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by Yuliya Romanyuk

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Abstract

An objective function is a key component of a strategic portfolio management model used to determine the optimal allocations of assets and, possibly, their associated liabilities over some investment horizon. The author discusses investment philosophies and perspectives for the management of foreign reserves, and investigates how to translate the three common policy objectives for reserves (liquidity, safety, and return) into objective functions for strategic reserves management. Stochastic programming is identified as an advantageous modelling framework to capture the objectives of foreign reserves management, and a strategic reserves management model is illustrated that trades off expected net returns with costs and liquidity issues related to a potential liquidation of a portion of the portfolio.

JEL classification: G11
Bank classification: Foreign reserves management

Résumé

Une fonction objectif est une composante essentielle d'un modèle stratégique de gestion de portefeuille visant à déterminer la répartition optimale des actifs et, éventuellement, des passifs connexes sur un horizon de placement donné. L'auteure se penche sur diverses philosophies de placement et des démarches adaptées à la gestion des réserves de change, et étudie la façon de traduire les trois objectifs poursuivis traditionnellement dans la gestion stratégique des réserves de change (soit la liquidité, la sûreté et le rendement) en des fonctions objectifs facilitant celle-ci. Elle établit que la programmation stochastique est un cadre de modélisation intéressant pour saisir ces objectifs et, pour illustrer son propos, elle présente un modèle stratégique de gestion des réserves qui permet d'exercer un arbitrage entre le rendement net attendu et les coûts et problèmes de liquidité associés à la liquidation potentielle d'une partie du portefeuille.

Classification JEL : G11
Classification de la Banque : Gestion des réserves de change

1 Introduction

An interesting statistic: global reserves increased in size to over $7 trillion by the end of 2008, and over half of this amount was accumulated in the previous five years (Juckes and Simmonds, 2009). The sheer size of foreign reserves held worldwide, together with the fact that this decade has witnessed an unprecedented growth of reserves worldwide, demand that research attention be given to the topic of foreign reserves management. In particular, issues such as asset and liability management, modelling frameworks, liquidity provision, funding and investment constraints, the tremendous earnings potential, and the opportunity costs of not investing the reserves optimally should be carefully examined.

Clearly, the management framework for reserves is inextricably linked to the objectives for having a reserve fund. Countries hold reserves for different reasons, which include the following: supporting and maintaining confidence in monetary and exchange rate policies, including the capacity to intervene in support of domestic currency; limiting external vulnerability during times of crisis when access to borrowing may be curtailed; maintaining a fund for national disasters or emergencies (IMF, 2004); granting emergency liquidity assistance to sectors of the economy, particularly the banking sector; and underpinning investor confidence that a country is able to meet its foreign exchange commitments, thereby lessening the probability of financial crises (Borio et al., 2008).

As pointed out by Nugée (2000), while the rationale for holding reserves varies, their management has the following features in common: being public funds,[1] reserves must be invested prudently and safely; since the need to use reserves may arise suddenly, liquidity matters; and, due to their size, the potential for generating returns cannot be ignored. Therefore, there are three established objectives for foreign reserves management: liquidity, safety, and return. The relative importance of each depends heavily on the economic environment and the risk preference of the country (its treasury, central bank, etc.). Within a modelling framework for reserves management, the chosen objective function should quantify a trade-off between liquidity, preservation of capital value, and return, in addition to capturing the associated risks, risk tolerances, and constraints.

Managers of central bank reserves face challenges beyond those typically encountered by private sector fund managers. For example, an increased focus on transparency and accountability has contributed to the pressure of generating higher returns from foreign reserves (Bakker and van Herpt, 2007a), while at the same time reserve managers must also meet the somewhat conflicting objectives of providing liquidity and preserving the capital value of the assets in their portfolio. In addition to dealing with public scrutiny in their own country, reserve managers must meet international disclosure standards.[2] The administrative structure for the reserves may lead to full or partial separation of the asset and the liability management functions, resulting in obstacles for assessing the net risk position of the reserves and making strategic funding and/or investment decisions. Finally, various constraints on the universe of investment assets and permissible risks, as well as risk preferences that may be difficult to quantify due to political or other reasons, further complicate foreign reserves management.

Trends in reserves management are examined in a number of central banking and sovereign

[1] Different countries have different administrative arrangements for the ownership of foreign reserves; typically, central banks hold reserves on their balance sheets. In Canada, the foreign reserves are held in a separate entity – the Exchange Fund Account, which the Bank of Canada manages on behalf of the federal government.

[2] Of note, Canada was one of the first countries to fully meet the new format requirements of the International Monetary Fund and the G-10 for the presentation of international reserves data, aimed at increasing transparency for reserves management (De León, 2003).

wealth fund publications.[3] Not surprisingly, the recent global financial crisis has affected the focus and the policies for reserves management. For example, whereas earlier studies (Rigaudy, 2000; Pringle and Carver, 2003) discuss the focus on income generation, more recent ones point out the flight-to-quality movement resulting in tighter credit constraints and reduced counterparty exposure (Pringle and Carver, 2009), and discuss the re-emergence of liquidity as a primary objective for reserves management (Hansen, 2009). In terms of the level of international reserves, Juckes and Simmonds (2009) indicate the projected reversal in the rapid growth of reserves levels from 2009 onwards.

The optimal size of international reserves, the demand for reserves, and the motives for accumulating reserves have been and remain important research questions (see, for example, Frenkel and Jovanovic, 1981; Ben-Bassat and Gottlieb, 1992a,b; Bahmani-Oskooee and Brown, 2002; and Aizenman and Lee, 2007). While some studies question the accumulation of reserves for precautionary purposes (Green and Torgerson, 2007), others find that large reserves appear reasonable in view of their self-insurance role during crises (García and Soto, 2004; Aizenman, 2009) or when accounting for foreign liabilities (Wyplosz, 2007). Discussion regarding the optimal size of foreign reserves is beyond the scope of this paper: the level of the reserves is assumed to be given exogenously.

Our purpose is to build a modelling framework for the management of Canadian international reserves, held primarily in the Exchange Fund Account (EFA). In particular, the aim is to develop a strategic model that would determine the optimal allocation of assets in the EFA and their associated liabilities over a given investment horizon. The output of such a model would help guide policy decisions regarding the target allocation of reserves in the EFA for the future. Specifying an objective function is an important component of the strategic model, for which one must evaluate which goals, risks, and constraints are the most significant to be included in the objective function, and which metrics best capture these factors. This paper focuses on the specification of objective functions in academic and central banking literature, which are relevant to the management of assets and liabilities associated with foreign reserves.[4]

2 Specifying Objectives in View of Investment Philosophies and Management Perspectives

2.1 Investment philosophies: asset-only vs. asset-liability

Investment philosophies for managing reserves typically belong to one of three categories, described by Cardon and Coche (2004). The first philosophy, the individual currency approach, separates the decision regarding the allocation of overall reserves to individual currencies from the decision regarding the allocation to individual asset classes within each currency. The objective function for a subportfolio of a given currency might focus on maximizing expected returns subject to liquidity constraints and risk tolerances. To determine the optimal currency distribution, the objective function might seek to minimize specified risks subject to minimum currency allocations. The drawbacks of this investment approach include establishing overall levels of risk tolerance or required return, not accounting for the ability to bear financial risks (liabilities), and disregarding potential diversification effects from different currencies.

[3] See the studies in Cassard and Folkerts-Landau (2000); Pringle and Carver (2003); Johnson-Calari and Rietveld (2007); Bakker and van Herpt (2007b); Rietveld (2008); Pringle and Carver (2009).

[4] More general issues related to portfolio management, such as asset-liability management strategies, modelling frameworks, and commonly used risk metrics, are reviewed in an earlier paper (Romanyuk, 2010).

The second investment philosophy, the base currency approach, is also an asset-only approach that denominates reserves in some numeraire currency, such as the domestic currency, the U.S. dollar, or special drawing rights (SDRs). This view determines the optimal asset allocation within currency subportfolios and the currency distribution simultaneously; therefore, objectives need to be specified at the level of overall returns in the chosen numeraire currency. While the resulting allocations may be more extreme within subportfolios, this approach improves over the first in that diversification benefits are taken into account, the risk-return profile may be enhanced in the numeraire currency, and goals and risk tolerances have to be established only for the level of aggregate reserves, as opposed to individual subportfolios.

The third investment philosophy is to integrate the management of reserve assets of a country/central bank with its liabilities. Here the interesting question is the definition of foreign reserve liabilities itself. One may think of the cost of funding of reserves as a liability; this cost of funding may be direct foreign borrowing or swapped domestic borrowing. However, perhaps a more useful way to think about reserve liabilities is to associate them with the purpose of reserves. For instance, if reserves are used for foreign currency-denominated expenses or debt repayment, then the liability should be the expected cash flows of such expenses/debt payments. In this setting, the objective may be defined as minimizing the shortfall of reserve income with respect to the expected liability cash flows.

If reserves are held primarily as insurance – that is, the central bank/treasury would use them to intervene during some disastrous market event – then one should probably think of liability as a potential call or multiple calls on reserves. In this situation, the size and the probability of these calls should be estimated, and the objective function may penalize the value of the reserves falling below some level deemed sufficient to cover the calls. The investment of reserve assets should be conducted with regard to their purpose; unless this purpose is pure return generation, then the liabilities associated with the reserves should be accounted for. In this lies the primary advantage of asset-liability management (ALM) over asset-only investment approaches: ALM provides a comprehensive view of benefits, obligations, and risk exposures associated with the reserves.

2.2 Reserves management perspectives: macro vs. micro

There are two approaches to reserves management: macroeconomic and microeconomic (Claessens and Kreuser, 2004). The macroeconomic literature views reserves as an instrument to smooth short-run shocks in external transactions; one resulting practical rule for selecting an adequate reserves level is to have reserves that are at least equivalent to 12 months' worth of imports. Also within the macroeconomic approach, government liabilities and reserves can be examined together in a balance-sheet-type approach, where reserves may be used to support the execution of government payments, such as debt management operations; in this case, a common rule of thumb is to ensure that external debt repayments falling due in the next 12 months do not exceed foreign reserves. Benefits and examples of balance-sheet-oriented approaches to reserves management are presented in Nugée (2000), Boertje and van der Hoorn (2004), and Gray (2007).

A more microeconomic-type perspective for managing reserves, such as a greater emphasis on active management, may be feasible for countries where macroeconomic issues and financial sector vulnerabilities are of less concern (for example, countries with well-developed financial markets and flexible exchange rate policies). In this case, Claessens and Kreuser (2004) suggest that it might be appropriate to divide reserves into active and passive parts, where, as their names suggest, the former would be used for profit generation, and the latter would be held to achieve the macroeconomic objectives and managed primarily with liquidity in mind. Of note, as outlined by the Department of Finance Canada (2008), the investment policy for the Exchange Fund Account splits its investments into two tiers: the liquidity tier and the investment tier, with the former serving to meet core liquidity requirements in foreign currencies.

An example of a macroeconomic view for modelling foreign reserves is provided by Coche et al. (2006), where a central bank has a policy objective of preventing the exchange rate from falling below a certain barrier (see section 3.3 for more details). Following this policy goal, the objective function used to determine the optimal allocation of reserve assets minimizes the probability of the exchange rate falling below some target level. An illustration of the microeconomic perspective is given in Bauer et al. (2004), where the objective of a central bank is to achieve maximum and stable returns subject to liquidity restrictions and limits on market and credit risks. However, to achieve this goal, the authors find that minimizing losses within the objective function works better than maximizing profit.

