i,j)} & if \ s_i = 1 \\ F^2_{(i,j)} & if \ s_i = 0 \end{cases}, \tag{6}$$

where $F^k_{(i,j)}$ is the (i,j) element of $F^k, k = 1, 2.$

The prices in the model are computed using the probabilities specified above. The expected values are computed as

$$\rho^s = \sum_{i=1}^{n} \bar{\pi}_i R_i^s, \quad \rho^b = \sum_{i=1}^{n} \bar{\pi}_i R_i^b, \tag{7}$$

where $\bar{\pi}_i$ denotes the unconditional probabilities associated with the stationary transition matrix Γ. The key difference between these computations and the ones in the model with distorted beliefs is that these probabilities depend on the transition matrices used by agents, whereas in the model with distorted beliefs there is no link between these and the agent's beliefs. Indeed, one may interpret the distortions in beliefs as systematic deviations of unconditional probabilities associated with the distorted beliefs from the unconditional probabilities associated with the true transition matrix. There is no such discrepancy in the model with rational beliefs. The rationality conditions specified above put restrictions on the unconditional state probabilities as well. In effect, the

[16] In the experiments in section 4.5, we assume that $\lambda_i = \lambda_j, \forall i, j$, and for simplicity refer to this parameter as λ. This assumption implies that all states are subject to equal revisions.

transition matrices used by agents imply the unconditional probabilities of the stationary matrix.

4. Rational Expectations, Rational Beliefs, Distorted Beliefs, and the Equity-Premium Puzzle

4.1 Parameterization

4.1.1 Consumption (endowment) process

CLM's specification of the endowment process differs from MP's and this has significant implications for the model performance.

Both specifications assume that the endowment process follows a two-state Markov process with the transition matrix:

$$\Phi = \begin{bmatrix} \phi_1 & 1-\phi_1 \\ 1-\phi_2 & \phi_2 \end{bmatrix}.$$

The difference is that in MP it is assumed that $\phi_1 = \phi_2$. The difference in specification is due to the difference in the way the consumption growth process is identified. In MP, the growth rates and transition probabilities are chosen to match the following sample properties of the U.S. consumption series:

- average growth rate of per capita consumption ($\mu = 0.018$),
- standard deviation of the growth rate of per capita consumption ($\delta = 0.036$), and
- first-order serial correlation of this growth rate ($\rho = -0.14$).

The growth rates in two states are:

$$\left[\begin{array}{c} g_1 \\ g_2 \end{array}\right] \equiv \left[\begin{array}{c} 1+\mu+\delta \\ 1+\mu-\delta \end{array}\right] = \left[\begin{array}{c} 1.054 \\ 0.982 \end{array}\right]$$

$$g_1 = 1+\mu+\delta, \;\; g_2 = 1+\mu-\delta,$$

while the transition probabilities are

$$\phi_{11} = \phi_{22} = \phi, \; \phi_{12} = \phi_{21} = 1-\phi,$$

with the resulting transition matrix

$$\Phi = \left[\begin{array}{cc} 0.43 & 0.57 \\ 0.57 & 0.43 \end{array}\right].$$

CLM, rather than selecting the transition probabilities indirectly, estimate them directly using a regime-switching model, and use these estimates in their specification of the transition matrix. They get the following estimates of transition probabilities and growth rates:

- transition matrix:

$$\Phi = \left[\begin{array}{cc} 0.978 & 0.022 \\ 0.484 & 0.516 \end{array}\right],$$

- growth rates:

$$\left[\begin{array}{c} g_1 \\ g_2 \end{array}\right] = \left[\begin{array}{c} 1.02251 \\ 0.93215 \end{array}\right].$$

The values used by CLM differ considerably from those used by MP. For example, in MP, the unconditional probabilities of each state are 0.5, the unconditional probability of the good state in CLM is 0.96, and the probability of the bad state is only 4 per cent.

