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Asset-Liability Management: An Overview

by Yuliya Romanyuk

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Please note another way to spell the author’s name: Yulia Romaniuk.

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Abstract

Relevant literature on asset-liability management (ALM) is reviewed and different ALM approaches are discussed that may be of interest to the Bank of Canada for the purpose of modelling the Exchange Fund Account (EFA). The author describes the general idea behind ALM, its pros and cons, risk measures and strategies, as well as some applications. Particular attention is paid to central bank reserves management. Ideas and suggestions are offered for future research on the ALM framework for the EFA.

JEL classification: G11
Bank classification: Foreign reserves management

Résumé

Se fondant sur les travaux pertinents réalisés dans le domaine, l'auteure analyse différentes approches de gestion actif-passif dont la Banque du Canada pourrait s'inspirer pour la modélisation du Compte du fonds des changes. Elle décrit les principes de la gestion actif-passif, ses avantages et ses inconvénients, certaines de ses applications ainsi que les mesures de risque et les stratégies utilisées. Elle accorde une attention particulière à la manière dont les banques centrales gèrent les réserves de change. L'auteure propose des pistes que pourrait prendre la recherche portant sur le cadre de gestion actif-passif du Compte du fonds des changes.

Classification JEL : G11
Classification de la Banque : Gestion des réserves de change

1 Introduction

1.1 Overview

Asset and liability management (ALM) deals with the optimal investment of assets in view of meeting current goals and future liabilities. The keyword of ALM is the *joint* evaluation of risks and benefits for assets and liabilities. The more traditional view of managing risks separately depending on the risk type (the so-called *silo* approach) is no longer satisfactory to financial institutions, and research shows that there are gains from managing risks at the global level (Rosen and Zenios, 2006). Among the different risks faced by an institution (such as market, credit, liquidity, operational, and business), ALM focuses on financial risks, as indicated in the following definition by the Society of Actuaries (SOA, 2003):

> ALM is the ongoing process of formulating, implementing, monitoring and revising strategies related to assets and liabilities to achieve an organization's financial objectives, given the organization's risk tolerances and other constraints.

The issue of jointly managing assets and liabilities arises in a number of industries, such as banking, insurance, and pension funds, as well as at the level of individual households.[1] The definitions of assets, liabilities, and risks are specific to each institution, but, very generally, assets may be viewed as expected cash inflows, and liabilities as expected cash outflows. Although short-term risks arising from the possibility that an institution's assets will not cover its short-term obligations are important to assess and quantify, ALM is usually conducted from a long-term perspective. As such, ALM is considered a strategic discipline as opposed to a tactical one (Choudhry, 2007).

Consider a central bank in a particular country that has a responsibility to manage that country's foreign reserves. The purpose of the reserve fund could be to provide liquidity in case of an intervention in foreign exchange markets; to cover the country's payments denominated in foreign currencies; a combination of these; or to generate a return. The day-to-day operations related to the reserve fund, such as risk assessment over short horizons and trading activities, fall into the domain of tactical risk management. When senior representatives from the central bank decide on the allocation of the fund's assets and liabilities, subject to the various constraints over, say, a year, they establish a strategic asset-liability target for the fund. From this point on, the foreign reserves and risk management teams at the bank would be responsible for keeping the actual asset-liability allocation within permissible bounds of the strategic target. Therefore, ALM sets out a long-term position for investing assets and covering liabilities, whether at a single future point in time or over multiple future periods.

Before discussing the motivation for undertaking the current project, let us consider some of the pros and cons of ALM. The benefits of ALM are rather obvious: an understanding of the company's overall position in terms of its obligations; comprehensive strategic management and investment in view of liabilities; the ability to quantify risks and risk preferences in the ALM process; better preparation for future uncertainties; and, ideally, gains in efficiency and performance from the integration of asset and liability management. Recognizing these benefits, banks and other institutions have implemented their own ALM methodologies.[2] There is also

[1] See Berger and Mulvey (1998) and Consigli (2007) for examples of ALM models for individual households.

[2] Examples include the CAP:Link system of Towers Perrin (Mulvey, 1996) and the CALM model (Consigli and Dempster, 1998) for pension funds; the Russell-Yasuda Kasai ALM model (Cariño et al., 1994) in the insurance industry; and, among banks, the MIN-MAD model developed by Brodt (1978) for a Canadian chartered bank, the BESMOD model of the First Austrian Bank (Grubmann, 1987), a goal programming model at the Commercial

a significant volume of academic literature on the subject of bank ALM models.[3]

Despite the widely accepted benefits of ALM, the associated challenges may prevent an institution from adopting an ALM framework. These difficulties lie primarily in the implementation of ALM. First, each institution has its particular objectives, risk tolerances, and constraints, and it would be difficult to devise an optimization algorithm that would realistically account for these specific characteristics when evaluating portfolio allocation decisions. Second, long-term strategic decisions depend on factors whose forecasts may not be readily available to the institution. Third, risk preferences and their changes over time must be translated into mathematical language, which is far from trivial.[4] Finally, a reasonable ALM model must put all of its different components (assets, liabilities, goals, institutional and policy constraints, etc.) together in a meaningful manner, which is difficult.[5]

For a central bank, there may be additional challenges in building ALM models for foreign reserves management. First, it may be difficult to define the benefits and risks associated with foreign reserves from a very high level: should the reserves be treated as a purely financial portfolio (hence the financial risk metrics and management strategies apply), or does the portfolio represent a public good (hence non-traditional approaches apply)? Second, any ALM objective function that specifies a central bank's (treasury's, or government's) appetite for return, tolerance for risk, liquidity preference, or other goals, would be highly sensitive to internal and external criticism; as such, the bank (treasury, government) may be reluctant to state its risk preferences. Third, it may be challenging to reach a consensus between researchers and policy-makers in the interested institutions regarding the relevant structure, risk measures, assumptions, and inputs of the model.

Nevertheless, the importance of developing a reasonable and useful ALM model that takes into account an institution's assets, liabilities, goals, and constraints cannot be overestimated. The next section discusses the motivation for undertaking a research project on ALM frameworks at the Bank of Canada.

1.2 A brief review of the Exchange Fund Account

The Bank of Canada is the fiscal agent for the Minister of Finance in Canada. In this role, Bank staff conduct funds management activities for the Government of Canada, such as wholesale and retail debt issuance, and investment and operations related to foreign reserves. Together with the Department of Finance, Bank staff are responsible for strategic planning of Canada's foreign reserves, in particular the Exchange Fund Account (EFA).

According to the Currency Act, the legislative objective of the EFA is to aid in the control

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Bank of Greece (Giokas and Vassiloglou, 1991), and a 5-year ALM model at the Vancouver City Savings Credit Union (Kusy and Ziemba, 1986).

[3] See, for instance, Kosmidou and Zopounidis (2002), Oğuzsoy and Güven (1997), Simonson, Stowe, and Watson (1983), Seshadri et al. (1999), Booth and Koveos (1986), Langen (1989a,b), and Fielitz and Loeffler (1979).

[4] Risk preferences are implicitly assumed when pricing assets in a complete market, such as the pricing of options by means of replicating strategies, while incomplete markets require risk preferences to be stated explicitly (see Rosen and Zenios, 2006, and the references therein); for instance, when evaluating pension fund claims or insurance liabilities related to mortality. For an example of ALM applied in the context of the latter, see Melnikov and Romaniuk (2006); Melnikov and Romanyuk (2008).

[5] Accounting poses a potential obstacle for ALM as well. Traditional book (actuarial) accounting is problematic, since losses due to variation in asset/liability prices can be amortized over long periods and therefore hidden (Ryan and Fabozzi, 2002; Ryan, 2004). In view of this fact, regulators have been pushing companies to use mark-to-market accounting, which does capture the volatility in price movements. But switching from book to market-based accounting may complicate the implementation of ALM: risk measures based on accounting ratios may not be adequate for strategies that rely on an economic (market-based) value of assets and liabilities.

and protection of the external value of the Canadian dollar.[6] The strategic objectives for the EFA are to:

- maintain a high standard of liquidity,
- preserve capital value, and
- maximize the return on the portfolio assets, while respecting the liquidity and capital preservation objectives.

The EFA represents the largest component of Canada's official international reserves, and is a portfolio of liquid foreign currency assets. It serves as a type of 'insurance for a rainy day,' when the government may have to intervene in the foreign exchange market to strengthen the value of the Canadian dollar by liquidating foreign assets.

The EFA is managed using a set of policies[7] that apply to asset-liability management, investment, and risk and performance management. In terms of investment policy, the assets in the EFA are allocated to one of two tiers: the *liquidity tier* or the *investment tier*. The investment policy specifies which assets are eligible for the EFA.[8] The rules for eligibility incorporate external credit ratings, and the investment policy sets out limits pertaining to issuers and all counterparties, and defines high standards of liquidity of reserve asset classes.

The funding for the EFA assets comes primarily from borrowing in domestic currency and swapping the proceeds into foreign currency; for instance, for the fiscal year 2007/08, swapped domestic borrowing comprised 76.8 per cent of total foreign currency borrowing for the EFA (Department of Finance Canada, 2008). But funding can also result from direct borrowing in foreign currencies (Canada bills and Canada notes, euro medium-term notes, and global bonds). Historically, the funding mix changed from mainly foreign currency borrowing to primarily swapped domestic issuance due to cost savings available in the domestic markets (De León, 2003). The domestic borrowing for the EFA is aggregated with general borrowing for the Government of Canada.