Putnam (2004) also argues for the separation of a foreign reserves portfolio into two sections: liquid and liquidity-challenged. He indicates that central banks need their reserves to be liquid precisely in times when domestic markets may be facing liquidity problems, and that, in their role as a lender of last resort, the banks may have to place a substantial call on foreign reserves to support currency or borrowing needs. As such, liquidity should be the overriding principle for managing the liquid tier, with all the relevant asset class, issuer, and maturity constraints and investment objectives. The liquidity-challenged portion of the portfolio, on the other hand, could be managed using straightforward risk-return criteria with different or fewer constraints and objectives than the liquid tier. Such a division of a reserves portfolio may be preferable to using a complicated set of investment guidelines to achieve the multiple purposes for holding foreign reserves.

3 Objectives for Reserves Management: Liquidity, Safety, Return

3.1 Capturing liquidity

The recent global economic meltdown has sharply highlighted the need to shift the focus from returns to liquidity, for portfolio management in general and central bank reserves in particular. Central banks are among those re-evaluating the merits and dangers of the pursuit of returns at the expense of liquidity, as mentioned in section 1. Unfortunately, liquidity risk is more difficult to monitor and measure than, say, credit or currency risk. For example, it may be possible to estimate reasonably the sensitivity of a portfolio to movements in foreign exchange rates, but how does one evaluate the effect on the value of a portfolio of a change in liquidity?

Some indicators of liquidity may include a widening of bid-offer and/or credit spreads, a sharp rise in the correlation of certain asset classes, a reluctance of institutions to trade with each other, and a complete disappearance of quotes from brokers' screens. However, it would be quite challenging to specify precise metrics for some of the above and other symptoms of liquidity problems. Even if one manages to do so, it would be difficult to incorporate these metrics into a *strategic* reserve allocation model: liquidity issues are usually observed in real time, which does not lend itself easily to being packaged in expected terms over some specified investment horizon. Also, as pointed out by the International Monetary Fund (IMF, 2004), potential requirements for liquidity in crisis conditions, precisely when reserves are supposed to supply liquidity, are extremely difficult to anticipate.

The challenge of capturing liquidity for reserve modelling is further exacerbated by the fact that the commonly employed value-at-risk (VaR) metric and its variations fail to reveal any information regarding the liquidity of a portfolio. The first problem with VaR is that when everyone relies on it, during times of rising market volatility, the risk limits of some investors are hit, who then sell their assets at the same time, which increases market volatility and covariances, and then risk limits of more investors are hit, who then sell, and so on, creating a vicious cycle of falling asset prices, higher market volatility, and investor panic (Persaud, 2000). Another problem with VaR is that it is a statistical process that relies on past data, and so all the traditional arguments apply (whether history can predict the future, how far back to look, which data to use and how to weigh them, etc.; see Dwyer and Nugée, 2004, for more details). Paraphrasing Putnam (2004), VaR is great at providing insight about the frequency of returns falling within certain bounds (the middle part of the distribution), but bad at answering how large losses can be, should an event fall outside of the distribution (tails).

Liquidity management among central banks involves some common practices, such as tiering reserves, with a view that more-liquid tiers should be sufficient for potential calls on the reserves; imposing requirements on amounts of assets that are perceived to be more and less liquid; requiring a certain quantity of highly liquid securities that can be transacted and settled on the same day; and diversifying assets and asset classes by issuer and credit rating, so as not to be exposed to any single counterparty or asset class. Additional tools for liquidity management, made possible by market developments, are other funding options, such as repurchase agreements and swaps. But the cost of using these tools (margins/haircuts) also rises during market turmoil (IMF, 2004).

Liquidity risk management practices do not necessarily *measure* liquidity or provide answers regarding the quality of a reserves portfolio in distress. Nugée (2009), for instance, points out that liquidity is not a continuous variable such as those typically assumed in risk control, but an 'on-off' variable that is not easily adapted within traditional portfolio-allocation models. He also states that, at times of stress, market participants have found not that there was less liquidity, but that it was possible to have none at all. To aid in risk management, the author suggests that central banks should employ stress tests and simulations that do not rely on normal distributions; extreme scenario tests, which address the questions of what could cause a given loss in a portfolio value, and the probability of this occurring; and maximum drawdown analysis, where the manager examines the maximum loss of a reserves portfolio.

There is a new stream of financial literature that examines the effects of liquidity on asset prices, such as Fontaine and Garcia (2009); see also the references therein. Their paper identifies and measures the value of funding liquidity by adding a liquidity factor into an arbitrage-free term-structure model. The authors show that liquidity has a large and pervasive impact on risk premia and bond prices during a crisis and in normal times. In the context of reserves management, liquidity affects assets, liabilities, and their interactions. Going forward, we should consider incorporating liquidity directly into a reserves management model, as opposed to relying on constraints, which may or may not be sufficient to provide the desired level of liquidity in a reserves portfolio.

3.2 Capturing risk and return

One of the reasons it is difficult to capture liquidity within an objective function for portfolio management is that traditional approaches to asset allocation do not consider liquidity explicitly, focusing instead on quantifying risk and return. One of these is the mean-variance (MV) efficiency of Markowitz (1952a, 1959), and Roy (1952); it has become a classical approach for portfolio optimization and the foundation of modern portfolio theory. MV efficiency evaluates assets by their contribution to the risk/return profile of the portfolio, and relies on assets being less than perfectly correlated to reduce the overall portfolio risk.

The idea of MV efficiency is to maximize portfolio return μ_p subject to a specified level of risk, measured by portfolio variance σ_p^2 (alternatively, one can minimize σ_p^2 subject to a given μ_p). Frequently, the two metrics are combined into a single objective function and weighted by some parameter λ, which reflects the risk tolerance of the portfolio manager: maximize $f(w) = \mu_p - \frac{\lambda\sigma_p^2}{2}$, subject to $\sum_{n=1}^{N} w_n = 1$, $0 \leq w_n \leq 1$ (portfolio weights must sum to 100 per cent and short-selling is not allowed). Practical suggestions for improving MV optimization include stabilizing the optimizer with respect to inputs and constraining/adjusting the output (Black and Litterman, 1992; Michaud, 1998; Hensel and Turner, 1998; Ziemba, 2003).[5] Nyholm (2008) discusses strategic asset allocation with fixed-income portfolios in the context of MV efficiency and provides a useful guide for model implementation.

Another classical portfolio-allocation approach is the expected utility (EU) theory, developed in an economic context by Von Neumann and Morgenstern (1944) and Markowitz (1952b), where one maximizes the expected utility of wealth subject to some constraints. While risk preferences differ for each individual, and infinitely many possibilities exist for the shape of a utility function, a typical utility is increasing and concave. Some common families of utility curves U used for objective functions in asset allocation are: quadratic ($U(w) = w - \alpha w^2$), exponential ($U(w) = 1 - e^{-\beta w}$), log ($U(w) = \log(w)$), and negative power ($U(w) = \frac{w^{1-\gamma}}{1-\gamma}$), where w denotes wealth of the investor. Ziemba (2003) provides references on how to estimate an individual's utility function, and discusses the riskiness of allocations resulting from the different families of utility curves.

MV efficiency and EU theory typically rely on normality assumptions about asset returns for tractability.[6] Also, because the two approaches are equivalent in a one-period setting under quadratic utility, this particular utility function is frequently assumed to be the approximating function to the true utility of the investor. Therefore, the (multi-)normal distribution, along with an MV- or EU-type objective function, is often seen in the literature in the context of portfolio management.

3.3 Examples of objective functions of/for practitioners

Pinpointing the 'right' objective function for the joint evaluation of assets and liabilities is challenging, because the benefits, costs, and liabilities associated with the reserves are difficult to capture. Of course, there is the obvious return generation, but what about the social benefit of having large reserves as insurance during crises? And what is the opportunity cost of investing funds elsewhere? Large liquid reserves are costly during normal times, but invaluable during market turmoil; how can this effect be translated into the objective function? These and other issues related to costs and benefits of reserves are discussed by Wyplosz (2007) and Green and Torgerson (2007). Sections 2.1 and 2.2 indicate that, within the objective function, the definition of reserve liabilities should be specific to each country and dependent on the use of reserves.

In some industries, it is arguably easier to define the objective function, because the assets and the liabilities are easier to specify. For example, in the pension fund industry, the

[5] Additional developments in asset allocation, with relevant references, are given in Romanyuk (2010).

[6] Expected returns matter most within an MV-type framework, since errors in expected returns are about ten times more important than errors in variances, and the latter are about twice as important as errors in covariances (Chopra and Ziemba, 1998).

assets are the contributions collected from individuals, and the liabilities are cash outflows to retired members or their spouses. In this context, actuaries traditionally prepare projected cash outflows and inflows, and then investment managers allocate the funds accordingly. Some examples of objective functions (Martellini and Milhau, 2009) are:

$$\max E\left[u\left(A_T - L_T\right)\right] , \tag{1}$$
$$\max E\left[u\left(\frac{A_t}{L_t}\right)\right] ,$$

where A_t and L_t denote the assets and the liabilities indexed by time. The authors evaluate related costs and constraints, such as regulation, funding ratio requirements, and max/min constraints.

The insurance industry is another example where assets and liabilities are relatively straightforward to define conceptually (although this does not mean that they are easily quantified). Traditionally, the investment of assets and the evaluation of liabilities functions have been performed separately by investment officers and actuaries, respectively. In the past two decades, there has been work done to promote joint evaluation of assets and liabilities. Tilley (1980) presents a sample asset-liability framework where the objective function (very simplified) is roughly defined as:

$$A = \sum_t \gamma \left[CF_t^{\text{in}} - CF_t^{\text{out}}\right] , \tag{2}$$

where CF denotes cash flows (in/out), with cash flows in being the premiums collected from participants, and cash flows out the projected liability cash flows, depending on the function of the insurance firm (life, casualty, property).

For central banks, the management perspective comes into play when determining the objective functions. For example, central banks that care about risk-return efficiency in local or foreign currency often establish their strategic asset allocation using mean-variance analysis (Cardon and Coche, 2004), facing a reduction on potential returns because of policy constraints, which may be mitigated to some extent by active management. The objective function is a variation of the one given in section 3.2, but since it captures only assets, liabilities may be added by examining excess returns over the liabilities and adding a shortfall constraint. MV-type objective functions reflect the microeconomic perspective of managing reserves (section 2.2).

From a macroeconomic perspective, Coche et al. (2006) examine reserves allocation following the objective of the exchange rate not falling below a certain level:

$$\min \sum_t \delta^t \text{Prob}\left(e_t < \rho P\right) , \tag{3}$$

where e is the exchange rate, δ a time preference parameter, and ρ calibrates some (assumed) time-invariant level of exchange rate P. One valuable aspect of Coche et al. (2006) is that the optimal asset allocation is examined in both single- and multi-period settings, since the authors find that, with one period only, the optimal allocation affects the exchange rate only marginally.

Another example of a macroeconomic perspective for managing reserves is Alfaro and Kanczuk (2007), where foreign debt and reserves are viewed as a means to smooth consumption. Here, consumption is constrained by the level of external debt and reserves, and the sovereign can choose to default on debt (and, in that case, the consumption in the economy is constrained by the size of the reserves only). The objective function of the sovereign is the expected utility of consumption, c_t:

$$\begin{aligned} U &= E\left[\sum_t \beta^t u\left(c_t\right)\right] , \quad (4) \\ u(c) &= \frac{c^{1-\sigma}-1}{1-\sigma} , \end{aligned}$$

where σ measures the curvature of utility and β is the discount factor. The sovereign has to weigh the decision to default against having a bad credit history in a stochastic dynamic game. Interestingly, the authors find that, in their setting, with reserves used to smooth consumption, it is not optimal to hold reserves at all.