Whereas in both cases the probabilities of staying in the bad state, once there, are around 0.5, the probability of staying in a good state, once there, in CLM, is more than twice that for MP.

Taking into account the growth rates, in the CLM's economy good states are moderately good, although they dominate by far, whereas the bad states, while infrequent, are quite bad, with a drop in the growth rate of over 6 per cent and a greater than 50 per cent chance of lasting for more than one period. In MP, the situation is quite different. Both states are equally likely and, once in either one of them, equally persistent, with a higher probability of exiting the current state in the next period than staying in it. The growth rates in the good state are over 5 per cent; in the bad state the decline is by roughly 2 per cent. In summary, the MP economy is *much more volatile*. This will be an important clue in interpreting the results.

To make the results of our experiments comparable to those of CLM, their specification of the endowment process will be used.

4.1.2 Other parameters

MP restrict the value of the risk-aversion parameter, γ, to be less than 10. The subjective discount rate is restricted to the range $\beta \in < 0, 1 >$. These restrictions are standard.

The values of the other parameters are given and discussed in sections 4.2 to 4.5.

4.2 Recovering the equity-premium puzzle

In the first set of experiments, we report the values of the risk-free rate and the equity premium in the special case when both the DB model and the RB model coincide with the MP model. This is accomplished by setting the parameters to the following values:

$$\alpha_1 = \alpha_2 = 0.5,$$

$$a_i = 1,\ i = 1, 2,$$

$$\lambda = 1,$$

$$g_1 = 1.02251,\ g_2 = 0.93215,\ \phi_1 = 0.978,\ \phi_2 = 0.516.$$

In this case, the stationary matrix, Γ, and the matrix used by CLM coincide. In addition, the parameters of the DB matrix have been set to the following values:

$$\mu_p = 0.978,\ \mu_q = 0.516,\ \delta_p = \delta_q = 0.$$

This means that the agent in the DB model is using the stationary matrix, so that in this special case the DB model reduces to the MP model. The results are given in Table 1.

Table 1: Equity Premiums with CLM Specification of Endowment Process

Cases	$\boldsymbol{\beta}$	γ	$\boldsymbol{\rho}^b$	$\boldsymbol{\rho}^s$	**ep**
1	0.95	1	0.0719	0.0722	0.003
2	0.95	2	0.0913	0.0914	0.001
3	0.95	15	0.344	0.699	0.356
4	0.90	3	0.173	0.172	-0.001

The expected returns on bonds and stocks are represented by ρ^b and ρ^s, respectively, and ep is the implied equity premium. As the last column of the table shows, the equity premium is below 0.5 per cent for low values of the risk-aversion parameter. As case 3 illustrates, the high values of the risk-aversion parameter move the equity premium in the desired direction. For reasonable values of the risk-aversion parameter, the equity-premium puzzle remains.

4.3 Equity-premium puzzle with distorted beliefs

In this section, we modify the probabilities in accordance with the CLM specification and present the results of the experiments. The results are used to explain the source of explanatory success of the model.

Table 2: Equity Premiums in the DB Model (expected returns computed using true probabilities)

Cases	$\bar{\mathbf{p}}$	$\bar{\mathbf{q}}$	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	$\boldsymbol{\rho}^b$	$\boldsymbol{\rho}^s$	**ep**
1	0.5	0.2	0.933	1.831	0.0245	0.0790	0.0545
2	0.5	0.4	0.918	2.371	0.0239	0.0778	0.0539
3	0.6	0.2	0.923	3.159	0.0262	0.0795	0.0533
4	0.7	0.1	0.930	4.691	0.0309	0.0829	0.0520

Columns 2 to 5 of Table 2 give the parameter values. These values are taken from CLM[17] and are the parameter values which, according to CLM, solve the equity-premium puzzle nearly exactly, in the sense that the mean risk-free rate is around 2.5 per cent and the equity premium is around 5.5 per cent. $\bar{p}$ and $\bar{q}$ are the mean values of subjective transition probabilities used by agents in good and bad states, respectively.