The asset-liability framework consists of managing assets and liabilities such that they are currency- and duration-matched to minimize the net exposure to foreign exchange and interest rate risks.

1.3 Motivation for the current study

Modelling of EFA assets and liabilities is an essential component of foreign reserves management, since it provides insight into key policy decisions regarding the strategic target and daily tactical operations related to the fund.

Several years ago, the Department of Finance engaged Fischer Francis Trees and Watts (FFTW) to evaluate the management of Canada's foreign exchange reserves. In July 2006, FFTW released the Evaluation of the Exchange Fund Account (FFTW, 2006).[9] The evaluation covered all aspects of the management of the asset side of the EFA, but the asset-liability

[6] For details, see Department of Finance Canada (2008).

[7] The policies governing the EFA are outlined in the Statement of Investment Policy (SIP), which can be found in Department of Finance Canada (2008).

[8] The assets permitted by the policy include U.S.-dollar- and euro- and yen-denominated securities issued by sovereigns and their agencies or supranational organizations, cash deposits with financial institutions, commercial paper and certificates of deposit from private sector entities, International Monetary Fund Special Drawing Rights, repurchase agreements (repos), and gold.

[9] The Evaluation of the Exchange Fund Account (FFTW, 2006), and the response from the Department of Finance, can be found on the website of the Department of Finance.

framework was outside the scope of the evaluation. As such, the EFA was evaluated as a portfolio of assets, and the resulting recommendations from FFTW were given from this perspective. FFTW indicate that most countries completely separate the asset and the liability functions, and that the Bank of Canada and the Bank of England are notable exceptions within the central bank community, in that both have adopted some type of a matched-book portfolio management approach. Some FFTW conclusions relevant for ALM modelling are as follows:

- The EFA is managed prudently and with full regard for the key strategic objectives (liquidity and capital preservation). Subject to these, the return becomes a tertiary consideration, although, historically, the EFA has generated an excess return and thus a positive contribution to the public purse.

- The management approach of Canada's international reserves is among the more conservative within the central bank community.

- The reserves management structure is unusual in that liabilities are matched against assets. The usual (although not necessarily superior) approach is to manage assets against a benchmark and not a cost of funding.

- Liquidity is very high in all investments in the EFA. The need for the highest level of liquidity for aggregate reserves should be reviewed, since the cost of the liquidity premium may be lowered by reducing liquidity marginally within a component of reserves.

- It is suggested to consider allowing an unmatched duration on a portion of the portfolio and to carefully evaluate the yield curve exposure that would result from this unmatched duration exposure.

To make meaningful policy recommendations for the strategic planning of EFA investments, and to address the possibility of deviating from the current ALM approach, as suggested by FFTW, we need to investigate different views of managing portfolios within the asset-liability framework and the alternatives to the matched-book approach. Once we understand their risks and benefits, we will see more clearly which key issues should be accounted for in the ALM framework of the EFA. Moreover, we would be better equipped to address the question of whether we should have a fully matched foreign reserves portfolio.

In particular, FFTW's evaluation highlights the fact that, currently, EFA liabilities are defined as cost of funding. The issue of providing liquidity in case of an intervention is addressed primarily by constraints on the size of the liquidity tier, currency, and asset classes. As a result, the liability to the Government of Canada arising from a potential liquidation of EFA assets and the likelihood of such liquidation are not explicitly quantified in the objective function for long-term strategic asset-liability planning.[10] For the purpose of comparison, the liquidity model of the Bank of England (discussed in section 4, which is used for strategic planning, views liabilities of their equivalent of the EFA, the Exchange Equalisation Account (EEA), as expected liquidation costs in case of an intervention, assuming a fixed cost of funding for EEA assets. From a tactical perspective, the EEA liabilities are defined in a similar way, as the cost of funding of the EEA assets.

In terms of risk management, one should be aware of the following common fallacies about risk: that it is always bad, that some risks are so bad that they must be eliminated at all costs,

[10]Such a definition of the EFA liabilities is used in the tactical management of the portfolio, and the EFA is closely monitored on a daily basis to ensure that it meets the desired liquidity profile by means of various risk measures, such as value-at-risk.

and that playing safe is the safest thing to do. The key is to *manage* risk to ensure that one's actual risk exposure corresponds to what one thinks it is and to what one is willing to bear (Culp, 2001). In light of this, and FFTW's comment about the conservative management of Canada's international reserves, a structured approach to the strategic management of foreign reserves would aid in assessing the constraints and investment policies, aimed at capturing the risk attitude of reserve managers.

This paper takes a step towards building a strategic model for the EFA by reviewing different ALM approaches and risk measures, and trying to identify the ones pertinent to foreign reserves management in Canada. In the process of setting a long-term target for investing EFA assets and managing the corresponding liabilities, a customized ALM model would quantify the risk exposure of the EFA and help identify whether the various investment policies and constraints, aimed at mitigating liquidity, credit, and other risks, actually accomplish this purpose.

2 Portfolio Management: Risks, Asset Allocation, and Asset-Liability Strategies

2.1 Defining risk

2.1.1 *Event-driven risk*

Risk at financial institutions is commonly viewed from an event-driven perspective (Culp, 2001): it is categorized depending on the type of event that can cause a potential loss. Using the classification system proposed by the Global Derivatives Study Group (1993), the following types of risk fall under the category of event-driven risk.

- Market risk: Risk that stems from changes in market-determined prices, rates, or indexes. Interest rate, exchange rate, and asset-price movements are all market risk factors, as are changes in the prices of derivatives or contracts dependent on market rates or indexes. Market risks include *correlation risk*, which reflects the exposure to unexpected changes in the correlation of underlying risk factors, and *basis risk*, which refers to the difference between the spot and the derivatives price of an asset.

- Liquidity risk: Risk that arises when current assets and cash inflows are insufficient to cover cash outflows. The inability of an institution to raise cash to fund its business activities is known as *funding liquidity risk* (Rosen and Zenios, 2006). *Market/trading liquidity risk*, on the other hand, is the risk that it would be costly, or perhaps even impossible, to liquidate certain asset types or establish new positions to hedge existing exposures when such needs arise. While market risk generally affects the present value of assets and liabilities, and the management of market risk is evaluated in the context of the expected value of assets and/or liabilities over all future periods, liquidity risk has to do with cash flows per period, and the ability (or inability) to meet cash flow needs as they occur in time.[11]

- Credit risk: Risk that refers to the potential non-performance of a firm of its obligation. Credit risk occurs (i) when a firm actually fails to deliver assets/funds when required

[11] A general discussion of the principles of liquidity-risk management, which includes specific recommendations for firms regarding the different types of liquidity risk, is provided in a report by the Institute of International Finance (2007). An earlier document by the Basel Committee on Banking Supervision (2000) outlines sound practices for liquidity management in banking.

(*default risk*), and (ii) when there is a perceived probability that such a failure will occur in the future (*downgrade risk*); both of these are types of *direct credit risk*. There is also *indirect credit risk*, which occurs when the present value of cash flows changes due to the credit quality of the third party not directly involved in the transaction; for instance, a credit downgrade of the government of a particular country would affect the value of contracts dependent on the term structure of interest rates in that country.

2.1.2 *Diversifiability*

This view of risk, popular among risk managers, academics, and portfolio investors, separates risk into two categories: the *idiosyncratic* risk that is particular to an asset or a bundle of assets and that can be diversified away by holding correlated assets (bundles of assets) to the one in question, and *systematic* risk that is driven by some risk factor(s) affecting all assets or cash flows.

The original single-factor capital-asset-pricing model (CAPM) explains the excess return of an asset over the risk-free rate by its covariation with the systematic risk factor (the market) and the idiosyncratic component (shock) uncorrelated with the market. The CAPM has been criticized for its unrealistic representation of systematic risk. Academic research has shown that excess market return is not the only factor that significantly affects excess asset returns (Culp, 2001), and many models have expanded the CAPM to include multiple risk factors and non-linear forms of dependence on risk factors. The basic idea of categorizing risk into systematic and idiosyncratic, or diversifiable and non-diversifiable, remains a useful approach for viewing, measuring, and managing risk in financial and non-financial institutions.[12]

For the purpose of identifying and measuring risks pertinent to the EFA, both risk paradigms are important. On the one hand, as with any portfolio holding foreign assets in the form of bonds, the EFA is exposed to event-driven risk: the market risk arising from fluctuations in interest rates and currency movements, the liquidity risk arising from the potential need to sell off assets to support the Canadian dollar, and the credit risk of the issuers of bonds held in the fund. As mentioned in section 1.2, numerous constraints are in place to minimize or even eliminate interest rate, currency, and credit risks and to guarantee sufficient liquidity in the fund. In addition, the explicit constraints on asset types and amounts aim to maximize the benefits of diversification by reducing the idiosyncratic risks of the portfolio. Diversification is the fundamental principle behind the theory of portfolio allocation; the fundamentals of asset allocation are briefly reviewed in the next section.