Yet another example of a macroeconomic perspective for managing reserves is evaluating the net position of a central bank in a balance sheet approach, where the objective function is (Boertje and van der Hoorn, 2004):

$$\max E\left[P\right] , \quad (5)$$

$$\text{subject to } \alpha PL \leq R \text{ and other constraints,}$$

$$\text{where } PL = \text{VaR} - E\left[P\right] . \quad (6)$$

Here, $E\left[P\right]$ is the expected profit of a central bank (a function of its assets – such as gold, foreign assets, lending, and other assets – and liabilities, such as bank notes and reserves); PL is the potential loss, defined using the familiar VaR and expected profit; α determines the proportion of the potential losses; and R is the bank's risk tolerance. In this setting, though, due to normality assumptions, the problem turns into a familiar MV-efficient allocation, where the central bank maximizes a linear function of decision variables subject to a quadratic constraint.

Bauer et al. (2004) examine several objective functions for managing reserves in the context of the Czech Republic, and with two foreign currencies only: the U.S. dollar and the euro. They consider minimizing VaR (using normality assumptions for returns), minimizing the volatility of anticipated returns, and minimizing the exchange rate risk should the reserves be used to repay some liabilities (they discuss several possible scenarios necessitating such a use of reserves). The authors find that the first two objectives (minimize VaR or minimize volatility) give similar results, but when liabilities are considered, results vary, based on different scenarios.

A summary of the types of objective functions and constraints that may be implemented by reserve managers in the stochastic programming modelling framework is provided by Claessens and Kreuser (2004). Illustrating a variety of reserves management perspectives and ways to translate the policy objective for the reserves into an objective function and constraints, they discuss incorporating the ratio of reserves to short-term debt, total debt less reserves to some scaling variable (such as exports), and improving returns on reserves.

3.4 Lessons learned so far: traditional approaches lacking

As can be seen from the foregoing examples of objective functions considered by practitioners, central bank portfolio managers have a difficult time getting away from the traditional mean-variance and expected utility approaches, whether in a single- or a multi-period setting, and usually with the underlying assumptions of normality. For normal market times, such approaches may be acceptable, especially if the probability of extreme outcomes has been minimized. This is particularly true of central bank reserves portfolios, in which the risk of extreme outcomes is mitigated by investment constraints on the investment universe. Putnam (2004), for example, points out that the commonly used VaR may perform reasonably well when the outcomes are constrained to be in the middle of the distribution. This is somewhat reassuring, since many central banks, including the Bank of Canada and the Danmarks Nationalbank, rely on VaR to measure market and other risks.[7]

However, in turbulent market environments and during crises, the diversification paradigm for risk fails. Suddenly, there are event-driven risks that are not accounted for, such as liquidity, credit, and behavioural, that push the situation into extreme scenarios. For example, institutions liquidate large portions of their portfolios, causing asset prices to drop or trading to cease altogether; affected institutions' credit spreads rise sharply, limiting their access to funding and further affecting their creditworthiness; demand for quality assets changes correlation patterns between stocks and bonds; and herding behaviour/flight to quality introduce behavioural elements that are not accounted for within the context of diversification.

The direct consequence for reserves management is that the diversification paradigm is too limiting to capture the risks that become relevant in bad times. As a result, approaches and risk measures that are based on normality assumptions and average scenarios/expectations (such as MV- and EU-type objective functions and VaR-related metrics discussed in section 3.1) do not seem to be adequate for adopting within a strategic modelling framework for reserves management *moving forward*. Of course, one possibility is to improve and enhance these approaches, but perhaps a better solution is to recognize that a different framework is needed to develop an asset(-liability) reserves management model, which accounts for event risk (such as stock market crashes or liquidity crises) and explicitly incorporates low-probability, high-impact scenarios in the determination of optimal portfolio weights. The next section addresses this issue.

3.5 Formulating objectives within a suitable modelling framework

In general, models for asset(-liability) management can be static or stochastic, and single- or multi-period. Of the possible model types, stochastic multi-period (dynamic) models are arguably the best for portfolio management under uncertainty. While static models have their use in stress/scenario testing, they are too simplistic to provide insight into allocation decisions with only one view of the future. And single-period models are too limiting, in practice, for a variety of reasons.[8] Because medium- and long-term outcomes of reserve performance depend on decisions taken in the near future, and changing regimes necessitate that decisions be made dependent on the future states of the world, realistic portfolio models should be stochastic and dynamic (Claessens and Kreuser, 2007).

Therefore, to make our strategic asset-liability management model for the Exchange Fund Account useful, we should formulate an objective function in the context of a multiperiod framework that incorporates uncertainty. While the task of building a dynamic stochastic model may sound daunting, the process need not be overly difficult. For example, a number of models implemented in practice use only two to four periods and reduce the state space by examining a few meaningful scenarios. The important point is to allow the portfolio manager to react to the realization of uncertainty and adjust the portfolio accordingly, and to ensure that outcomes that matter for the purpose of holding reserves are evaluated by the optimizer.

Among dynamic stochastic models, we can identify four broad modelling frameworks:

[7] See Department of Finance Canada (2008) and Danmarks Nationalbank (2004).

[8] For example, they do not capture long-term investment goals; do not allow for future investment decisions once uncertainty has been realized; tend to produce high portfolio turnovers; and misrepresent transaction costs, since each period is treated without consideration for new and existing assets (Mulvey and Vladimirou, 1989).

stochastic optimal control, decision rules, simulation, and stochastic programming. These approaches are summarized below; they are examined in more detail in Romanyuk (2010), with relevant references for theory and applications provided.

- Stochastic optimal control is a popular approach to dynamic-portfolio allocation problems in academia, but, generally, it is too stylized and insufficiently flexible to capture the dimensionality and complexity of reserves management problems.[9]
- Decision rules are simple to communicate and useful when investment objectives are clearly defined and/or guidelines are strict.[10] However, this approach may overlook other factors critical to reserves management, such as liquidity risk or potential for return generation.[11]
- Simulation is widely used by practitioners, and frequently adapted within other modelling frameworks.[12] However, sampling the entire state space becomes difficult in higher dimensions, which is of particular concern if one wants to analyze low-probability, high-impact scenarios. Also, there are challenges related to aggregating scenarios and averaging in a consistent manner.
- Stochastic programming is quite flexible in the specification of goals, constraints, and penalty/transaction costs. It is intuitive in representing uncertainty (future states of the world can be summarized in several scenarios, such as 'normal,' 'volatile,' and 'crisis'), and it is computationally tractable.[13] One of the challenges of this approach is translating the values from the factors driving model variables (asset prices, interest/exchange rates, etc.) into discrete outcomes. Another is that a stochastic programming model is highly specialized to the problem at hand; as such, it may seem somewhat ad-hoc, and traditional portfolio theory concepts such as utility may not be applicable. Nevertheless, it appears to be ideally suited for complex reserves management problems, precisely because the model can be tailor-made.

The objective functions of the types identified in section 3.3 can be adapted in any of the above modelling frameworks, although utility maximization is frequently presented as stochastic optimal control problem, and mean-variance analysis often relies on resampling and simulation for single and multiple periods. Objective functions in the stochastic programming framework are usually of the type summarized by Claessens and Kreuser (2004) (see section 3.3).

4 A Sample Model for the Exchange Fund Account

Foreign reserves in Canada are held primarily as insurance for a rainy day; that is, the assets in the Exchange Fund Account may be used for intervention purposes in times of market turmoil or

[9] If the problem can be solved analytically, then it probably lacks realism, and when a numerical solution is needed, it is difficult to sample the entire state space, because uncertainty grows exponentially in the number of state variables, and approximation errors arise.

[10] Black and Telmer (2000) provide an example in the context of debt issuance, while Coche et al. (2006) examine reserves management subject to a policy objective of exchange rates staying above a certain level.

[11] In addition, computational issues may arise due to non-linearities, and a global search algorithm may be needed for optimization.

[12] At the Bank of Canada, simulation is used in the modelling of domestic debt management (see Bolder, 2003), and Coche et al. (2006) use simulation for reserves management with a given objective.

[13] The size of the problem is polynomial in the number of branches.

crises. In such circumstances, the inclination may be to sell the most-liquid assets first; within an objective function, this may be quantified by minimizing the transaction (liquidation) costs should the call on reserves occur. However, Nugée (2009) argues that, while such a decision may be instinctive, it may also be the worst response, leaving a portfolio of less-liquid (or illiquid) assets. He indicates that losses at which the less-liquid assets would be sold off may be compensated by gains on liquid assets going into a liquidity storm. Discussing the hedging of tail risk by central banks, Reveiz (2009) also favours the idea of managing reserves to maximize the value of the portfolio when a shock arrives over minimizing the impact of a shock on the value of the portfolio.

Hence the objective function of a strategic model for the EFA should capture *both* the provision of liquidity *and* the preservation of value in the remaining portfolio. The model should explicitly incorporate volatile and crisis scenarios, particularly because intervention would most likely be needed in times of crisis. Also, it should be flexible enough to consider returns. Successful applications of stochastic programming in insurance (Cariño et al., 1994, and developments), pension funds (Hilli et al., 2007), and central bank reserves management (Claessens, Kreuser, and Wets, 2000; Bhattacharya, Kreuser, and Sivakumar, 2009) give confidence that our own asset-liability model for reserves management can be developed within this framework. In particular, they give confidence that the complex objectives of the EFA – providing liquidity, preserving capital value, and generating a return – can be quantified and evaluated.

4.1 A brief review of stochastic programming

The literature on programming models for asset-liability management in the banking industry dates back to the 1960s, and it has been developing steadily since.[14] Most recently, stochastic programming models have been adapted in the context of foreign reserves management and sovereign wealth funds (Kreuser, 2002; Claessens and Kreuser, 2009). The steps in developing a stochastic programming model include[15] the generation of sparse trees of stochastic variables, and the formulation and solution of a dynamic stochastic optimization model.

The generation of sparse trees of stochastic variables is done to match historical information, satisfy expert opinion, fit values implied by derivatives, agree with theoretical pricing models, or a combination of these aims. The universe of all possible outcomes is represented by a branching tree, where nodes are events with future states branching out of them. A typical approach involves formulating stochastic differential equations for driving factors, estimating their scenario-dependent parameters, translating these into discrete scenarios, and assigning probabilities to the scenarios in the tree. Figure 1 shows sample scenarios over a three-stage (two-period) tree with a 5-3 node structure.

Two observations should be made here. First, as with any pricing model, the driving factors should produce forecasts that are free of arbitrage and consistent with derivative-implied values where applicable; these features should be preserved when discrete scenarios in the tree are formed. The modelling structure should also be consistent: decisions taken at any given node should be based on the current information at that node independent of future paths (non-anticipativity constraints must be added). Second, determining regime transition probabilities

[14] See, for example, Chambers and Charnes (1961), Cohen and Hammer (1967), Booth (1972), Brodt (1978), Fielitz and Loeffler (1979), Booth and Koveos (1986), Kusy and Ziemba (1986), Giokas and Vassiloglou (1991), and Cariño et al. (1994) and references therein.