[17] See CLM, Table 4.

These are the “distorted probabilities” used by the agent; β is the subjective discount rate and γ represents the risk-aversion parameter.

The results in Table 2 indicate that the equity premium is roughly in the desired range, according to CLM’s criteria. These results are obtained by the judicious choice of parameters. It is important to emphasize that the above returns were calculated using the **correct** probabilities associated with the transition matrix, Γ, whereas prices are computed using the distorted probabilities.

What happens when distorted probabilities are used to compute expected returns? Table 3 gives the results. Both prices and expected returns are computed using the same set of probabilities.

Table 3: Equity Premiums in the DB Model (expected returns computed using distorted probabilities)

Cases	$\bar{\mathbf{p}}$	$\bar{\mathbf{q}}$	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	$\boldsymbol{\rho}^b$	$\boldsymbol{\rho}^s$	**ep**
1	0.5	0.2	0.933	1.831	0.0431	0.0470	0.0039
2	0.5	0.4	0.918	2.371	0.0333	0.0391	0.0058
3	0.6	0.2	0.923	3.159	0.0451	0.0534	0.0083
4	0.7	0.1	0.930	4.691	0.0528	0.0658	0.0130

The results in Table 3 are in sharp contrast to those in Table 2. They are very similar to the MP model: the difference in return is quite small. In order to increase it, we need to increase the value of the risk-aversion parameter. Changing the method of computing the expected values has resulted in drastically deteriorated performance of the model. These findings imply that the distortion of beliefs is **not** in itself sufficient to generate the differences in returns observed. The key to the explanatory success of the DB model is in the way the expected values are computed: while prices are computed using the DB matrices, the expected values are computed using the true transition matrix. This type of problem was illustrated by the example in section 2. Note that the results in Tables 2 and 3 were obtained without any change in the parameter values of the model. In an inconsistent model, one can obtain different results with the *same* parameter values.

4.4 Rational beliefs and distorted beliefs

The problems illustrated in section 4.3 arise because the DB model does not satisfy the basic consistency criterion described in section 2. Rationality of beliefs is a weak-form consistency condition that resolves this problem without the need to resort to rational expectations. In this section, the relationship between distorted and rational beliefs is examined in more detail.

Taking the parameters of the consumption process used by CLM as the description of the true data-generating process, we determine the restrictions that the rationality of beliefs imposes on the agent's transition matrices. These restrictions will be used to assess whether the transition matrices specified by CLM could satisfy the RB condition.

Let Γ_c denote the true transition matrix that is computed using CLM's parameters. The rationality conditions then require that

$$\alpha F_1 + (1-\alpha)\, F_2 = \Gamma_c.$$

Conditional probabilities are selected according to the following rule:

$$\pi_{ij} = \begin{cases} F^1_{(i,j)} & if\ s_i = 1 \\ F^2_{(i,j)} & if\ s_i = 0 \end{cases} .$$

Since π_{ij} are used to compute prices, and since CLM use Γ_c to compute expected values, the question is whether, given Γ_c, the probabilities that correspond to CLM's specification of distorted beliefs can be obtained. This will depend on the extent to which the matrices F_1 and F_2 can deviate from Γ_c.

The deviation is regulated by the value of the parameter λ. The RB condition implies a number of restrictions on the value of this parameter, such as:

$$\lambda \leq \frac{1}{\phi_1},\ \lambda \leq \frac{1}{\phi_2},$$

$$\lambda \leq \frac{1}{a_1\alpha\left(1-\phi_1\right)},\ \lambda \leq \frac{1}{\left(1-a_1\alpha\right)\left(1-\phi_1\right)}.$$

These and other restrictions follow from the requirement that the entries of matrices F_1 and F_2 satisfy the properties of transition probabilities. Which of these conditions will be binding depends on the values of the parameters of the process. Given the specification of the consumption process, with $\phi_1 = 0.978, \phi_2 = 0.516, \lambda \in \left(\phi_1, \frac{1}{\phi_1}\right)$ guarantees that the entries in the transition matrix will be non-negative.