2.2 Asset allocation foundations and recent developments

2.2.1 *The classical mean-variance efficiency*

The mean-variance (MV) efficiency (Markowitz, 1952, 1959; Roy, 1952) is the classical paradigm for portfolio optimization and the foundation of modern portfolio theory. Let us introduce the following notation:

- $w = [w_1, w_2, \ldots, w_N]'$: N-dimensional vector of weights for the assets in consideration;
- $\mu = [\mu_1, \mu_2, \ldots, \mu_N]'$: N-dimensional vector of expected returns of the assets;

[12] For instance, the Principles Underlying Asset Liability Management (SOA, 2004) use this particular categorization for risk, identifying systematic risk as one that cannot be eliminated by aggregation or pooling of assets that are less than perfectly positively correlated, but that can be reduced by hedging.

- $\mu_p = w'\mu$: expected portfolio return;
- Σ: $N \times N$ matrix of covariances of expected returns;
- $\sigma_p^2 = w'\Sigma w$: portfolio variance.

Returns of individual assets are assumed to be normally distributed with mean μ and covariance matrix Σ.

In the classical setting, we maximize μ_p subject to a given σ_p^2 (alternatively, minimize σ_p^2 subject to a given μ_p). By plotting the possible combinations of risk/return levels, we obtain the so-called *mean-variance efficient frontier*, a curve[13] on which, for a given level of return, the portfolio variance is minimal (or, for a given variance, the return is maximal). We can combine return and variance into a single objective function, weighing their relative importance: maximize

$$f(w) = c\mu_p - \frac{\sigma_p^2}{2}\,, \quad c \in [0, \infty)\,, \tag{1}$$

subject to $\sum_{n=1}^{N} w_n = 1$, $0 \leq w_n \leq 1$, the typical constraints for most investors (portfolio weights must sum to 100 per cent and short selling is not allowed). The value of $c \to 0$ means that we care mostly about minimizing risk; $c = 1$ implies that we are indifferent between a 1-basis-point squared decrease in variance and a 1-basis-point increase in returns.

In his early works, Markowitz does not specify which portfolio along the efficient frontier should be selected by the investor (Markowitz and van Dijk, 2006); Roy (1952), on the other hand, recommends choosing the portfolio along the efficient frontier that maximizes $(\mu_p - d)/\sigma_p$, where d is a disastrous level of portfolio return. Tobin (1958) includes 'cash' as a 'risk-free asset' (with zero variance), citing 'liquidity preference' as a reason for holding this relatively low-yielding instrument. He shows that portfolios containing cash consist of cash and specific combinations of risky securities, now known as *tangent portfolios*.[14] Later work by Sharpe (1964) and Lintner (1965) assumes that investors can borrow at the risk-free rate, and shows that efficient portfolios consist of either tangent portfolios, tangent portfolios and positive cash holdings, or tangent portfolios and negative cash holdings (leveraged portfolios).

MV-efficient asset allocation takes advantage of correlations between asset returns to minimize the portfolio variance.[15] As pointed out by Rubinstein (2002), this was a key insight of Markowitz: the idea to evaluate securities not in isolation but as a group, and to decide whether to hold individual securities based on their diversification benefit to the portfolio. However, correlations among assets in bear markets tend to be higher — and diversification benefits lower — than in bull markets; this effect should be accounted for in an asset allocation model (see, for example, Ang and Bekaert, 2002). Hence the correlation risk factor should be incorporated into the ALM framework for the EFA: the expected liquidation costs may be magnified by potentially higher correlations among asset classes since, presumably, the need to sell off EFA assets would arise during some kind of market turmoil.

[13] The efficient frontier is a parabola in the mean/variance space and a hyperbola in the mean/standard deviation space (Merton, 1972).

[14] Graphically, the tangency portfolio is the point on the efficient frontier at which the line drawn through the risk-free rate on the y-axis is tangent to the efficient frontier.

[15] The degree of risk reduction from diversification depends on the actual correlation between expected asset returns. With a simple example of two assets, theoretically speaking, it is possible to eliminate the portfolio risk if the correlation between asset returns $\rho = -1$. If, on the other hand, $\rho = 1$, there are no diversification benefits. For $|\rho| < 1$, as correlation decreases, the total portfolio risk diminishes.

2.2.2 *The principles of expected utility theory*

A major criticism of the MV approach is that investor risk preferences are not taken into account. Expected utility theory, which directly incorporates the degree of investor's risk aversion, is a leading alternative framework for making portfolio allocation decisions under uncertainty.[16] The principles underlying the expected utility theory state that, as a rule, wealth is always better than no wealth, wealth (consumption) never reaches satiation, and marginal utility of wealth decreases for a typical risk-averse individual: $U(W) > 0$, $U_W(W) > 0$, and $U_{WW}(W) < 0$ for all W, where W denotes the wealth (consumption opportunities), $U(W)$ the utility of wealth, and U_W and U_{WW} the first and the second partial derivatives with respect to W. The above relations need not hold for each and every individual: a gambler may be a risk-*taker* with a convex utility, or a person may be risk-*indifferent* with a linear utility.

Since wealth is typically a random variable, it is difficult to deal with the utility of wealth directly, so, instead, the expected utility of wealth $E[U(W)]$ is the function of interest. Expanding the utility of wealth using a Taylor series about the utility of expected wealth $U[E(W)]$ and taking expectations, we obtain

$$\begin{aligned} E[U(W)] &= U[E(W)] + \frac{1}{2}U_{WW}E[W - E(W)]^2 \\ &\quad + \frac{1}{3!}U_{WWW}E[W - E(W)]^3 + \frac{1}{4!}U_{WWWW}E[W - E(W)]^4 + \dots . \end{aligned} \tag{2}$$

This equation involves the mean, the variance, and higher moments of the distribution of W, which may not be readily available or easy to compute. This explains the frequent assumption of normality for modelling asset returns: with a (multivariate) normal distribution, the first two moments are sufficient to characterize the entire distribution. Thus an expected utility function with a normal underlying random variable depends solely on the mean and the variance of this random variable, eliminating the higher-order moments from all subsequent computations (Culp, 2001).

Another way to eliminate higher-order moments is to consider a quadratic utility of the type $U(W) = W - \frac{a}{2}W^2$: its $U_{WWW} = U_{WWWW} = 0$, leaving only $E(W)$ and $E(W)^2$ in equation (2). Markowitz (1959) shows that, in a single-period optimization, choosing optimal portfolio allocation based on mean and variance only is equivalent to having a quadratic utility function for risk preference. He also shows that maximizing the expected utility function, which is a linear combination of the expected return and a risk measure (not necessarily variance), is equivalent to calculating the efficient frontier and making portfolio allocation decisions based on these two metrics only. In other words, in a single-period setting, either we assume normality where variance is sufficient to describe risk, or we choose a quadratic utility function that makes variance alone relevant. If we do not assume normality but want to use the MV approach, the efficient frontier should be specified in terms of another risk measure.

[16] The development of expected utility theory can be traced back to the mathematicians of the 1700s, when Bernoulli's St. Petersburg paradox highlighted the problem of relying solely on expectations when making decisions under uncertainty. The paradox shows that, if one cares only about expected values, one should be willing to pay any (read: infinite) price for a payoff whose expectation is infinity, even if this payoff is very unlikely. To overcome this problem, Bernoulli proposed to correct the expected value approach by using a mathematical function to account for the risk aversion of the person making the decision. For details of the paradox problem statement and related derivations, see Melnikov (2004). The expected utility theory was formalized in the economic context by Von Neumann and Morgenstern (1944).

2.2.3 *Developments in asset allocation*

For the purpose of the current project, reviewing the foundations of portfolio theory is useful in two ways. First, knowing the classical asset allocation approach, with its assumptions and limitations, helps us to understand and motivate developments in this area in academia and among practitioners, and in the field of asset-liability management. Second, being familiar with advances in portfolio theory and knowing how other researchers address the challenges of specifying risks, their metrics, and risk preferences should help us to determine the preferred method for developing an asset-liability framework for the EFA.

One area of research in portfolio allocation deals with measuring risk and the ability to incorporate investors' risk preferences. In the classical setting, the investor's measure of return is the expected portfolio return μ_p, while its variance σ_p^2 is the risk metric. Markowitz (1959) identifies and discusses six alternative risk measures: standard deviation, semi-variance, expected value of loss, expected absolute deviation, probability of loss, and maximum loss; he recommends semi-variance, stating that it is a realistically superior risk measure (Rubinstein, 2002). Similarly, Harlow (1991) focuses on minimizing the downside risk, as measured by semi-variance. Different risk metrics are also examined in Markowitz and van Dijk (2006) and Mulvey (2001).[17]

In many practical applications, investment managers are rewarded for generating portfolio returns equal to or greater than certain benchmarks. There is a stream of literature that studies the incorporation of benchmarks into the objective function. The idea is to measure the *tracking error*, generally defined as the absolute (or squared) difference between the portfolio and the benchmark returns, and to penalize large tracking errors. Studies in this area include Chow (1995), Grinold and Easton (1998), and de Pooter, Martens, and van Dijk (2008).

A large volume of research extends the classical single-period setting to a multiperiod framework or long horizons. Studies in this area include Samuelson (1969) and Merton (1969), who work in discrete- and continuous-time settings, respectively. Later developments, dealing with different asset allocation structures and programming techniques, include Brennan, Schwartz, and Lagnado (1997) (multiperiod optimal control, utility); Dantzig and Infanger (1993) and Grauer and Hakansson (1998) (multiperiod stochastic programming, utility); Cariño and Turner (1998) (multiperiod stochastic programming, MV-type objective function); and Brennan and Schwartz (1998) (multiperiod optimal control, utility).