[15] In addition to previous references on stochastic programming, please see Birge and Louveaux (1997), Mulvey (2001), Kreuser (2002), Dupačová, Hurt, and Štepán (2002), Ziemba (2003), and Wallace and Ziemba (2005).

is a challenging task; they may change depending on the length of periods and the time horizon covered by the model. While some researchers assume that the probabilities of ending up in a particular regime are independent of the previous regime (Geyer and Ziemba, 2008), this does not have to be the case as long as reliable regime transition probabilities are available to the modeller.

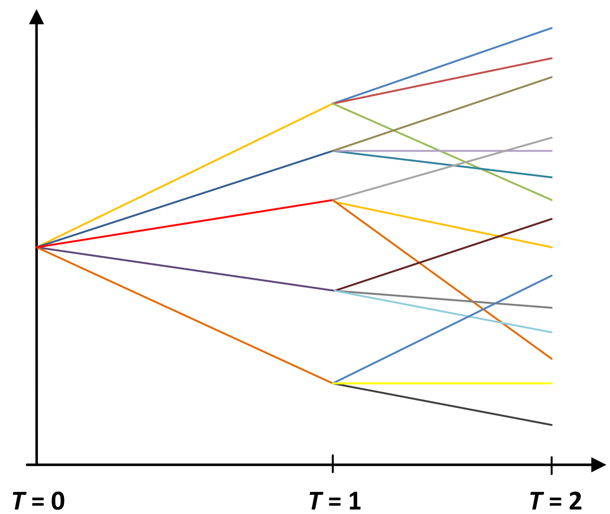

Figure 1: An example of a 5-3 scenario tree for stochastic programming.

The second step involves building a model to derive the decision variables. Times at which decisions have to be made help set the number of stages in the scenario tree; this number does not necessarily correspond to the number of periods which the model covers. For example, one may be analyzing a two-year investment horizon, with an initial stage (at time zero), a second stage in a half-year, a third stage in another half-year, and a fourth, final stage at the end of year two. Then, decision variables are defined for each event regarding the levels of assets, liabilities, currencies, etc. (the decision variables for future events depend on the realization of uncertainty). The model is then optimized with respect to the indicated objective function and constraints.

Sometimes, it is preferable to have densities – as opposed to discrete points – for the outcomes of the value function, in which case the model can be set up to optimize with respect to the shape of the density function of the variable of interest. Wets (1999) provides techniques to translate discrete distributions resulting from stochastic models into density functions, and Claessens and Kreuser (2007) discuss optimization in the context of preferences of density functions.

4.2 Sample model set-up

This section illustrates how EFA assets might be allocated optimally in view of future uncertainty and the associated liabilities. A caveat must be made at this point: in practice, additional considerations would apply, due to the asset-liability matching framework governing the EFA. For our simplified modelling exercise, we suppose that, at time zero, all EFA assets have to be allocated to the available asset classes, and that no call on reserves arrives at time zero. At the end of the investment horizon, however, depending on the economic regime, there may be a call on reserves, with probability depending on the economic regime. The objective function trades off expected returns, funding costs, liquidation costs in case of a call on reserves, and so-called liquidity factors, which are meant to capture the preference of the portfolio manager for a given asset class in each economic regime.

4.2.1 *Model assumptions*

The general assumptions and key ideas of the model are summarized below.

- The investment horizon is one year. There are three possible outcomes for economic regimes in one year: *normal*, *volatile*, and *crisis*. There are no intertemporal decisions between time zero and the end of the investment horizon. This assumption serves only to simplify the exposition of the problem statement; incorporating decisions over several periods, as proposed in stochastic programming models for reserves management in other central banks (see Bhattacharya, Kreuser, and Sivakumar, 2009; Claessens, Kreuser, and Wets, 2000), is possible and will be addressed in future research.

- The 'benefits' of the fund are captured by the expected portfolio return in one year. The random return outcomes are generated using the estimates of expected returns and regime-dependent covariance matrices for four asset classes (European stocks, U.S. stocks, European bonds, and U.S. bonds), following Geyer and Ziemba (2008). Because the estimates are provided in a single numeraire currency, the euro, the portfolio allocation is conducted across the four asset classes (and two currencies), as opposed to separate allocation decisions for U.S.-dollar and euro portfolios.

- The liabilities associated with the fund are twofold. First, there are the funding costs that are assumed to be known today.[16] Second, since the EFA is a type of insurance for a rainy day, a potential claim on this insurance – the possibility of a call arriving at the end of the investment horizon – is quantified. This is the 'low-probability, high-impact' event that is accounted for.

- As previously discussed, liquidity is very challenging to capture in portfolio-allocation problems, and models are insufficient to assess the true exposure to liquidity risk (see section 3.1). Central banks follow a variety of practices to mitigate liquidity risk; in particular, the United Kingdom employs an asset-allocation model that explicitly trades off liquidity and return (IMF, 2004). Here, liquidity is measured by two components: bid-ask spreads (liquidation costs) and *liquidity factors*. Bid-ask spread estimates during different economic regimes are meant to be a more objective market measure of liquidity issues, whereas liquidity factors are used as a subjective risk metric indicating the preference of the portfolio manager for the asset class in consideration.[17]

- The objective function trades off expected excess returns (returns net of funding costs), liquidation costs, and the liquidity value of the remaining portfolio. The idea is to maximize expected excess returns, and – should a call on reserves arrive – minimize the

[16] This is not unrealistic; if the liabilities are issued directly in foreign funds, their coupons are known, and if the liabilities are in Canadian funds, the payments of cross-currency swaps would be known as well. Of course, to account for already-issued liabilities and the fact that valuation would occur at points other than maturity, we would have to incorporate uncertainty in funding costs, but for this simplified example we assume that these are known.

[17] Alternatively, or in addition to the method proposed here, liquidity could be incorporated into the model directly, as in Fontaine and Garcia (2009).

liquidation costs of the portion being sold while ensuring that the remaining portion remains liquid.

- The decision regarding asset allocation occurs at time zero in view of uncertain outcomes in one year. At the end of the year, the portfolio manager either does nothing or liquidates a portion of assets, depending on the economic regime and the realized state of the world.[18]

4.2.2 *Some specifics*

The objective function for the problem is the following:

$$\max_{x,y} E_\phi \Big[\alpha \left(\mu(\phi) - fc \right)' x - \left[\beta lc' y(\phi) + \gamma lf' \left(x - y(\phi) \right) \right] I_{\{\text{call}\}} \Big] , \tag{7}$$

where E_ϕ denotes the expectation with respect to uncertainty (summed up by ϕ) at investment horizon T ($T = 1$ year in this example); fc, lc, and lf denote funding costs, liquidation costs, and liquidity factors ($lf \in [0, 1]$, with lower values denoting the more desirable asset class in terms of liquidity); α, β, and γ (all $\in [0, \infty)$) indicate the relative importance of each objective (maximizing return vs. minimizing liquidation costs vs. being left with a liquid portfolio); and the control vector x gives proportions for initial asset allocation, and $y(\phi)$ is the (random) control vector with proportions of assets to be sold in case of a call on reserves at T (indicated by $I_{\{\text{call}\}}$).

Optimization is subject to the following constraints:

$$\begin{aligned} x_1 + \cdots + x_N \quad &= \quad R \,, \\ y_1(\phi) + \cdots + y_N(\phi) \quad &= \quad dx'(1 + \mu(\phi) - fc) \,, \\ 0 \le lb_{x_i} < \quad x_i \quad &< ub_{x_i} \le R \,, \\ 0 \le lb_{y_i} < \quad y_i(\phi) \quad &< ub_{y_i} \le x_i(1 + \mu_i(\phi) - fc_i) \,. \end{aligned} \tag{8}$$

There are N asset classes in the portfolio; the initial reserve value is R; d is a regime-dependent drawdown parameter indicating the proportion of reserves to be liquidated should a call on reserves arrive. We assume that, at the investment horizon, each asset class i will earn the (random) excess return $(\mu_i(\phi) - fc_i)$, and that the dollar amount allocated to asset class i, x_i, will increase in value to $x_i(1 + \mu_i(\phi) - fc_i)$. The constants lb_x, lb_y and ub_x, ub_y are lower and upper bounds, respectively, for x and y that may be set by the portfolio manager.

In terms of future uncertainty, only the returns that will be generated for each node in the model are truly stochastic (recall that uncertainty is captured by a branching tree, with nodes representing realized outcomes), since they will be based on random draws and regime-dependent covariance matrices, as described below. For the other inputs, the funding costs are assumed to be static, while the liquidation costs (bid-ask spreads), the liquidity factors, the probabilities of a call on reserves at T, and the size of a call (determined by parameter d), are all regime- but not scenario-dependent. The values of all parameters are given in the next section.

We start at time zero (initial stage), and after a period of one year (the final stage is the end of this year), we suppose that S scenarios representing future outcomes are possible (these will be nodes in the tree). The outcomes are from the normal, volatile, or crisis regime with

[18] This is known as stochastic programming with recourse; see, for example, Birge and Louveaux (1997) for an introduction.

probability p_n, p_v, or p_c, respectively. The probability estimates may be obtained in a number of ways. For example, they can result from the output of an economic model; they can be established using expert opinion or economic surveys; in scenario testing, they can be specified according to the values of interest. Given the regime (normal, volatile, or crisis), a call on reserves may arrive with probability $p_{\text{call}|n}$, $p_{\text{call}|v}$, or $p_{\text{call}|c}$, respectively. These probabilities can also be obtained in various ways: another model, historical analysis, expert judgment, a combination of these, etc.

Thus, each of the S nodes in the final stage represents a random outcome from one of six distinct states of the world with its respective probability: $p_n^{\text{call}} = p_n p_{\text{call}|n}$ (normal, call), $p_n^{\text{no call}} = p_n(1 - p_{\text{call}|n})$ (normal, no call), and so on ((volatile, call), (volatile, no call), (crisis, call), (crisis, no call), with probabilities defined similarly). Clearly, the number of nodes assigned[19] to each state reflects its likelihood; for example, a (normal, call) state takes $S_n^{\text{call}} = Sp_n^{\text{call}}$ nodes.

The return scenario for each node is generated following the specifications in Ziemba (2003) and Geyer and Ziemba (2008), who provide estimates of the regime-dependent covariance matrices Σ_n, Σ_v, and Σ_c, for asset-class returns in the normal, volatile, and crisis regimes.[20] The first step in scenario generation is to draw S standardized (mean zero, variance one) random variables. In our setting, we can use one of the four possible distributions: normal, Student t (scaled or skewed[21]), or extreme value, as shown in Figure 2; Geyer and Ziemba (2008) use normal, Student t, or historical draws. The four distributions above should suffice for illustrative purposes, since they allow for fatter tails and skewness typical of asset returns.

Once S standardized random variables are drawn, the second step is to transform them into correlated variables using the estimates of expected returns and covariance matrices Σ_n, Σ_v, or Σ_c. This is done by multiplying the random draws by the Cholesky decomposition of the respective covariance matrix. Then the expected-returns estimate is added to produce a realized outcome of asset returns for a node in consideration. Out of S random draws, $S_n = Sp_n$ are transformed using the Σ_n covariance matrix, and so on. Furthermore, out of, say, S_n nodes, only $S_n^{\text{call}} = Sp_n^{\text{call}}$ have liquidation calls arriving in a normal regime, so liquidation occurs in a total of $S_n^{\text{call}} + S_v^{\text{call}} + S_c^{\text{call}}$ outcomes only.