With this information, the deviations of the transition probabilities in F_1 and F_2 from the stationary probabilities specified in Γ_c can be determined. Given the large value of ϕ_1, the admissible range is quite narrow.

The transition matrix, Γ_c, is given as follows:

$$\Gamma_c = \begin{bmatrix} 0.489 & 0.489 & 0.011 & 0.011 \\ 0.489 & 0.489 & 0.011 & 0.011 \\ 0.242 & 0.242 & 0.258 & 0.258 \\ 0.242 & 0.242 & 0.258 & 0.258 \end{bmatrix},$$

with the unconditional state probabilities $(0.47826, 0.47826, 0.02174, 0.02174)$. The unconditional probability of the high-dividend state is 0.95652, or roughly 96 per cent, which corresponds to CLM.

The rationality conditions imply the following restrictions on the unconditional probabilities:

a. Matrix F_1:

(i) $\Pr(s = h) \leq 1$, $\Pr(s = l) \geq 0$, when $\lambda = 1.02249$
(ii) $\Pr(s = h) \geq 0.92$, $\Pr(s = l) \leq 0.08$, when $\lambda = 0.978$

b. Matrix F_2:

(i) $\Pr(s = h) \geq 0.91$, $\Pr(s = l) \leq 0.08$, when $\lambda = 1.02249$
(ii) $\Pr(s = h) \leq 1$, $\Pr(s = l) \geq 0$, when $\lambda = 0.978$

These results indicate that any matrices satisfying the RB condition would imply the unconditional state probabilities in the range $(0.92, 1)$ for the h state and $(0, 0.08)$ for the l state. The unconditional state probabilities for the DB matrices are given in Table 4. The matrices that solve the equity-premium puzzle in CLM are outside of this range and therefore do not satisfy the rationality criteria.[18]

Table 4: Unconditional State Probabilities (DB matrices)

Cases	$\bar{\mathbf{p}}$	$\bar{\mathbf{q}}$	$\mathbf{Pr}(\mathbf{s} = \mathbf{h})$	$\mathbf{Pr}(\mathbf{s} = \mathbf{l})$
1	0.5	0.2	0.62	0.38
2	0.5	0.4	0.55	0.45
3	0.6	0.2	0.67	0.33
4	0.7	0.1	0.75	0.25

[18] Additional cases are given in CLM, Table 4. The unconditional probabilities for all cases are in the range $(0.2, 0.82)$ for state 1 and $(0.25, 0.75)$ for state 2.

4.5 Equity-premium puzzle with rational beliefs

In this section, we use the RB model presented in section 3.3 and report the magnitude of the equity premium under CLM's parameterization of the endowment process. The results are reported for four different combinations of values of (β, γ) .These are the values used by CLM.

The belief matrices are characterized by parameters $\lambda, \alpha,$ and a. λ regulates the deviations from the stationary matrix Γ. Setting $\lambda = 1$ means that agents use the stationary matrix in all periods. Parameter a regulates correlations of beliefs; setting it to $a = (1, 1)$ makes the beliefs uncorrelated. The value of α regulates the proportion of times the agent uses matrix F_1 in computations of expected values. Setting $\lambda = 1, a = (1, 1)$, and $\alpha = 0.5$ gives us the stationary RE model used by MP. The results of section 4.2 are applicable.