Examples of recent research topics in this field are multiperiod expected utility maximization with regime switching for asset correlations and volatilities (Ang and Bekaert, 2002), multiple rebalancing periods with many asset classes in a stochastic programming setting (Infanger, 2007), and receding-horizon[18] control techniques for constrained portfolio optimization (Herzog, Dondi, and Geering, 2007; Primbs and Sung, 2008). Herzog, Dondi, and Geering (2004) investigate strategic asset allocation in a discrete-time setting, maximizing a mean-variance objective function over terminal wealth several steps into the future. Herzog, Geering, and Schumann (2004) examine strategic portfolio management using coherent risk measures[19] in a

[17] Section 2.3 defines some of these measures and mentions others used within portfolio and asset-liability management contexts.

[18] A receding-horizon technique uses a finite constrained horizon to replace an infinite constrained horizon, which makes the optimal control problem easier to solve.

[19] The problem with the commonly used value-at-risk measure is that it is not subadditive: the risk of the total portfolio may be larger than the sum of risks of the portfolio parts. Using this risk metric in practice implies that a risk manager may underestimate to an unacceptable degree the total risk exposure of a portfolio by adding the individual risk exposures of the portfolio parts. Subadditivity is one of the requirements for a measure to be *coherent*, as defined in a seminal paper by Artzner et al. (1999) on the topic of coherent risk measures. Artzner

dynamic setting (see also Rockafellar and Uryasev, 2000). These studies provide evidence of the usefulness of coherent risk measures — something to consider for the EFA model.

Kallberg and Ziemba (1984) study the effects of misspecification in a normally distributed portfolio from different sources and find that errors in means are about an order of magnitude more important than errors in variances/covariances. Chopra and Ziemba (1998) find that errors in variances are about twice as important as errors in covariances, and that the significance of mean errors increases, relative to variance/covariance errors, with the risk tolerance of the investor. Ortobelli et al. (2003) study asset allocation with non-Gaussian distributed returns, and Meerschaert and Scheffler (2003) investigate portfolio modelling with heavy tailed random vectors.[20]

A significant limitation of the MV framework is its instability: small changes in the estimates of inputs may lead to large variations in the optimal portfolio allocations (Michaud, 1998; Scheel et al., 2001). Because the method relies on statistical estimates of expected returns and covariances, the variability in the estimates makes many portfolios statistically equivalent to those on the efficient frontier.[21] A number of studies address the issue of stabilizing the MV weight estimates; a well-known extension by Black and Litterman (1992), for example, in which a Bayesian-type approach is used to specify investors' views and the CAPM prior to estimating asset returns, leads to more stable and more diversified portfolios than the traditional MV approach (Walters, 2009). Chopra, Hensel, and Turner (1993) apply the Stein estimation technique[22] to reduce the sensitivity of the MV approach to inputs, and find that this improves the MV optimization.

Hensel and Turner (1998) indicate that applying the MV approach naively does not generally lead to favourable results, and that, in order to obtain stable MV-optimal weights over time (to reduce transaction costs), one can either methodically massage the inputs and/or constrain the optimal weights. One method to stabilize the weights is to use Tikhonov regularization (Tikhonov and Arsenin, 1977), where a target (or, in some manner, 'reasonable') vector for asset distribution is specified within the objective function, and deviations from this target vector are penalized. Should an MV-type framework be adopted for the EFA model, its sensitivity and stability must be carefully investigated.

2.3 Asset-liability management strategies and applications

Following Rosen and Zenios (2006), asset-liability management strategies can be grouped into four broad categories:

- single-period static models,
- single-period stochastic models,
- multiperiod static models, and

et al. (2004) extend the tail (conditional) value-at-risk coherent risk measure to a multiperiod setting.

[20] Additional studies on financial modelling using heavy tailed distributions are provided in Rachev (2003).

[21] A stream of literature analyzes statistical properties of the estimated portfolio weights and the sample efficient frontier (for example, Kozubowski, Panorska, and Rachev, 2003; Jobson and Korkie, 1980; Kan and Smith, 2008; Okhrin and Schmid, 2006), and several studies present tests for the efficiency of a given portfolio and its weights (Huberman and Kandel, 1987; Gibbons, Ross, and Shanken, 1989; Jobson and Korkie, 1989; Britten-Jones, 1999; Kan and Zhou, 2008; Bodnar and Schmid, 2008).

[22] Stein estimation relies on using data sets for several indexes that are similar to estimate parameters for one of these indexes, as opposed to using a single index to obtain its parameters. Using Stein estimation, an estimated mean for an individual country index might be adjusted to reflect parameters of a global index. This technique is referred to as 'shrinkage towards the mean' (Hensel and Turner, 1998).

- multiperiod stochastic models.

As their name suggests, single-period models are concerned with optimal investment over a single time horizon. Their length can vary widely depending on the application. For example, central bank policy-makers may set a strategic target for foreign reserves investment over a year. A commercial bank trader might be concerned with which assets to purchase today to generate a certain target return over a week. Electronic trading seeks out strategies to maximize profit over very short time periods. Multiperiod (dynamic) models allow investors to rebalance their portfolios over several periods, adjusting to market conditions and perhaps new investment goals. While single-period strategies may perform well in some settings (see Hakansson, 1971, 1974; Mossin, 1968; Hakansson and Ziemba, 1995), they are generally too restrictive for most practical applications.

In this section, we examine common ALM risk measures and strategies within the four above categories, following Brennan, Schwartz, and Lagnado (1997), Mulvey (2001), SOA (2003), Rosen and Zenios (2006), Mulvey and Ziemba (1998), Kosmidou and Zopounidis (2008), and Mulvey and Vladimirou (1989). Note that many risk measures can be, and have been, extended from single- to multiperiod models, and various ALM objectives (maximizing profit minus transaction costs, attaining certain goals, etc.) can be, and have been, used in both static and stochastic environments.

2.3.1 *Single-period static models*

The models in this category hedge against small well-defined changes from the current state of the variables of interest, such as interest or exchange rates. Portfolios are structured to behave in a predictable and acceptable manner to the investor. ALM strategies in this category include the following:

- Immunization: Introduced by Redington (1952) and analyzed recently by de la Grandville (2007), portfolio immunization aims to make a portfolio insensitive to small changes in a specified factor, most frequently interest rate movements.[23]

- Dedication: A dedicated portfolio is one in which asset and liability cash flows are fully matched to eliminate exposure to changes in the factor(s) of interest.

- Gap/surplus management: The gap measure usually refers to the difference between the value of assets and liabilities, and an institution may be interested in minimizing the gap or keeping it within an acceptable boundary. Surplus may be defined in the same manner as the gap metric, but the terminology is used typically when there is excess wealth to be invested.

Immunization and dedication frequently utilize *duration* and *convexity* risk measures. Duration, $-\frac{\partial P}{P \partial r}$, measures the sensitivity of a bond portfolio with value P to changes in the interest rate r (a related metric, convexity, $\frac{\partial D}{\partial r}$, measures changes in duration relative to changes in r). The shortcoming of duration is that it captures price changes due to small parallel shifts in the yield curve only, and does not reflect optionality that may be embedded in bonds comprising the portfolio. Variations of the standard duration have been developed to overcome these limitations, including Macaulay, modified, effective, and key rate durations (see Ho, Chen, and

[23] There are numerous sources on measuring interest rate risk, which will not be listed here; see Milgrom (1985) on this topic within the ALM context.

Eng, 1996; Ho, 1992; Reitano, 1990, 1991). Leibowitz et al. (1989) extend the duration metric to equities and show that they have two durations: one with respect to real rates and the other with respect to inflation. Waring (2004a) and Barrie, Turnbull, and McCulloch (2006) examine how immunization can fail if the different durations of bond and equity portfolios are not accounted for.

Immunization, dedication, and gap/surplus management are standard within the banking and insurance industries. To allow greater flexibility and more realistic modelling frameworks, they have been extended to accommodate uncertainty and multiperiod investment horizons. For example, Zenios (1995) discusses how to adapt the static immunization and dedication methods to a stochastic environment by means of Monte Carlo simulations; Monfort (2008) studies immunized portfolios using the surplus measure with random processes for assets and liabilities; Albrecht (1985) and Gajek (2005) investigate portfolio immunization under stochastic interest rates; and Waring (2004a,b) defines an efficient frontier for surplus wealth.

2.3.2 *Single-period stochastic models*

These models describe the distribution of returns of assets and liabilities due to random market movements. Unlike the static models, stochastic models explicitly incorporate and quantify risk, but they are concerned with uncertainty at the end of a single investment horizon only (versus dynamic ALM models with multiple or even continuous portfolio rebalancing opportunities). The classical MV approach discussed in section 2.2.1 is a prime example of single-period stochastic modelling, where the risk is measured by the variance of the portfolio. Most risk measures used in this category aim at minimizing downside asset movements; some of them are defined as follows:

- Absolute deviation: $|w - E(w)|$, where w is the (random) wealth of the investor and $E(w)$ its expectation.
- Semi-variance: $(\min[(w - E(w)), 0])^2$.
- Downside formula: $(\min[w - w^*, 0])^2$, where w^* is a target portfolio return.
- (Conditional) value-at-risk: There are several variants of the value-at-risk metric, discussed in detail in Jorion (1997); a common variant is the dollar loss relative to the mean, measured by the lowest portfolio value $\bar{w}(c)$ at a given confidence level c: $E(w) - \bar{w}(c)$. In general, it is the worst possible realization $\bar{w}(c)$ such that the probability of exceeding this value is c. Conditional value-at-risk (expected shortfall, or tail value-at-risk) is the expectation of all outcomes in the c tail of the distribution (the expected value of bad outcomes).