At this point, we will have generated S nodes to represent random return outcomes in a year's time. In other words, we have captured the uncertainty using a branching tree structure with an initial stage (time zero, a single node), an investment period (one year), and a final stage (the end of one year, S nodes), where decisions are taken at the initial and the final stages, depending on the outcome. In a multi-period setting, scenario generation would be repeated after each period until the final stage. Next, we have to solve a deterministic control problem over S scenarios. Using the notation $\phi_n^{\text{call}}, \ldots, \phi_c^{\text{no call}}$ for states (normal, call), ...,

[19] Note that the allocation is random across S nodes (Geyer and Ziemba, 2008); this does not matter for a one-period two-stage model, as in our example, but it does matter in multi-period settings, and is meant to reflect the uncertainty of future states of the world.

[20] The model described in the cited papers is a stochastic programming model for an Austrian pension fund. We take the same asset classes for our example, so that we may use the estimates of expected returns, probabilities for different economic regimes, and their respective correlations and variances. Of course, the assets in the EFA are different than those of a pension fund, but this example merely illustrates a potential set-up for an EFA model, and, as such, the particulars do not matter at this point.

[21] We set the degrees-of-freedom parameter to a low value, $df = 10$, to better capture fat tails evident in asset returns.

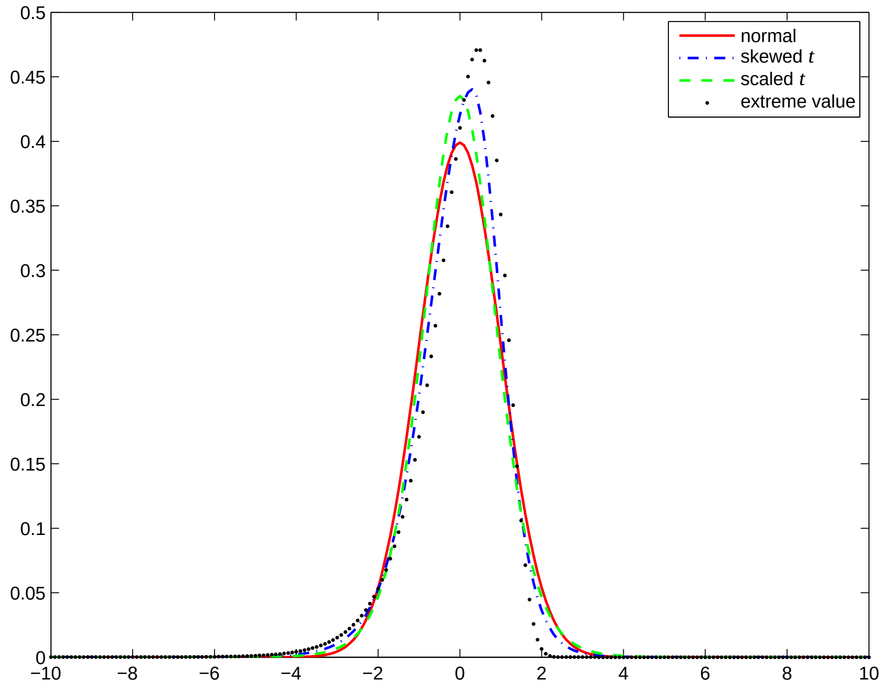

Figure 2: Four distributions (each with mean zero, variance one) used to generate random draws for scenarios in the model.

(crisis, no call), we can rewrite our objective function (7) in its deterministic equivalent form:

$$\max_{x,y}\Bigg[\sum_{s=1}^{S_n^{\text{no call}}}\alpha\big(\mu(\phi_{n_s}^{\text{no call}})-fc\big)'x+\sum_{s=1}^{S_v^{\text{no call}}}\alpha\big(\mu(\phi_{v_s}^{\text{no call}})-fc\big)'x+\sum_{s=1}^{S_c^{\text{no call}}}\alpha\big(\mu(\phi_{c_s}^{\text{no call}})-fc\big)'x$$
$$+\sum_{s=1}^{S_n^{\text{call}}}\Big[\alpha\big(\mu(\phi_{n_s}^{\text{call}})-fc\big)'x-\beta lc_n'y(\phi_{n_s}^{\text{call}})-\gamma lf_n'\big(x-y(\phi_{n_s}^{\text{call}})\big)\Big]$$
$$+\sum_{s=1}^{S_v^{\text{call}}}\Big[\alpha\big(\mu(\phi_{v_s}^{\text{call}})-fc\big)'x-\beta lc_v'y(\phi_{v_s}^{\text{call}})-\gamma lf_v'\big(x-y(\phi_{v_s}^{\text{call}})\big)\Big]$$
$$+\sum_{s=1}^{S_c^{\text{call}}}\Big[\alpha\big(\mu(\phi_{c_s}^{\text{call}})-fc\big)'x-\beta lc_c'y(\phi_{c_s}^{\text{call}})-\gamma lf_c'\big(x-y(\phi_{c_s}^{\text{call}})\big)\Big]\Bigg]/S\,. \tag{9}$$

Recall that liquidation costs (bid-ask spreads) and liquidity factors are regime-dependent. The constraints (8) can be rewritten in a similar manner, with regime-dependent drawdown proportions in case of a call, $d=[d_n,d_v,d_c]'$, and returns and controls corresponding to each of the six possible states.

The dimension of the control vector is $N+N(S_n^{\text{call}}+S_v^{\text{call}}+S_c^{\text{call}})$: N initial allocations x_i for the four asset classes in consideration (European stocks, U.S. stocks, European bonds, and U.S. bonds), and the remaining allocations for amounts y_i to be liquidated should a call on reserves arrive during a given economic regime. The above optimization problem was solved using MATLAB with $S=300$ scenarios for distribution comparison, and $S=100$ scenarios for the remaining analysis.

Table 1: Input estimates for different economic periods.

Economic period	Normal	Volatile	Crisis
Probability	$p_n = 0.7$	$p_v = 0.2$	$p_c = 0.1$
Drawdown proportion	$d_n = 0.1$	$d_v = 0.3$	$d_c = 0.7$
Conditional probability of call	$p_{\text{call}\|n} = 0.2$	$p_{\text{call}\|v} = 0.5$	$p_{\text{call}\|c} = 0.8$

4.3 Results and discussion

Suppose that a European-based[22] central bank has a foreign reserve fund of $R = 100$ million euros. Suppose that the reserve manager's name is John, and John has to make a decision regarding the allocation of his portfolio today, so that his objective function (7) is maximized over his investment horizon (one year, in this example). The flexible specification of the objective function allows John to focus on optimizing particular elements that matter to him; for example, maximizing expected returns net of funding costs, or minimizing liquidation costs in case of a call on reserves.

Due to the nature of the modelling framework (uncertainty represented by a branching tree), in addition to the usual concerns about the effects of return/covariance/cost estimates on modelling outcomes, our model results are sensitive to the estimates of probabilities of future economic regimes and liquidation likelihoods under each regime. In view of this, the robustness of investment and liquidation allocations resulting from this optimization exercise should be carefully investigated.

4.3.1 *Model inputs*

John faces the following upper and lower bounds, assumed to be binding, for each asset class i, $i = 1, \ldots, 4$:

$$\begin{aligned} lb_{x_i} &= 0.1R\,, \\ ub_{x_i} &= 0.5R\,, \\ lb_{y_i} &= 0\,, \\ ub_{y_i} &= x_i(1 + \mu_i(\phi) - fc_i)\,. \end{aligned} \tag{10}$$

As previously mentioned, the four asset classes are: European stocks, U.S. stocks, European bonds, and U.S. bonds. In addition to the values for the probabilities of different economic periods, mean annual returns, standard deviations, and correlations from Geyer and Ziemba (2008), John's expert team of researchers has provided the remaining estimates of model inputs; all are summarized in Tables 1 and 2.

The idea behind the parameters in Table 1 is that crisis times are least likely (only 10 per cent of the time), but, during crises, the likelihood of a call on reserves is highest ($p_{\text{call}|c} = 0.8$), as is the required proportion of reserves to liquidate ($d_c = 0.7$). These and all parameters should be carefully estimated or, if there are impediments such as lack of data, set/calibrated according to expert opinion.

[22] Because we use the specifications of the Geyer and Ziemba (2008) stochastic programming model for a European-based pension fund to generate random asset-class returns, all parameters used in the model, as well as all results, should be viewed from the perspective of a European investor.

Table 2: Input estimates for different asset classes.

Asset class	European stocks	U.S. stocks	European bonds	U.S. bonds
Funding cost	0.085	0.088	0.055	0.060
Mean annual return	0.106	0.107	0.065	0.072
Normal periods (70% of the time)				
Liquidation cost	0.0007	0.0006	0.0012	0.001
Liquidity factor	0.2	0.1	0.3	0.4
U.S. stocks correlation	0.755			
European bonds correlation	0.334	0.286		
U.S. bonds correlation	0.514	0.780	0.333	
Standard deviation	0.146	0.173	0.033	0.109
Volatile periods (20% of the time)				
Liquidation cost	0.012	0.014	0.005	0.003
Liquidity factor	0.4	0.5	0.2	0.3
U.S. stocks correlation	0.786			
European bonds correlation	0.171	0.100		
U.S. bonds correlation	0.435	0.715	0.159	
Standard deviation	0.192	0.211	0.041	0.124
Crisis periods (10% of the time)				
Liquidation cost	0.035	0.042	0.015	0.010
Liquidity factor	0.8	0.9	0.1	0.1
U.S. stocks correlation	0.832			
European bonds correlation	-0.075	-0.182		
U.S. bonds correlation	0.315	0.618	-0.104	
Standard deviation	0.217	0.271	0.044	0.129

Drawing scenarios using the correlations and the standard deviations reported in Table 2 allows us to capture the observed tendencies of stock correlations to increase, and of stock/bond correlations to change signs, during times of great market turbulence. The liquidation cost parameters are intended to convey that, during crises, stocks may become very expensive to liquidate. Also, low liquidation costs for U.S. bonds reflect the typical flight-to-quality behaviour of market participants during extreme market uncertainty. The funding cost values show that John has a comparative advantage purchasing euro-denominated assets, which should be reasonable given that he is a European-based investor.

Finally, low liquidity factors during normal times and high liquidity factors during crisis times for stocks show that John wants to own these high-yielding assets when things are going well, but that he wants to avoid these potentially difficult-to-liquidate assets during times of greater market volatility. Bonds, on the other hand, have higher liquidity factors during normal times (bonds are not as desirable when liquidation is not very likely and potential for profit generation is great), and very low liquidity factors during crisis times (flight-to-quality, reputation considerations, etc.). Note that, here, high liquidation factors serve as a *penalty function* for holding assets deemed bad during times of market turbulence. Conversely, investors are *rewarded* for holding assets with low liquidity factors. For practical implementations, liquidity factors could be estimated using a combination of credit ratings and expert opinion.

4.3.2 *Comparing distributions*

To compare investment strategies based on scenarios generated with different distributions (Figure 2), let us focus on returns net of funding costs and ignore, for now, liquidation costs and liquidity factors. This implies that we set $\alpha = 1$ and $\beta = \gamma = 0$ in the objective function (7), and the recourse strategy is zero (all control variables $y = 0$). In other words, John allocates his portfolio based on expected net returns only.[23] We use 300 nodes in the scenario tree. Figure 3 shows the results.

We observe that the sample scenarios generated by the four distributions preserve the same general shape of returns (U.S. stocks highest, European bonds lowest) as the mean returns used to construct these random scenarios. In this particular sample, normal and scaled t distributions produce lower-than-mean stock returns, while extreme value and skewed t values are higher than the mean. When we subtract the funding costs, we see that the attractiveness of the four asset classes in consideration changes significantly: European stocks are expected to be the best in terms of excess returns, but scaled t and skewed t scenarios produce different results.

