BANK OF CANADA
BANQUE DU CANADA
CELEBRATING 75 YEARS
CÉLÉBRONS 75 ANS

Discussion Paper/Document d'analyse
2010-8

Inventories, Stockouts, and ToTEM

by Oleksiy Kryvtsov and Yang Zhang

www.bank-banque-canada.ca

Bank of Canada Discussion Paper 2010-8

July 2010

Inventories, Stockouts, and ToTEM

by

Oleksiy Kryvtsov and Yang Zhang

Canadian Economic Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
okryvtsov@bankofcanada.ca

ISSN 1914-0568

Acknowledgements

We thank Stephen Murchison and Rhys Mendes for useful comments, and Glen Keenleyside and Jill Ainsworth for their editorial help with the paper.

Abstract

Inventory investment is an important component of the Canadian business cycle. Despite its small average size – less than 1 per cent of output – it exhibits volatile procyclical fluctuations, accounting for almost one-third of output variance. Procyclicality of inventories is somewhat smaller than that of sales, resulting in a counter-cyclical aggregate inventory-sales ratio. These salient inventory facts are matched in a partial-equilibrium version of Kryvtsov and Midrigan's (2010) model in which firms hold stocks of goods to buffer against stockouts. In booms, firms boost their inventories to avoid stocking out due to the rise in demand. The model combines the real marginal cost estimated by ToTEM with the convex cost of adjusting inventories to match the dynamics of the inventory-sales ratio in the data.

JEL classification: E31, E32
Bank classification: Business fluctuations and cycles; Transmission of monetary policy

Résumé

Les investissements en stocks forment un volet important du cycle économique au Canada. Malgré leur taille généralement modeste – ceux-ci représentent moins de 1 % de la production –, ils fluctuent fortement de façon procyclique et expliquent presque le tiers de la variance de la production. Les stocks ont un degré de procyclicité un peu moins marqué que les ventes, si bien que le ratio global des stocks aux ventes affiche un caractère contracyclique. Les auteurs reproduisent ces faits saillants dans une variante à équilibre partiel du modèle de Kryvtsov et Midrigan (2010), où des entreprises gardent des marchandises en réserve pour se prémunir contre les ruptures de stocks. En périodes d'expansion, ces firmes augmentent leurs stocks afin de ne pas subir de ruptures de stocks à cause de l'accroissement de la demande. Pour restituer la dynamique du ratio stocks/ventes illustrée dans les données, le modèle associe le coût marginal réel estimé par TOTEM et le coût d'ajustement convexe des stocks.

Classification JEL : E31, E32
Classification de la Banque : Cycles et fluctuations économiques; Transmission de la politique monétaire

1 Introduction

Inventory investment dynamics is an inherent part of business cycle fluctuations.[1] Despite its small average size, typically between half and 1 per cent of output, inventory investment is highly volatile, leading to fluctuations that are comparable in size to those in output itself. But what makes inventory investment important for business cycles is its co-movement with the rest of the economy. First, inventories are highly procyclical. This fact is consistent with a view that inventory dynamics may amplify the effect of demand disturbances on output. Procyclicality combined with high volatility of inventories accounts for almost one-third of output variance. Second, inventories are less procyclical than sales. As noted by Bils and Kahn (2000), this fact is indicative of the sources of business cycle fluctuations, in particular that demand shocks are important in driving the business cycles.

Models of inventory investment have evolved from the partial-equilibrium linear-quadratic models of Holt et al. (1960), which are able to match selected moments of inventory behaviour, to general-equilibrium business cycle models that match most inventory moments in addition to other standard business cycle moments. Only recent models of inventories have been able to match a combination of a procyclical inventory investment and a counter-cyclical inventory-sales ratio in a general-equilibrium framework. Procyclicality is typically achieved by firms adjusting inventories to facilitate their production or sales, economizing on the fixed cost of adjusting their stocks, or self-insuring against the incidence of stockouts.[2] In turn, the counter-cyclicality of the inventory-sales ratio stems from the strong procyclicality of the real interest rate (Khan and Thomas 2007a), counter-cyclical average markups (Bils and Kahn 2000), or the convex adjustment cost of inventories (Jung and Yun 2005).