2.3.3 *Multiperiod static models*

A multiperiod static environment is one where investors can rebalance their portfolios over several periods within a well-defined environment, or where changes in the factors driving model variables are well defined. Some work was done on multiperiod static models in the 1960s and 1970s (see, for instance, Chambers and Charnes, 1961; Cohen and Hammer, 1967). These models generally rely on variants of risk measures outlined in section 2.3.1 and industry-specific risk metrics. However, without the element of risk, such models are of limited practical use. They have been replaced by the models in the next category.

2.3.4 *Multiperiod stochastic models*

These models allow both assets and liabilities to evolve randomly over time following a probability distribution. Investors may change the compositions of their portfolios over the investment horizon, possibly reversing their previous decisions due to the evolution of the driving factors. Brodt (1978) is an early adaptation of the MV framework to the multiperiod setting in the context of a bank managing its balance sheet with the goal of maximizing profits. Another example is Bradley and Crane (1972), who maximize expected terminal wealth (over multiple stages) under constraints. There are a number of ALM frameworks in this category; the primary ones, described below, are decision rules, scenario analysis/simulation, stochastic optimal control, and stochastic programming.

- Decision rules

  Decision rules are strategies to determine portfolio allocation in each time period; they do not change over time. A well-known example is the *fixed mix strategy*, where at the end of each time period the portfolio manager adjusts the portfolio composition to keep the proportions of assets and liabilities in a fixed ratio (say, 30 per cent in bonds and 70 per cent in stocks). Some studies (for example, Merton, 1969, 1990) indicate that variations of this approach may be optimal, theoretically speaking, for long-term investors even in the presence of transaction costs. Moreover, fixed-mix strategies dominate the passive *buy-and-hold* strategies (Mulvey and Chen, 1996). Decision rules reduce the number of decision variables, which may improve computational efficiency.

  One drawback of the decision rules approach is its independence from the risk aversion of the investor. However, Mulvey and Ziemba (1998) point out that decision rules can be adapted to reflect changes in risk appetite; for example, they can be used to accommodate greater tolerance for risk with higher levels of wealth (see Perold and Sharpe, 1988). Alternatively, one could set a decision rule by which the surplus wealth should be positive over the investment horizon, or match a specified target wealth. A more serious disadvantage of decision rules is that the optimization model may not find a global solution, identifying local critical points only. The way to overcome this difficulty is to employ a type of grid search/global search algorithm.

- Scenario analysis/simulation

  Generating scenarios is a fundamental aspect of stochastic ALM models. A scenario represents a single coherent forecasted realization of random variables driving the model over the planning horizon. The number of scenarios can be small or large, depending on the application and the available computational power, and the scenarios may be weighted appropriately (for instance, extreme outcomes may be assigned higher probabilities during market turmoil). The goal is to construct a set of scenarios representing the universe of all possible outcomes, and the challenge is to sample the entire state space in higher dimensions.

  Simulation is a popular technique in reserves management; for example, Coche et al. (2006) use Monte Carlo simulation in a single-period and a multiperiod framework to study the implications of an explicit policy objective on the management of foreign reserves in a central bank. Bolder (2003) utilizes simulation in developing a domestic debt management model in Canada. An extended version of this model has been implemented at the Bank of Canada; it provides support in determining the optimal allocation of securities in the Canadian domestic debt portfolio.

Related simulation research topics include: generating consistent scenarios (see Mulvey, 1996; Mulvey and Thorlacius, 1998); estimating the fat tails that are typical for asset returns, which can be done using extreme value theory (Longin, 1996); reflecting co-movements and the covariance structure between assets and liabilities (studied by Simonson, Stowe, and Watson, 1983, for commercial banks; Hoevenaars et al., 2008, for investors who include real estate, commodities, and hedge funds along with bonds and stocks in their portfolios; DeYoung and Yom, 2008, for commercial banks using derivatives; Frauendorfer, Jacoby, and Schwendener, 2007, for pension funds); and selecting representative scenarios and aggregating these in a meaningful manner to prevent arbitrage opportunities (examined by Cariño, Myers, and Ziemba, 1998; Dupačova, Bertocchi, and Moriggia, 1998; Klaassen, 1998). This approach overlaps with other ALM strategies, such as decision rules and stochastic programming.[24]

- Stochastic optimal control

  This method, used by Samuelson (1969), Merton (1969, 1990), Brennan and Schwartz (1998), and others, relies on a small number of state variables whose evolution is modelled by a joint Markov process. Because the size of the stochastic optimal control problem grows exponentially with the number of state variables (think of a multi-dimensional mesh discretizing the state space within a numerical solver), this approach is limited to situations where the state of the world can be represented by few (three or four) factors.[25]

  For most practical applications, especially those with complex liabilities, other factors would have to be added; moreover, various constraints would most likely require the problem to be solved numerically (usually, by dynamic programming or finite-element methods). This becomes quite challenging computationally and may explain why there do not seem to be any practical or commercial applications of stochastic optimal control in asset management.[26] Additional challenges of control problems are: the output is sensitive to the model parameters, as in the MV framework, and approximation errors may arise due to the discretization of the state space.

- Stochastic programming

  The main difference between stochastic optimal control and stochastic programming is the way in which uncertainty is modelled. While in control problems the state space is frequently continuous (discretized by numerical solvers), stochastic programs usually describe uncertainty by using a branching tree in a discrete-time setting. Each node of the tree represents a joint realization of all random factors, corresponding to a particular outcome of the factors at a point in time in the optimal control set-up. Conditional decisions are made at each node subject to modelling constraints, hence the model expands

[24]For example, Kouwenberg (2001), Kim (2006), and Frauendorfer and Schürle (1998) discuss how to sample scenarios for the stochastic programming of ALM problems, and Kosmidou and Zopounidis (2004) use simulations within goal-programming models of bank ALM strategies.

[25]For example, Brennan, Schwartz, and Lagnado (1997) use short-term government rates, long-term government rates, and dividend yield as driving variables in their model; another dimension is added by the time remaining in the planning period. The authors point out that to describe the world with such parsimony would be inadequate if transaction costs were added, or if the exact composition of assets and liabilities at each point in time were important; the model could, however, accommodate a large number of assets (with values driven by the three key variables) or constraints.

[26]This observation was made by Brennan, Schwartz, and Lagnado (1997); however, advances in computational efficiency may have since changed the situation, to allow control methods to be practically useful for asset-liability management.

the decision space based on the conditional nature of the scenario tree. The size of the model depends polynomially on the number of outcomes at each node (4 time steps and 3 branches result in 81 distinct scenarios). A key requirement is *non-anticipativity*: scenarios with the same history up to a particular point should lead to the same portfolio allocations. Grebeck and Rachev (2005) and Tokat and Rachev (2003) are useful recent reviews of stochastic programming methods in the ALM context.

A clear advantage of stochastic programming over, say, simulation is that a tree structure can incorporate scenarios with low probability but high impact without having to generate thousands of new scenarios (Claessens and Kreuser, 2007). Another benefit is that it can accommodate a large number of random factors driving the model, since at each node of the tree the realizations of all factors are expressed by a given number of branches. The problem is selecting these few realizations to capture uncertain outcomes adequately (Mulvey and Vladimirou, 1992, and similar studies may answer some questions on this topic). Another disadvantage is that one requires highly efficient computational algorithms if the problem has many decision periods and variables. And, like stochastic control, it is not always easy to test this approach, although some studies appear to do this successfully (for instance, Jobst, Mitra, and Zenios, 2006).

One type of stochastic programming, called stochastic programming with recourse, models situations where investment decisions are reversible (see Grebeck and Rachev, 2005; Booth, 1972; Cariño et al., 1994; Kusy and Ziemba, 1986); this element makes modelling more realistic in most practical applications, in particular for modelling the EFA. Stochastic programming is also useful when the objective function cannot be defined using a single goal. In practice, most institutions face a number of goals of different priorities, some of which may be competing. The programming set-up of the ALM problem allows investors to specify multiple goals with their priorities, and to deal with competing objectives; see Kosmidou and Zopounidis (2008), Mulvey and Ziemba (1998), Korhonen (2001), and Langen (1989a,b) for model specifications and additional references in this area.

Stochastic programming has been popular in practice.[27] It appears to have many advantages over other ALM modelling structures: it can accommodate many decision variables and driving factors to reflect numerous investment objectives and constraints; it can incorporate low-probability, high-impact scenarios, which are of concern to all portfolio managers; it can allow reversible investment decisions; and it extends naturally to a multiperiod setting. Stochastic programming may be an ideal candidate to model the ALM framework for the EFA, although the pros and cons of all ALM approaches should be carefully evaluated.