Since, in this illustration, only excess returns matter for initial asset allocation, John's optimal investment strategies make sense. The greatest proportion is allocated to highest-yield assets (European stocks based on mean returns, and normal and extreme value distributions; U.S. stocks and U.S. bonds using skewed t and scaled t distributions, respectively), while investment in lowest-yielding assets is forced by lower-bound constraints.

4.3.3 *Optimizing factors of interest*

In this section, we examine how different preferences of the investor affect asset-allocation decisions. To do this, we look at three cases:

[23] It is relatively common to make portfolio decisions based on expected excess returns. For this particular set of results, this is done for clarity of exposition, to avoid the effects of potential liquidations.

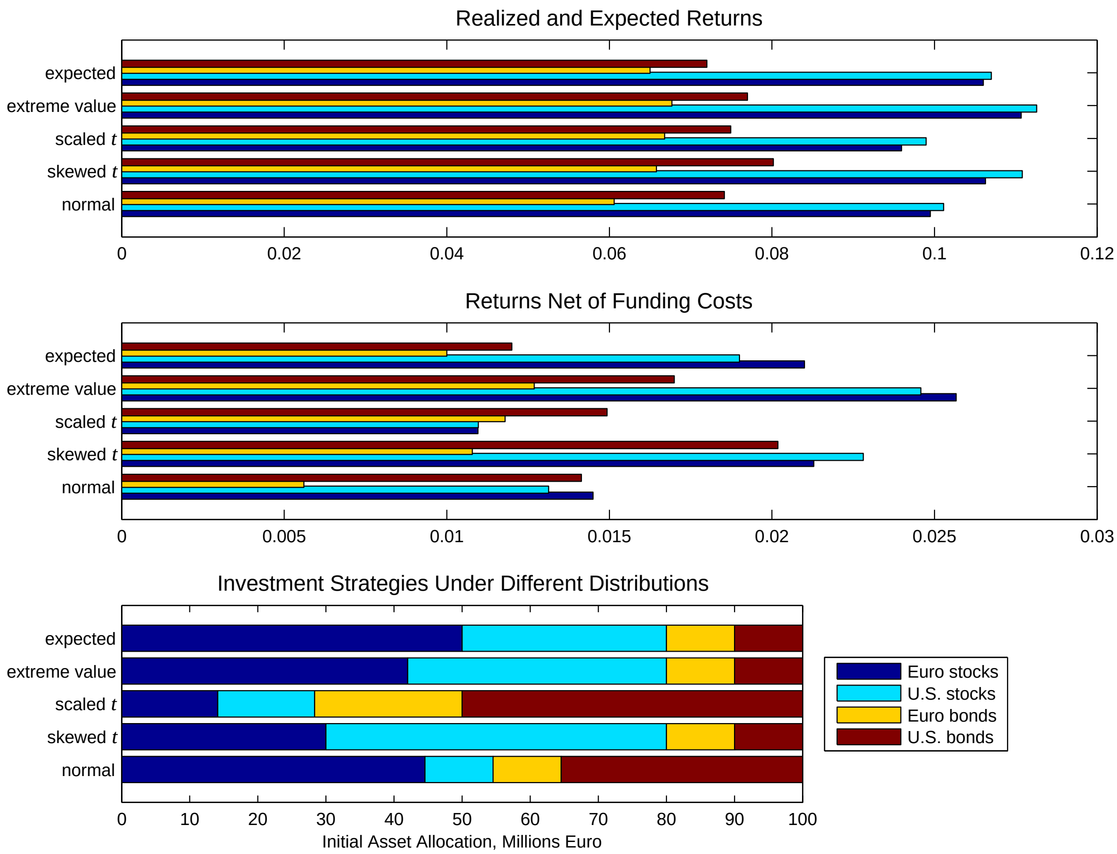

Figure 3: Sample mean returns, net returns, and initial investment strategies using different distributions. The label 'expected' denotes optimization performed with all future realized returns set equal to the estimated mean annual returns, given in Table 2.

- Case 1: focus on net returns and liquidation costs ($\alpha = 1, \beta \geq 0, \gamma = 0$), with subcases ($\alpha = 1, \beta = 0, \gamma = 0$), ($\alpha = 1, \beta = 1, \gamma = 0$), ($\alpha = 1, \beta = 10, \gamma = 0$);

- Case 2: focus on net returns and the liquidity of the remaining portfolio ($\alpha = 1, \beta = 0, \gamma \geq 0$), with subcases ($\alpha = 1, \beta = 0, \gamma = 0$), ($\alpha = 1, \beta = 0, \gamma = 0.1$), ($\alpha = 1, \beta = 0, \gamma = 1$); and

- Case 3: the 'everything matters' case, where net returns, liquidation costs, and the liquidity of the remaining portfolio are all accounted for ($\alpha = 1, \beta \geq 0, \gamma \geq 0$), with subcases ($\alpha = 1, \beta = 0, \gamma = 0$), ($\alpha = 1, \beta = 1, \gamma = 0.1$), ($\alpha = 1, \beta = 10, \gamma = 1$).

The results are generated using 100 scenarios from normal distribution. We use fewer scenarios than in the previous section due to higher computational requirements with non-zero α and β values. Normal distribution is selected for illustrative purposes only as the one commonly used in simulation, but a distribution that allows for fat tails and skewness may be more useful for practical implementations.

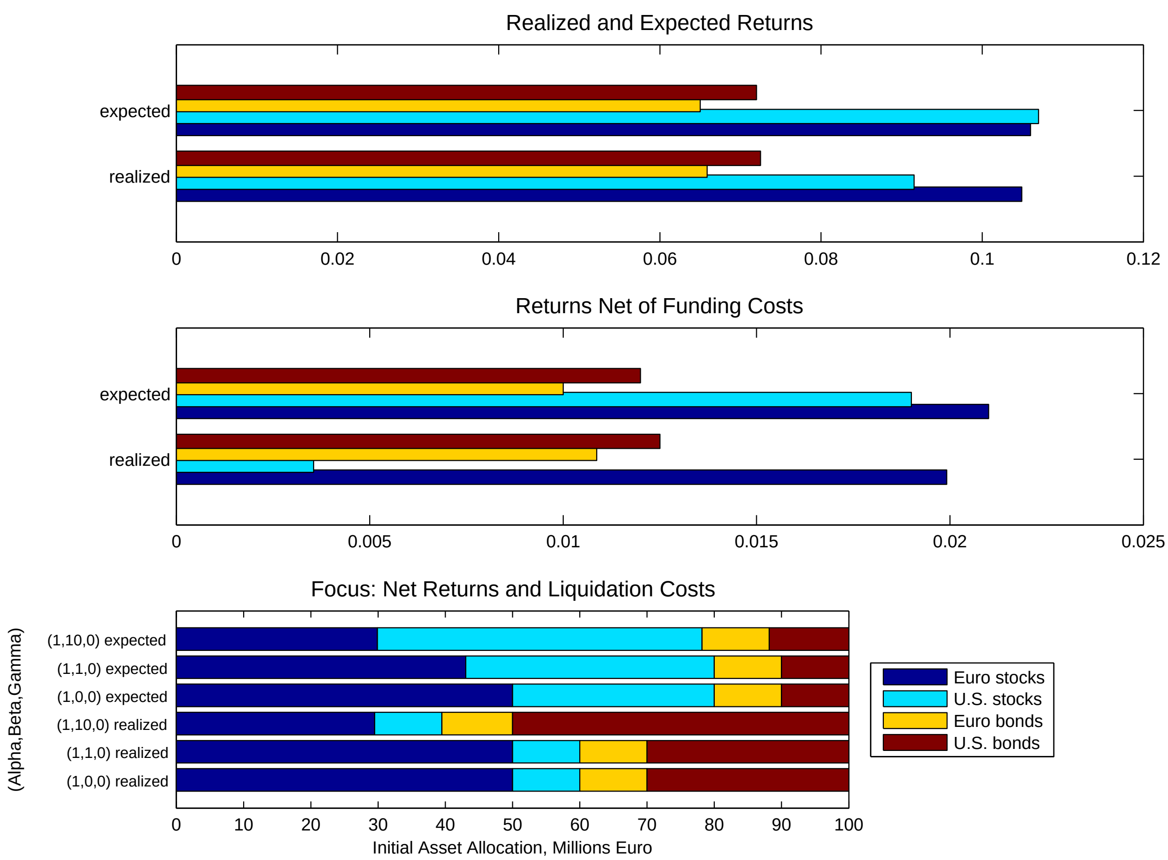

Figure 4: Mean returns, net returns, and initial investment strategies with priority given to net returns and liquidation costs (Case 1). The label 'expected' denotes optimization performed with all future realized returns set equal to the estimated mean annual returns, given in Table 2.

Observe that, due to liquidation costs for bonds being orders of magnitude lower than realized net mean returns during most economic periods (normal and volatile, see Table 2), the initial asset allocations are the same for cases ($\alpha = 1, \beta = 0, \gamma = 0$) and ($\alpha = 1, \beta = 1, \gamma = 0$).

Once the relative significance of liquidation costs is increased ($\alpha = 1, \beta = 10, \gamma = 0$), we see a move away from the costly-to-liquidate European stocks to the cheaper-to-sell U.S. bonds. Note also that reality turns out to be different than expected in this sample: U.S. stocks are more attractive if we look at expected returns, but European stocks have higher realized returns. In net terms, the relative attractiveness of asset classes changes from what were expected to be the highest-yielding European stocks and U.S. stocks to what were in fact the highest-yielding European stocks and U.S. bonds in the sample. Initial investment strategies shown in Figure 4 reflect this.

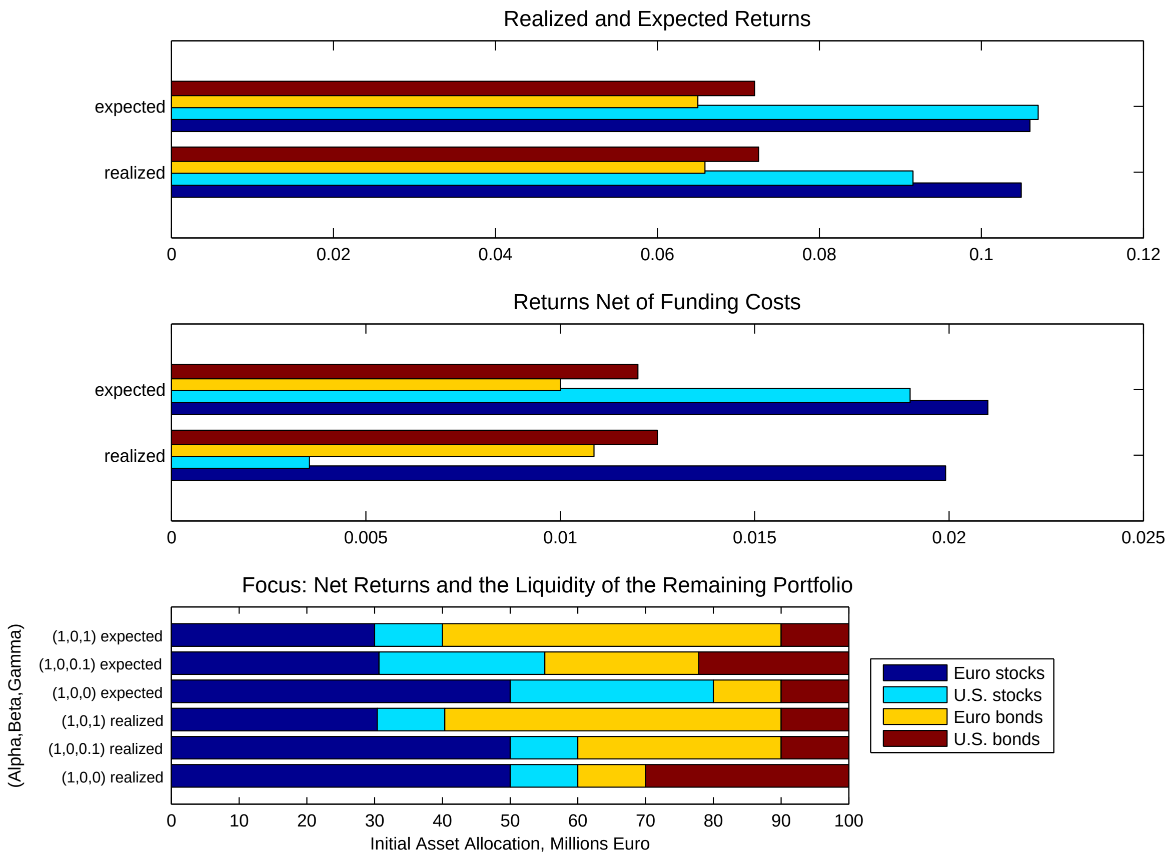

Figure 5: Mean returns, net returns, and initial investment strategies with priority given to net returns and the liquidity of the remaining portfolio (Case 2). The label 'expected' denotes optimization performed with all future realized returns set equal to the estimated mean annual returns, given in Table 2.