To determine whether the model can explain the puzzle, $\lambda \neq 1$ was chosen. In this case, $F_1 \neq F_2$, so that the agent will be using different matrices at different points in time. Setting $\lambda > 1$ means that the agent is more optimistic relative to the stationary measure. Setting $\alpha > 0.5$ means that the agent is optimistic more than half of the time. For this experiment, λ was set at its maximal value of 1.02249. We then searched for a combination of values of parameters (α, a) that would result in the riskless rate in the desired range of around 2.5 per cent, and computed the rate of return on risky assets and the implied equity premium. The results are given in Table 5.

The prices were computed using the probabilities selected according to (6), and the expected returns were computed using (7). As stated earlier, the rationality conditions guarantee that the stationary probabilities used to compute expected values are

Table 5: Equity Premiums in RB Model ($F_1 \neq F_2$)

Cases	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	$\boldsymbol{\alpha}$	**a**	$\boldsymbol{\rho}^b$	$\boldsymbol{\rho}^s$	**ep**
1	0.933	1.831	0.96	0.140	0.0254	0.0579	0.0325
2	0.918	2.371	0.96	0.122	0.0254	0.0513	0.0259
3	0.923	3.159	0.95	0.125	0.0254	0.0281	0.0027
4	0.930	4.691	0.93	0.076	0.0253	-0.013	-0.0383

implied by the transition matrices used as the agent's belief. This means that there is no inconsistency in the computations.

The results in Table 5 show that the agent's use of transition matrices that differ from the stationary transition matrix does make a difference in terms of the model's performance. The riskless rate is lowered to the acceptable region, and in three out of four cases the equity premium increases. Using different transition matrices over time increases the volatility of prices, and this increases the equity premium required. Another common characteristic of these cases is that, in all of them, the acceptable riskless rate is obtained by setting α close to 1. Recall that α represents the probability of using the first transition matrix. Since $\lambda > 1,$ the transition probabilities associated with F_1 are greater than the transition probabilities associated with the stationary transition matrix. The agent displays optimism relative to the stationary transition matrix. High values of α indicate that the agent is using this matrix most of the time. This is what one might expect given that, according to CLM's parameterization, the unconditional probability of the economy being in a good state is 0.956. Although the theory of rational beliefs allows the agent's transition matrix to differ from the stationary measure, the above results indicate that, given the nature of parameterization, the agent's beliefs cannot greatly differ from the stationary measure (implied by the maximum value of

$\lambda = 1.02249$), and that since the economy is most of the time in a good state, it does not seem to be reasonable to be pessimistic most of the time. The reason the model is not successful in explaining the equity-premium puzzle is simple: the parameterization of the endowment process implies that there is not enough volatility in the data! The RB condition then puts the restrictions on the set of admissible transition matrices that can be used to model the agent's belief. Since the rationality conditions require that the properties of the data, as represented by the stationary transition matrix, be reproduced, this will restrict the type of behaviour possible.

5. Conclusion

In this paper, we have investigated the problem of model consistency and the explanatory power of inconsistently specified models by examining a particular example that appeared in the literature. We believe that the problems illustrated here are more general and may apply to a variety of models in which the assumption of RE is replaced by alternatives that are more behaviourally plausible without demonstrating that the resulting model is consistent. These alternatives include, but are not limited to, various rules of thumb postulated as substitutes for learning, as well as a variety of irrational "noise trader" models.

We have interpreted Kurz's (1994) rationality-of-beliefs condition and demonstrated that it is a meaningful postulate when the assumption of rational expectations is deemed unsatisfactory. Although the representative-agent model with rational beliefs cannot generate the observed equity premium, it does provide the best-case scenario. Since the rationality of beliefs is a weak consistency condition, the model shows, *conditional on the observed data*, the best we can do in terms of explaining the puzzle within the framework of a consistently specified representative-agent model that relies on changes in beliefs. The results reported herein, in conjunction with the results surveyed in Kotcherlakota (1996), suggest that the way ahead most likely lies in consistently specified heterogeneous agent models, rather than modifications of the representative-agent model. The results of Kurz and Motolese (2001) are encouraging in this respect.

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