[27] Prominent examples of commercial stochastic programming models are those of the Yasuda Kasai Insurance Company (256 scenarios, insurance), the Mitsubishi Trust (2,000 scenarios, pension consulting), the Swiss Bank Corporation (8,000 scenarios, pension consulting), the Daido Life Insurance Company (25,600 scenarios, life insurance), and the Banca Fideuram (asset-only model for individual investors with 10,000 scenarios, banking); see Kallberg, White, and Ziemba (1982), Kusy and Ziemba (1986), Cariño et al. (1994), and Hilli et al. (2007). Another example is the CALM stochastic programming model (Consigli and Dempster, 1998), developed for general ALM applications and analyzed in the context of portfolio management within a pension fund.

3 Reserves Management in Central Banks/Sovereign Wealth Funds

3.1 Background

Nugée (2000)[28] indicates that there are three reasons why official reserves management has a relatively common structure across various countries. First, the managed assets are owned by the public;[29] this means that security is a primary concern for reserve managers, regardless of whether the foreign assets are on the central bank's balance sheet. Second, the reserves are generally held for a purpose, and not for their own sake: the central bank has to be able to supply cash as needed to maintain monetary policy, exchange rate policy, and/or transaction needs. Third, reserves are usually large, and their potential for generating profits cannot be ignored.

Therefore, the main objectives of managing a country's foreign reserves can be summarized as follows: liquidity, security, and profit.[30] The subjugation of the third objective to the first two is commonly accepted among the central banking community. Historically, though, this has not always been the case. For example, during the 1960s and early 1970s, reserves management focused mainly on liquidity, profit was generally ignored as a strategic objective, and the cost of holding foreign reserves was seen as a policy cost and often not even measured. Towards the end of the 1970s, however, due to volatile currency and interest rate markets, central banks became increasingly aware of currency and interest rate mismatches and the need to control the resulting risks.

With more active management of reserves in the 1980s, central banks realized that the costs of holding reserves could be mitigated and profits generated. With the development of various derivatives products to manage foreign exchange and currency risks, securities houses took the opportunity to attract central banks as a new and reputable class of customers. The 1990s were perhaps less favourable to managers of central bank reserves, in terms of profitability. The markets became increasingly sophisticated, and the complexities of risk analysis, derivatives pricing, and forecasting became a significant challenge for reserve managers at central banks in view of constraints on salaries for portfolio managers, and constraints on computer support and risk-control systems.

3.2 Recent trends

Despite the challenges of managing foreign reserves, research in this area continues actively. This section outlines some trends in reserves management in the 1990s and 2000s, following Goedhuys and Pringle (1996), Rigaudy (2000), Borio, Galati, and Heath (2008), and Carver and Pringle (2009). Foreign exchange reserves holdings have been growing at an increasing rate in the past two decades, with industrialized countries accumulating reserves at a slower rate compared to emerging-market economies in Asia, Eastern Europe, and Latin America.[31]

[28] Section 3.1 follows primarily Nugée (2000).

[29] While the administrative arrangements for the ownership of foreign reserves — whether they are on the balance sheet of the central bank, treasury, debt office, or another government agency — differ by country, the duty of managing the reserves is frequently entrusted to the country's central bank.

[30] The policy governing the investment of Canada's international reserves follows these principles: the objectives for the EFA are liquidity, preservation of capital value, and, subject to these two, maximization of return (see section 1.2).

[31] For specific numbers and further details, please refer to the tables and figures in the primary sources of this section.

This growth can be attributed to factors such as accumulation of foreign reserves interest, the foreign exchange policies of central banks, and increased trade. As a result, the question of the optimal size of foreign reserve holdings has generated much interest within the central banking research community (see, for example, Ben-Bassat and Gottlieb, 1992a,b; García and Soto, 2004; Roger, 1993). Aizenman (2009) discusses the value of having large reserves due to their insurance role during a crisis and their potential to ease the recovery from a crisis.

The composition of foreign reserves has been relatively stable. U.S.-dollar holdings comprise the majority of aggregate reserve holdings, although there has been discussion about the diversification of currencies in reserves in view of recent depreciation of the U.S. dollar. The other two major currencies in aggregate reserves are the euro and the Japanese yen. Such composition is unsurprising, since the U.S. dollar, the euro, and the yen are the most liquid international currencies and, as such, satisfy most central banks' overriding concern for liquidity (Cassard and Folkerts-Landau, 2000). For the sake of comparison, as at March 2008, the distribution of liquid assets (which exclude gold and IMF Special Drawing Rights) within the EFA was about 53 per cent in euro-, 46 per cent in U.S.-dollar-, and 1 per cent in yen-denominated securities (Department of Finance Canada, 2008).

Changes to the objectives and investment policies for foreign reserves management include a progressive increase in transparency,[32] changes to the objectives and constraints of the management policies (for example, changes to credit constraints for eligible securities and proportions of securities in total foreign reserve holdings for liquidity purposes), and an increased appetite for return before a market crisis.

Shifts in the relative weight of the three objectives of foreign reserves management (liquidity, safety, and profit) have been identified. Up until the late 2000s, central banks placed increasing importance on generating a return and on the active management necessary to achieve that goal, feeling pressure from governments to maximize income from foreign reserves. Steps taken by central banks to achieve higher returns included hiring external portfolio managers (motivated by their additional expertise), diversifying portfolios and increasing the use of derivatives products for purposes of return generation/risk management, and adjusting objectives for foreign reserves management and investment policies.[33]

Prior to the global financial crisis, reserve managers assessed their potential liquidity needs and evaluated the costs of large liquid holdings in view of the trade-off between liquidity and return. Since the global financial turmoil, rapid changes have occurred in risk management, as reserve managers seek to improve liquidity and credit-risk modelling, and investment guidelines, where structured products have lost their appeal as diversification instruments. Hansen (2009) indicates that, in terms of lessons learned from the crisis, reserve managers will pay more attention to liquidity and liquidity risk. He states that the occurrence of this low-probability, high-impact event will force central banks to examine closely the performance of their reserve holdings "rain or shine."

3.3 Issues of interest

The move towards more active management further highlights the need for research to provide clarity about such issues as the optimal level of return, the right balance between safety and

[32] Canada was one of the first countries to fully meet the requirements of the International Monetary Fund's and G-10's new format for the presentation of international reserves data, reflecting the importance of increasing transparency for reserves management (De León, 2003).

[33] See Bakker and van Herpt (2007) for a discussion of the shifting focus from liquidity to a return in reserves management in the Eurosystem.

risk, proper risk metrics, and appropriate methods to measure liquidity needs in the absence of sufficient data on interventions. And, of course, the question of managing foreign reserves separately as a portfolio of foreign assets, or jointly as an asset-liability portfolio, remains open. This section discusses recent studies that address these topics, following Risk Management for Central Bank Foreign Reserves (2004) and the indicated sources. Issues on more general topics, such as the demand for foreign reserves, their management structure, and their relation to official interventions, are discussed in Bahmani-Oskooee and Brown (2002), Borio et al. (2008), Dooley, Folkerts-Landau, and Garber (2004), and Neely (2000), among others.

3.3.1 *Separate versus joint management of debt and reserves*

So far, we have discussed the asset-liability framework for foreign reserves management focusing on asset management as the primary component. However, the problem of managing foreign assets and liabilities separately or jointly applies in reverse as well: many countries issue debt denominated in foreign currency, and not all hedge the resulting exposures. Cassard and Folkerts-Landau (2000) discuss the issues related to foreign debt in the context of developing countries, indicating that governments in these countries often need to access foreign debt markets due to shortages in local savings.[34] The authors also indicate that, due to the existence of liquid domestic markets and the risks associated with large foreign currency exposures, a number of industrial countries have limited the issuance of foreign debt, either by not issuing any at all (Japan, the United States), by issuing only a small percentage of the overall domestic debt (Canada), or by issuing foreign debt to replenish foreign reserves (Denmark, Belgium, New Zealand). Sweden and Spain, who issue foreign debt, hedge currency risk using swaps or swaptions.

Hedged and unhedged borrowing in foreign currencies using the MV framework for sovereign debt management are examined by de Fontenay and Jorion (2000). The authors show that it is possible to reduce risks while maintaining similar cost levels by hedging against currency and interest rate fluctuations. In their study, the variability in foreign debt is offset by a hedge in the form of forwards, futures, or swaps. Another way of hedging foreign borrowing is to use the foreign reserves in a balance-sheet-type approach, as proposed by Nugée (2000) in the context of the United Kingdom. In particular, Nugée (2000) shows that the balance-sheet approach leads to a better evaluation of risks and returns, since the portfolio manager is able to focus on net foreign exchange reserves to assess their risk and the appropriateness of their size, composition, and maturity profile.

A major advantage of the balance-sheet formulation is that the government (debt office, treasury, etc.) can evaluate the cost of holding reserves, and particularly net reserves, which may provide additional insight regarding the optimal size of net or gross reserves. Boertje and van der Hoorn (2004) present a balance-sheet approach to managing market risk from a central bank perspective, specifically measuring the two financial strengths of a central bank: its ability to absorb losses and its capacity to generate profits. Similarly, Gray (2007) proposes modelling risk exposures on a comprehensive economic balance sheet using contingent claims analysis and value-at-risk techniques, arguing that the former can reveal how risks are transferred across sectors and how they accumulate in the overall balance sheet, and that the latter can be used to evaluate the adequacy and risk-reduction benefits of holding liquid foreign currency reserves.

Regarding the management of foreign reserves in Denmark, Hansen (2000) points out that

[34] Cassard and Folkerts-Landau (2000) state that derivative instruments are available to most emerging markets to hedge currency risk, and that numerous studies show that investors can lower their risks by completely hedging their exposure to exchange rates without significantly affecting their return profiles.