Figure 5 shows that, as John becomes more and more concerned about the liquidity of the remaining portfolio, his initial allocations to the highest-yielding assets (European stocks and U.S. bonds in net realized return terms) are reduced to allow for more European bonds. With higher priority given to the liquidity of the remaining portfolio ($\alpha = 1, \beta = 0, \gamma = 1$), the maximum allowed amount is allocated to the highly desirable (as indicated by their liquidity factors in Table 2) European bonds.

Finally, Figure 6 shows that Case 3 allows John to consider simultaneously all factors that are of interest for reserves management: net returns, liquidation costs, and the liquidity of the remaining portfolio. We see that, based on the specified preferences for these factors and

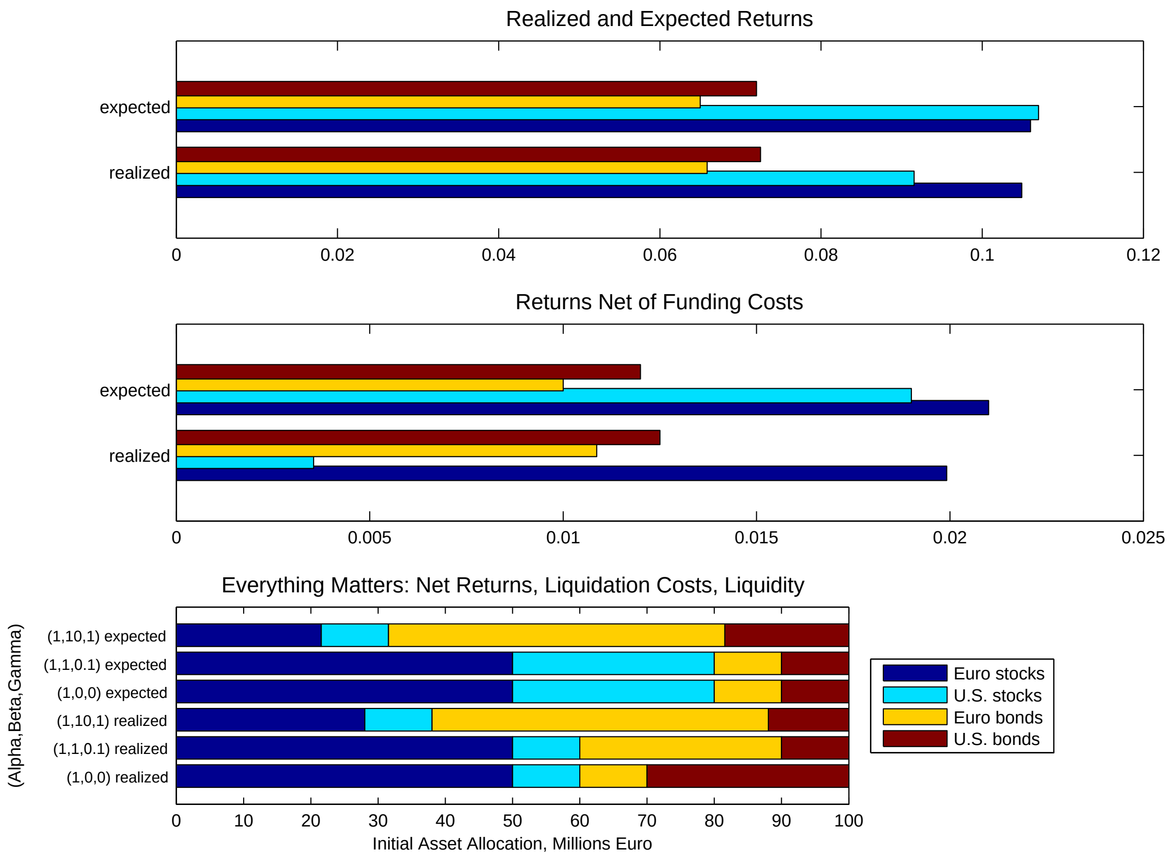

Figure 6: Mean returns, net returns, and initial investment strategies with all factors (net returns, liquidation costs, the liquidity of the remaining portfolio) considered simultaneously (Case 3). The label 'expected' denotes optimization performed with all future realized returns set equal to the estimated mean annual returns, given in Table 2.

sample realized net returns, the liquidity of the remaining portfolio dominates the concern for cheaper liquidation, because initial allocations are the same or similar for Cases 2 and 3 but not for Case 1 (subcases with $\beta \geq 0$, $\gamma \geq 0$). This result agrees with the opinions of practitioners in the field of reserves management (Nugée, 2009; Reveiz, 2009), and appears sensible for a central bank, which is particularly exposed to reputation risk.

4.3.4 *Examining recourse strategies*

For the final set of results, let us examine recourse strategies in different economic periods. Recall that recourse strategies are control variables representing decisions about the allocation of assets to be liquidated if a call on reserves arrives at the end of the investment period. Suppose that John's preferences for the three elements of the objective function (net returns, liquidation costs, and the liquidity of the remaining portfolio) are summarized by $(\alpha = 1, \beta = 1, \gamma = 0.1)$. Jane is another manager of central bank reserves, with the same amount to invest and facing the same constraints as John. She, however, values net returns less than John does, and her preferences are summarized by $(\alpha = 1, \beta = 10, \gamma = 1)$. Let us examine their respective recourse strategies and the remaining portfolio allocations.

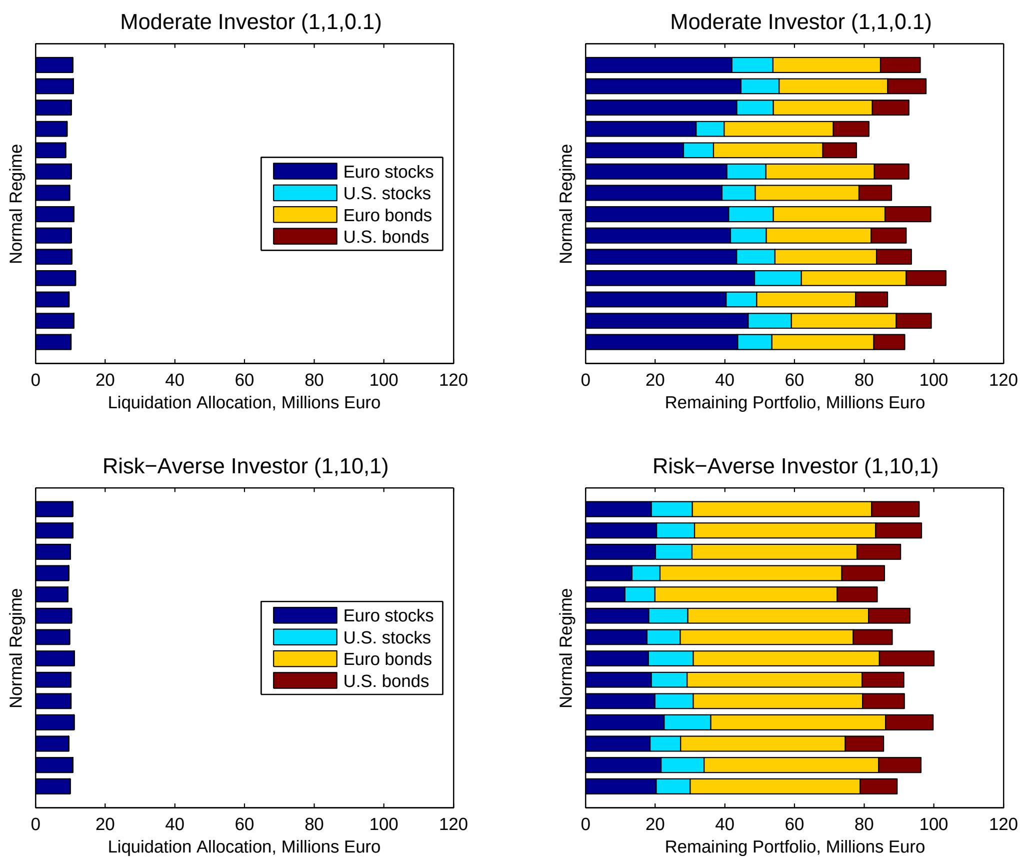

Figure 7: Recourse strategies for different realizations of returns in normal periods.

During normal periods of economic activity, characterized by low liquidation costs, low probabilities of liquidation, and low sizes of liquidation calls (Table 1), Jane's and John's liquidation allocations are very similar: only the cheap-to-liquidate European stocks are sold off. Their remaining portfolios differ due to the higher initial allocation to European bonds in

Jane's case, and European stocks in John's case (Figure 7). The sell-off of European stocks, as opposed to U.S. stocks, implies that the liquidity of the remaining portfolio matters more than liquidation costs under their preference specifications (lower-yielding U.S. stocks are slightly cheaper to liquidate, but also more desirable to have in normal times than European stocks).