In this paper, we develop a partial-equilibrium structural model of inventory adjustment that is based on real marginal cost and its expected growth estimated by the Bank of Canada's Terms-of-Trade Economic Model (ToTEM). Our model is an adaptation of models in Bils and Kahn (2000), Wen (2008), and Kryvtsov and Midrigan (2010), where firms hold

[1]See Blinder and Maccini (1991) and Ramey and West (1999) for reviews of inventory behaviour over the business cycle.

[2]Kydland and Prescott (1982), Jung and Yun (2005), and Iacoviello, Schiantarelli, and Schuh (2007) assume that inventory stocks affect demand for goods or are a production factor. Caplin (1985) and Khan and Thomas (2007a) emphasize non-convexities in the cost of acquiring or producing inventories. Kahn (1992), Bils and Kahn (2000), Khan and Thomas (2007b), and Wen (2008) study economies with a stockout-avoidance motive.

stocks of inventory to buffer against stockouts. In booms, firms boost their inventories to avoid stocking out due to the rise in demand. The model predicts that firms should hold more stock (relative to sales) when the markup is high, real marginal cost is expected to grow, or the real interest rate is low. The model is able to capture all of the salient business cycle facts for inventory behaviour in Canada during the inflation-targeting period.[3] The main determinant of inventory behaviour in the model is the expected growth of real marginal cost combined with the large cost of inventory adjustment.[4]

The only study of inventory behaviour in Canada that we are aware of is by Chacra and Kichian (2004). They estimate an error-correction model of short-run inventory investment behaviour. Our approach differs from theirs mainly in that we provide a structural interpretation of fluctuations in inventories that are based on observed aggregate time series for sales and real interest rates, as well as the estimated current and expected future real marginal costs. It is our view that this model should be used to provide tractable scenarios for the 1- to 2-year-ahead behaviour of inventories given ToTEM's forecasts of other pertinent variables over the same horizon. In this sense, our model is a complement to empirical models, such as that estimated by Chacra and Kichian (2004), which are designed for short-term forecasts.

The rest of this paper is organized as follows. Section 2 reviews the main facts of inventory behaviour in Canada. Section 3 describes the partial-equilibrium model of inventory adjustment. Section 4 provides the analysis of the model and discusses its advantages and limitations. Section 5 offers some conclusions.

2 Inventory Behaviour in Canada

In this section, we document several prominent facts regarding the cyclical behaviour of aggregate inventories in Canada. We employ two datasets: Statistics Canada's Quarterly Survey of Financial Statistics for Enterprises data on sales and inventories for manufacturing, wholesale, and retail from 1988Q1 to 2008Q1; and Statistics Canada's Monthly Survey of

[3]Secular improvement in inventory management is one of the theories behind a decrease in business cycle volatility, also called the Great Moderation; see Chacra and Kichian (2004) for Canada, and McConnell and Perez-Quiros (2000) for the United States. Since we focus on the relatively stable inflation period in Canada, we abstract from this discussion.

[4]Kryvtsov and Midrigan (2009, 2010) point out that New Keynesian models of business cycles that rely heavily on real rigidities to generate strong monetary non-neutrality predict that real marginal cost is growing in booms. This feature creates a very strong motive to accumulate inventories during expansions, and makes it difficult for these models to generate a counter-cyclical inventory-sales ratio.

Manufacturing data on shipments and inventories (by stage of fabrication) for manufacturing industries from January 1992 to April 2008.[5] Although we focus on a subset of the Canadian economy, our data account for most of Canada's aggregate inventory stock. Manufacturing and trade inventories (value added) comprise 85 per cent (74 per cent) of the total private non-farm inventory stock (value added); the remaining industries are mining, utilities, and construction.

Output is the sum of sales (shipments, in the Monthly Survey of Manufacturing) and the change in the end-of-period inventory stock. The inventory-to-sales ratio is defined as the ratio of the end-of-period inventory stock to sales in that period. For inventories by stage of fabrication, output and sales are the totals for manufacturing. Nominal variables for manufacturing and trade are deflated by the GDP deflator: for trade by core CPI, and for manufacturing by industrial producer price indexes (PPI). All data are seasonally adjusted and HP-filtered. Output, sales, and inventory-sales ratios are defined in per cent deviations from respective HP trends. Inventory investment is defined as a fraction of output, and we report it in percentage point deviations from its HP trend. Finally, to rule out breaks in time series, we restrict our analysis to the inflation-targeting period in Canada by starting both data samples in 1993.