Danish net foreign borrowing is specifically targeted to ensure adequate foreign exchange reserves (used for interventions to support the Danish krone). This setting lends itself to the coordinated management of Danish foreign debt and foreign reserves, and promotes integrated management of assets and liabilities. Hansen states that, while there is no direct coordination between the assets (on the balance sheet of Danmarks Nationalbank) and the liabilities (on the balance sheet of the Kingdom of Denmark), the short duration (about two years) of both marginal assets and liabilities ensures that there is no significant interest rate risk for either the government or the bank, or for both taken together. Australia also strives for a duration of about 30 months for their foreign reserves (Battellino, 2000), albeit for a different reason.[35]

In contrast to researchers who argue for a joint asset-liability management approach to foreign reserves (or debt), de Montpellier (2000) advocates managing assets and liabilities separately in the context of Belgium's public debt and foreign reserves. The reason for this is the different nature of asset and liability cash flows, resulting in incomparable sensitivity to financial variables and therefore difficulties in assessing the overall zero-risk position for the entire portfolio. Nevertheless, de Montpellier recognizes the benefits of an ALM-type framework for foreign debt and foreign reserves.

Overall, though, the vast majority of central banks see the benefits of managing assets and liabilities jointly, despite the possibly different characteristics of each. In the summer of 2002, Pringle and Carver (2003) surveyed managers of central bank reserves; the reserves of participating banks made up about half of the world's total external reserves. To the question of whether it is desirable to manage external liabilities alongside assets, 80 per cent of the respondents replied yes. Pringle and Carver recognize, however, some of the problems that may arise from managing assets and liabilities together on the national balance sheet; these include large fluctuations in the values reported, issues of profit retention, and the independence of the central bank. Taking the issue of managing assets and liabilities jointly even further, Dwyer and Nugée (2004) argue in favour of a global framework of risk management for a central bank: the so-called 'whole enterprise' risk management.

3.3.2 *Specifying objectives*

The issue of translating general policy guidelines into quantifiable investment rules and constraints for an asset-liability model is of particular concern for central bankers. Cardon and Coche (2004) introduce a 3-step process for strategic asset allocation, discussing how policy and institutional requirements should be translated into specific guidelines for investment and the optimal long-term risk/return profile. Putnam (2004) differentiates between investment objectives and investment guidelines, and shows that some of the common investment guidelines allow the existence of strategies that work against the central bank's long-term investment objective. Some researchers recommend specific approaches to incorporate investment guidelines into a mathematical modelling framework. For example, Coche et al. (2006) evaluate foreign reserves management in the context of an explicit policy objective: to prevent the undervaluation of the country's currency. Claessens and Kreuser (2004) show how to incorporate both macro (monetary and foreign exchange) and micro (portfolio benchmarks, evaluation of investment managers) policy objectives into an optimization model for foreign reserves management.

Gintschel and Scherer (2004) propose to model the currency allocation decision in a multi-objective framework, making explicit trade-offs between the standard reserves objectives of preserving wealth and maintaining liquidity. Their model also accounts for political constraints

[35] The logic is that a shorter duration could be achieved through repo markets, while a longer duration is not desirable, since risks of negative returns on foreign assets increase substantially.

in the decision-making process. Remolona and Schrijvers (2004) evaluate investment strategies with different objectives (duration, default risk, diversification with high-yield bonds) and discuss the issues arising in each situation. Finally, Fisher and Lie (2004) argue against exogenous restrictions of the investment universe; instead, they propose an objective function where portfolio returns should be maximized given a risk target, subject to liquidity, credit, and currency constraints.[36]

3.3.3 *Active management and diversification*

There are some central banks whose staff are allowed, or even have the mandate, to manage the public portfolios (debt/reserves) actively to generate cost savings/returns. For example, Rådstam (2000) discusses the cost savings resulting from the active management of Swedish foreign currency debt and illustrates that the Swedish National Debt Office is able to achieve higher savings than external managers due to the aggressive strategies of the former in the 1990s. Many central banks do not pursue aggressive positions to achieve a lower cost of debt or higher returns. For example, in the context of foreign reserves, the staff at the central banks in Canada and Australia (see Battellino, 2000, for a discussion of the latter) have a small amount of discretion when it comes to everyday trading for reserves; in both countries, foreign reserves are held for intervention purposes and liquidity is the primary consideration.

Here we should stress again that taking on calculated risk is not necessarily bad, just as being very risk-averse is not necessarily good. Sagun and Leiberton (2008) propose metrics to evaluate the activeness of sovereign portfolios and indicate that the concept of 'active management' suffers from a number of biases. Hansen, Olgaard, and Jensen (2003) discuss the reasons for undertaking certain financial risks; an example is the willingness of the Danish central bank to carry interest rate risk to achieve higher expected yields. Hansen, Olgaard, and Jensen indicate that reserves are diversified by currency (the euro, dollar, sterling, and yen) to reduce foreign exchange and liquidity risks, but point out that Danmarks Nationalbank does not undertake credit risk to generate returns on reserves. In contrast, the returns of the EFA in Canada are generated from credit spreads, not interest rate or currency positions.

The benefits of the additional credit risk that results from introducing equities into excess returns are investigated by de Beaufort, Berkelaar, and Petre (2008). The authors show that, for those countries willing to carry a portion of equities in their reserves portfolio (Switzerland and Korea, for example), there is room for cost savings. To further illustrate the benefits of diversification, Weinberger and Golub (2007) show that, in an MV framework, a wider array of asset classes (for example, emerging-market equity, real estate, and global bonds) leads to gains in expected returns for the same levels of risk, compared to the traditional portfolio of bonds and equities only. Lee (2007) builds on Weinberger and Golub's analysis by focusing on tactical asset-allocation techniques over short- and medium-term horizons.

Ferket and Zwanenburg (2004) compare risk-return characteristics of popular asset classes in the private industry (long-term and global bonds, high-yield bonds, and equities) to those of a lowest risk portfolio (cash), and suggest several diversification strategies that may have attractive risk-return trade-offs for central bankers. Grava (2004) examines diversification towards highly rated investment-grade corporate bonds, and finds that adding spread risk leads to better risk/return profiles than increasing the duration of the portfolio. He also indicates that, if reserve managers are willing to tolerate short-term underperformance, they may generate significantly higher returns in the long run. Other researchers (such as Gmuer and Cavegn,

[36] Interestingly, Fisher and Lie's (2004) analysis shows that a typical central bank can significantly increase the efficiency of its reserves portfolio by following the suggested methodology.

2003; De León, 2003) discuss the benefits of diversification in a reserves portfolio that result from the expansion of permissible investment classes.

3.3.4 *External managers*

As mentioned, one of the ways to enhance the generation of returns from foreign reserves is to hire external managers. Gmuer and Cavegn (2003), speaking of the Swiss National Bank, cite the following reasons for hiring external managers: to diversify currency reserves to enhance risk-adjusted returns on assets; to ensure that the central bank always has access to the latest state-of-the-art investment processes in the industry, and thus is forced to review external investment procedures; and to test the operational issues resulting from a multi-manager program in place.[37] Mohohlo (2008) discusses the general guidelines for selecting and managing external fund managers for central banks.

3.3.5 *Tactical asset-liability management*

Once the strategic (target) composition of the portfolio has been established, foreign reserves management staff are responsible for the tactical management of the fund on a daily basis to keep the portfolio on target. The risk metrics discussed in section 2.3.2 may apply to strategic planning and/or tactical decisions. Cassard and Folkerts-Landau (2000) discuss tactical issues such as operating within allowable margins for active management purposes, using derivatives to hedge currency and interest rate risks, and managing credit risk (not only at the strategic level by constraints on issuers, but also at the tactical level from entering into over-the-counter transactions, for example). The value-at-risk metric comes up frequently as the risk measure of choice among risk-control staff in central banks (in addition to Cassard and Folkerts-Landau, see, for instance, Rigaudy, 2000, and Rådstam, 2000).

3.3.6 *Accounting*

Finally, accounting is an issue for reserve managers as well. Rigaudy (2000) points out that accrual versus mark-to-market accounting methods may not matter much if the average duration of a portfolio is kept short (under one year), but they become much more of a problem if assets are invested in the long end of the yield curve, where capital gains and losses can be significant. Rigaudy indicates that, while mark-to-market accounting may generate higher volatility in the yearly profit-loss accounts due to market fluctuations (and this is usually regarded as undesirable), actuarial (book-cost) accounting may create profits and losses not related to market movements, and may interfere with the management of the portfolio. Kurtzig, Hemus, and Goodwin (2003) discuss the issue of accounting in foreign reserves management, and how this pertains to the trend towards greater transparency in disclosing data on foreign exchange reserves.

4 Foreign Reserves Management in the United Kingdom

4.1 General information about the Exchange Equalisation Account

The Exchange Equalisation Account (EEA) and the U.K. reserve tranche position at the International Monetary Fund (IMF) comprise the United Kingdom's official holdings of international

[37] The program was put in place at the time for part of the portfolio, but Gmuer and Cavegn (2003) could not comment about its costs and benefits, since the program was in place for three years before evaluation.

reserves.[38] Like the EFA, the EEA holds gold; foreign currency assets, denominated in euros, yen, and U.S. dollars; and IMF Special Drawing Rights. The Bank of England acts as agent for Her Majesty's Treasury (HMT) in day-to-day management of the EEA, such as executing foreign exchange transactions and investing reserves according to a framework set out in a service level agreement. The Bank of England manages both assets and liabilities issued to fund the EEA assets, held in the National Loans Fund (NLF).