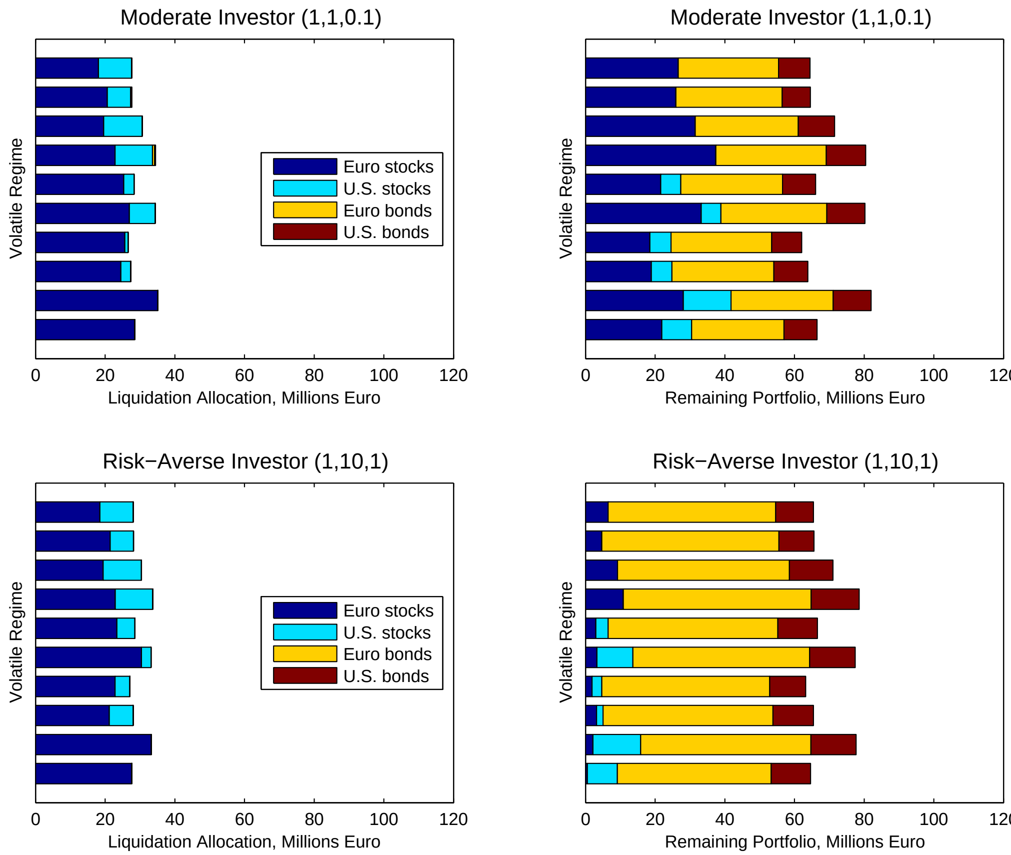

Figure 8: Recourse strategies for different realizations of returns in volatile periods.

Jane's and John's recourse strategies are also similar during volatile periods (Figure 8). Note, however, that compared to normal times, the bond proportions of their remaining portfolios are larger, due to greater liquidity preference for bonds during volatile times (see the liquidity factors in Table 2). Also, as expected with their relative preferences for maximizing net returns, John's remaining portfolio contains a larger portion of stocks than Jane's remaining portfolio.

Examining Jane's and John's liquidation allocations for crisis scenarios (Figure 9), we see that both investors sell off U.S. and European stocks, and are left with portfolios composed primarily of bonds. Small remaining portions of U.S. stocks reflect the fact that liquidating all U.S. stocks would be very expensive. As expected with Jane's higher risk aversion (or lower appetite for return), her portfolio contains a smaller proportion of stocks compared to John's. Unfortunately for him, because of his greater initial allocation to stocks and the need for liquidity in his remaining portfolio, John encounters greater liquidation costs from having to sell off a larger portion of stocks than Jane, whose liquidation portfolio is more evenly distributed between stocks and bonds.

Next, consider recourse strategies based on optimization with all future realized returns

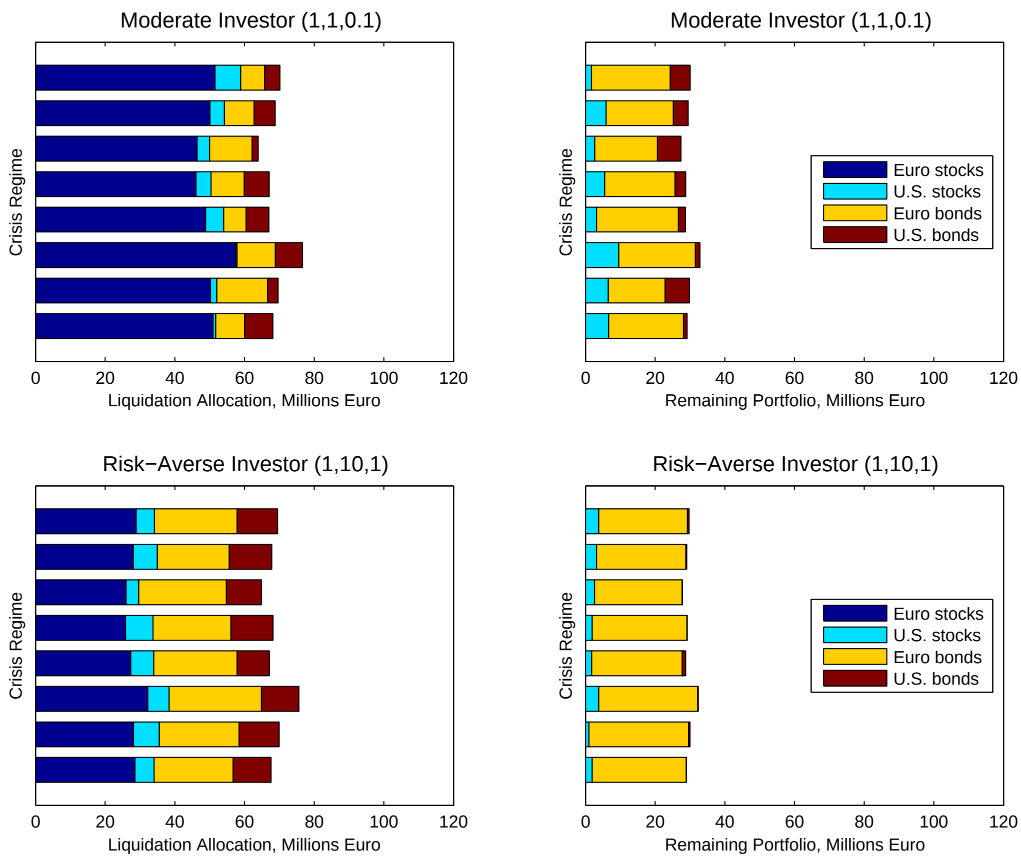

Figure 9: Recourse strategies for different realizations of returns in crises.

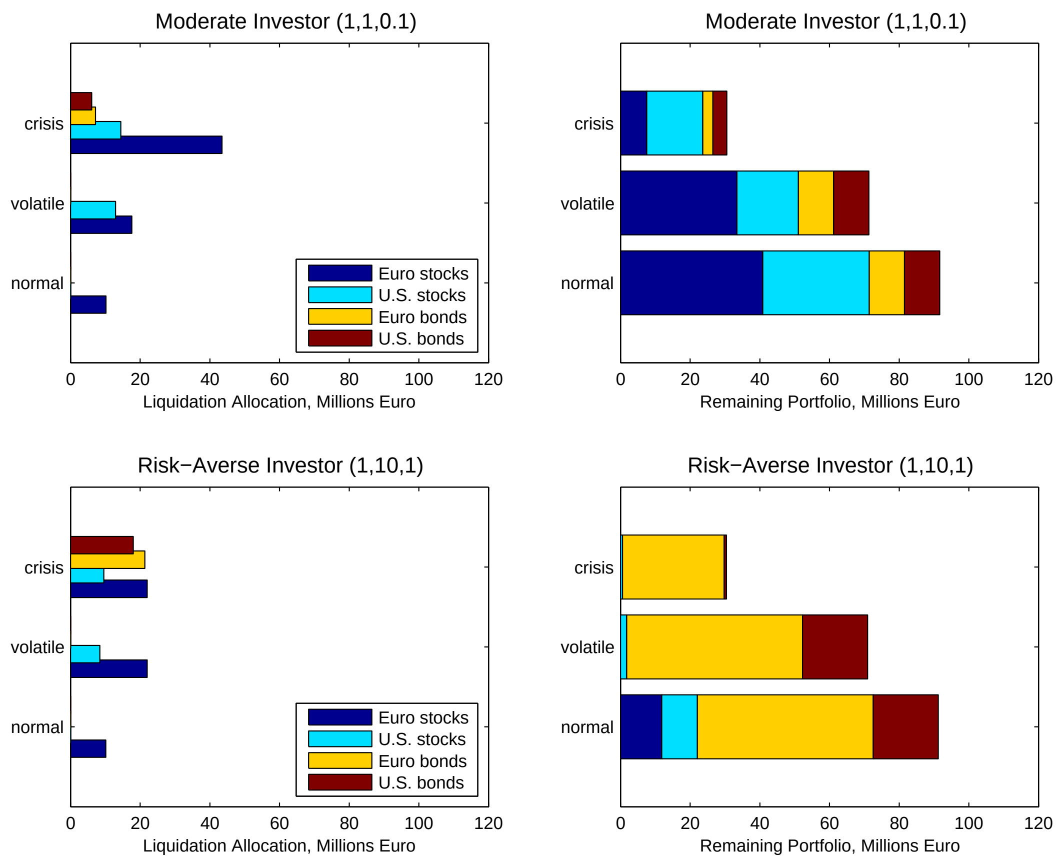

Figure 10: Recourse strategies in different economic periods based on all future realized returns set equal to the estimated mean annual returns, given in Table 2.

set equal to the estimated mean annual returns, given in Table 2. Since all realized return scenarios are the same, recourse strategies differ only by economic period, because of regime-dependent liquidation costs and liquidity factors (Figure 10). Also, due to higher estimated mean stock returns net of funding costs compared to realized net stock returns, proportions of stocks in John's remaining portfolio are higher than under realized sample scenarios. This difference highlights the danger of working with expectations: realized outcomes can be such that recourse strategy based on expected returns without regard for their stochastic nature may overstate the optimal holding of risky assets in one's portfolio.

5 Concluding Remarks and Future Work

Specifying an objective function for central bank foreign reserves management is a challenging task. There are many factors that can and should be considered; they include expected returns, the funding costs and opportunity costs of other funding sources, liquidation and/or transaction costs, the quality of the remaining portfolio, etc. This paper illustrates a potential objective function incorporating some of the above factors in the context of a particular modelling framework: namely, stochastic programming.

A key lesson learned from this research project, and highlighted by the 2007–09 market crisis, is the following. We may not know when crises will arrive, but we know what they look like when they occur; periods of market turmoil are typically characterized by large bid-ask spreads, high volatility, correlations becoming higher and changing signs, and behavioural elements such as flight to quality and hoarding of liquid assets. The model selected for the Exchange Fund Account should account for such scenarios explicitly in the optimization, as opposed to relying on constraints to ensure a high-quality portfolio, diversification, and liquidity.

There are several future research directions that would be useful for EFA modelling. First, we can examine funding costs in more detail in order to incorporate different possibilities for funding foreign reserve assets. For instance, in Canada, the funds for new reserve assets come from swapped domestic borrowing, due to the current cost savings with swapped borrowing over direct foreign issuance. However, this situation may change, making direct foreign issuance more appealing, and such a possibility should be evaluated within the objective function. As a further extension, we could incorporate stochastic funding costs in our model.

Another research direction is to extend the model to several periods over some investment horizon. This would allow us to make intertemporal allocation/liquidation decisions in response to changing economic scenarios. This extension would be useful for keeping the reserves portfolio in line with its strategic target by adjusting the allocations periodically in some optimal manner.

Finally, the joint optimization of asset and liability allocations should be investigated in future research. A joint asset-liability management model would be very useful for the EFA, where the portfolio manager may choose how to invest *and* fund reserve assets. This task is challenging, because the size of the optimization problem increases significantly: in addition to the larger size of the control vector, the number of constraints linking assets and liabilities grows. To tackle this task, it is crucial to choose a suitable modelling framework that is: flexible to incorporate factors of interest, constraints, etc.; computationally advantageous dealing with high-dimensional problems; and able to guarantee that a global minimum/maximum can be found. Stochastic programming appears particularly well suited to handle such problems, and should be examined further in the context of guiding reserves management decisions, as illustrated in this paper.

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