The purpose of the EEA is very similar to that of the EFA in Canada: the EEA is held on a precautionary basis — to meet any potential liquidity needs, should the United Kingdom choose to intervene in the foreign exchange markets (it has not intervened for the purposes of influencing the sterling exchange rate since 1992). While the EEA is also used to provide foreign currency services for government departments and agencies, for foreign payments, and payments related to IMF membership, the prime objective for managing the reserves is to meet current policy objectives and any potential future changes in policy.

The U.K. official reserves are divided into two parts: the *hedged* reserves (about 17 billion pounds sterling as of 2008), and the remaining *unhedged* reserves (about 14 billion pounds sterling as of 2008). The unhedged reserves comprise U.S.-dollar- and euro-denominated bonds, gold, the reserve tranche position, and forward yen purchases; they are financed primarily using sterling through accumulated retained earnings and historic sterling financing. The benchmark foreign currency allocation for the unhedged reserves is 40 per cent U.S. dollars, 40 per cent euros, and 20 per cent yen. This benchmark is structural and it reflects HMT's investment policy. The liquidity model, discussed in section 4.2, is not used to determine a strategic benchmark allocation for unhedged reserves.

The hedged reserves, as their name suggests, are hedged for currency and interest rate risk; they comprise eligible U.S.-dollar-, euro-, and yen-denominated bonds. These assets are hedged against foreign exchange risk because either the liabilities funding them are denominated in the same currency or the liabilities are currency-swapped, and interest rate swaps are used to hedge against interest rate risk. The hedged reserves are financed in two ways: either by issuing foreign currency securities (with proceeds sold for sterling to the EEA via the NLF), or directly using sterling liabilities. The primary determinant for the funding method is cost: the cheapest way of financing EEA assets is determined by comparing, on a swapped basis, the estimated costs of issuing in domestic versus foreign currencies. For example, based on interest and exchange rates, the United Kingdom's reserves are currently funded by majority swapped domestic borrowing: for the fiscal year 2007/08, there was swapped borrowing of 10 billion pounds sterling versus outstanding foreign currency securities of 1.5 billion pounds sterling.

4.2 The liquidity model

As mentioned in section 1.3, there is a difference between strategic and tactical models for reserves management. The tactical management relies on a balance-sheet-type framework, discussed in section 3.3, with liabilities defined as cost of funding, and appropriate risk metrics used to monitor and control risk exposures (see Her Majesty's Treasury, 2008, for more details). The strategic liquidity model examines the optimal benchmark allocation of hedged assets in the EEA portfolio. As such, the liquidity model defines assets, liabilities, and risks according to the strategic objectives, which are potential interventions in the foreign exchange market.

[38] Sections about foreign reserves management in the United Kingdom and the liquidity model are based on Her Majesty's Treasury (2008) and on the information provided on the websites of the Bank of England and Her Majesty's Treasury.

The objective of the model is to balance the return on, and the cost of holding, the hedged EEA assets: to find an optimal trade-off between maximizing expected returns and minimizing expected liquidation costs. The optimal asset allocation, produced by the model, is across broad asset classes. As with the EFA, credit constraints are imposed on EEA assets, and the portfolio typically holds sovereign bonds with low credit risk. In order to account for settlement terms (that some bond transactions can be settled, at the earliest, in two days, for example), the EEA is constrained to hold a minimum amount of zero-day settlement bonds (U.S. treasuries and agencies).

The assets in the model are the swapped yields of euro-, yen-, and U.S.-dollar-denominated bonds, while the liabilities are defined as expected liquidation costs resulting from a potential call on the EEA assets for intervention purposes. If there are no liquidation calls, the benchmark portfolio earns the asset-swap spread. In an asset swap, coupons on a bond are exchanged for the London Interbank Offered Rate plus a spread, which can be positive or negative (Hull, 2002). For modelling purposes, it is assumed that asset-swap spreads are driven by liquidity; if no liquidation calls are made, the EEA turns into a buy-and-hold portfolio. The model determines the optimal allocations of reserve assets for investment and potential liquidation in view of their expected returns and liquidation costs, providing useful insight for investment and management decisions.

5 Foreign Reserves Management in Canada

The liquidity model seems to provide a good starting point for the development of a strategic asset-liability framework for the EFA, since there are so many similarities between the U.K. and Canadian objectives, risks, and foreign reserves structure. Before discussing some ideas for an ALM model for Canadian reserves (section 6), let us briefly examine the main historical developments related to the EFA, following De León (2003). These developments are helpful to understand the EFA's current structure and operations, outlined in section 1.2; the reasons why certain policies, constraints, and management practices were established; and avenues for future research (discussed in section 6).

5.1 A point-form summary of EFA developments since the 1970s

Traditionally, foreign reserves were used for interventions in the foreign exchange market for the Canadian dollar. Over the years, the use of interventions and their frequency changed. Current policy, as described in a Department of Finance Canada (2008) report, is as follows:

> Intervention in the foreign exchange market for the Canadian dollar might be considered if there were signs of a serious near-term market breakdown (e.g. extreme price volatility with both buyers and sellers increasingly unwilling to transact), indicating a severe lack of liquidity in the Canadian-dollar market. It might also be considered if extreme currency movements seriously threatened the conditions that support sustainable long-term growth of the Canadian economy. The goal would be to help stabilize the currency and to signal a commitment to back up the intervention with further policy actions, as necessary. Since September 1998 the Bank of Canada, acting as agent for the Government, has not undertaken any foreign exchange market intervention in the form of either purchases or sales of US dollars versus the Canadian dollar.

- The level of foreign reserves increased from about 2 to 3 billion U.S. dollars in the 1970s and early 1980s to about 43 billion U.S. dollars in 2008.
- A number of changes took place in reserves management practices, reflecting financial markets developments, changing approaches to foreign exchange intervention, and government policies.
- 1970s and early 1980s: The level of reserves was relatively low, and the funds were used primarily to support and calm a depreciating currency market. The majority of funding and almost all investments were made in short-term U.S.-dollar-denominated assets. Liquidity and capital preservation were the overriding objectives and the portfolio was managed under very rigid constraints.
- Late 1980s: The level of foreign reserves started growing. Greater emphasis was placed on enhancing investment returns, and a fixed portion of the reserves was invested in Japanese yen and German marks.
- Early 1990s and restructuring following the 1999 review of the EFA: Changes were made to permit a broader range of investment alternatives. The flexibility helped to reduce the cost of carry of the reserves.
- Mid-1990s: A tiered approach to EFA investments was implemented, where the first tier was dedicated to meet core liquidity needs, the second was used to match outstanding liabilities maturing within a year, and the remaining assets were managed to maximize investment returns within strict guidelines. This served as the first step for the EFA's current asset-liability framework.
- 1997: An asset-liability framework was introduced for the tactical management of foreign reserves, motivated in part by past successes in immunizing some of the portfolio's liabilities. The goal of the framework is to minimize currency and interest rate risks by matching assets and liabilities.
- early 2000s: The funding mix changed towards swapped domestic borrowing, due to cost savings from issuing in domestic markets.
- 2000s: The composition of EFA holdings shifted to include more euro holdings, to reflect funding advantages in euro markets.

6 Concluding Remarks and Future Direction

To summarize, the EFA (and the entire debt portfolio) is on the balance sheet of the Government of Canada, and the Bank of Canada acts as the government's agent. The assets in the EFA are actively managed to take advantage of credit spreads, while the liabilities are managed passively. The assets are matched against the liabilities funding them in terms of currency and maturity to minimize exchange and interest rate risks. The fund is closely monitored for risk, and a number of risk measures are reported. The need exists for a strategic model to help guide policy decisions regarding the joint management of the EFA's assets and liabilities.

In light of our review of asset-liability management strategies and modelling frameworks, two research directions appear particularly important at this early stage of developing a strategic ALM model for Canadian reserves. The first direction is to choose an objective function, and the second is to select a suitable modelling framework. To specify an objective function for the strategic EFA model, we should determine how to define liabilities associated with the assets; in view of the recent market crisis and focus on liquidity, the Bank of England's approach of viewing liabilities as expected liquidation costs in case of a potential intervention seems particularly well suited. We should investigate which risk metrics are most appropriate for capturing the risks of greatest concern to the EFA. We should also consider how to account for the various explicit and implicit constraints on, and the cost of carry of, the reserve assets.

Regarding the second research direction, the selection of a modelling framework, it seems that stochastic programming is intuitively appealing and practically useful for a strategic ALM model. Using this approach, one is able to group uncertainty and examine outcomes under a few possible economic states in the future (for example, normal, volatile, and crisis), which, one may argue, are the natural scenarios to consider in portfolio management. This modelling structure is used by Claessens and Kreuser (2004, 2007) to develop a framework for strategic foreign reserves management; the authors also document the benefits of a tree structure for modelling uncertainty. The stochastic framework also accommodates portfolio rebalancing and multiperiod investment decisions, and allows for the specification of multiple or competing goals, which may be necessary to capture the complex objectives of foreign reserves management.

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