Table 1 provides moments for aggregate sales, output, and change in inventories. There are several main facts to note regarding the behaviour of the change in inventories over the business cycle. First, the change in inventories is very small: on average, around 0.5 per cent of the output level. In contrast, the volatility of the change in inventories is very large: the standard deviation is 0.70 percentage points of output for manufacturing and trade, and 1.24 percentage points for retail. Hence, despite being very small on average, inventory investment is very volatile, exhibiting a standard deviation that is around one-third of the standard deviation of output. In addition to high volatility, inventories exhibit significant procyclicality: the correlation with output ranges between 0.27 and 0.56 for manufacturing and trade and their major sectors, except for finished-good inventories in manufacturing, for which this correlation is around zero. These three prominent facts for inventories have

[5] The surveys collect inventory and sales data based on potentially different accounting methods by firms. To our knowledge, most Canadian firms (except oil producers) use first-in, first-out. The data from the national income and expenditure accounts (NIEA) are more limited, but are based on an inventory-valuation adjustment to revalue inventory holdings to replacement cost. For the data that are available in NIEA, we will cross-check the facts with those we obtained using survey data.

been well documented in the business cycle literature[6] and are robust for both monthly and quarterly frequencies, for goods with a different durability or stage of fabrication in our data. The importance of inventories as a business cycle component can be summarized by the output variance: 27.5 per cent for manufacturing and trade, ranging from 20.7 per cent for manufacturing (quarterly data) and 15.6 per cent for durables (monthly data) to 29.2 per cent for retail (quarterly) and 30.6 per cent for non-durables (monthly).

An additional set of facts concerns the dynamics of the inventory-to-sales ratio. Bils and Kahn (2000) emphasize that the behaviour of the inventory-to-sales ratio is directly related to the dynamics of marginal costs and markups over the business cycle, and hence that it sheds light on the nature of business cycle fluctuations. Table 2 reports the moments for the aggregate inventory-to-sales ratio in the Canadian data. On average, firms hold an inventory stock of around half of the quarterly sales. The ratio indicates a volatility similar to that for output. It is quite persistent: the serial correlation is 0.56 for manufacturing and trade at a quarterly frequency. Finally, the inventory-to-sales ratio is strongly counter-cyclical: its correlation with output ranges between -0.43 and -0.74, and with sales from -0.53 to -0.81.[7] The counter-cyclicality of the inventory-to-sales ratio is also well documented; for example, for the United States see Bils and Kahn (2000), Jung and Yun (2005), and Kryvtsov and Midrigan (2009, 2010).

Kryvtsov and Midrigan argue that, in addition to counter-cyclicality, measuring the elasticity of the ratio with respect to output is important for gauging the flexibility of the real marginal cost to output over the business cycle. For the U.S. data, Kryvtsov and Midrigan's measure of elasticity is around -0.8, which leads them to conclude that the aggregate real marginal cost is more flexible than is commonly found in the business cycle literature.[8] Table 3 reports the elasticities of the inventory-to-sales ratios in manufacturing and trade and their major sectors. For manufacturing and trade, the elasticity with respect to output is -0.51 (with respect to sales, it is -0.61). In manufacturing, the elasticity is lowest for non-durables, -0.34 (-0.44), and the highest for finished goods, -0.89 (-0.90). Hence, our elasticity estimates

[6] See Blinder and Maccini (1991) and Ramey and West (1999).

[7] Counter-cyclical inventory-to-sales ratios and procyclical inventory investment imply that inventory stock is less procyclical than sales.

[8] Kryvtsov and Midrigan's analysis implies that the elasticity of real marginal cost to output approximately equals the inverse of the elasticity of intertemporal substitution in order to account for the counter-cyclical inventory-to-sales ratio in the data.

are similar to those found in Kryvtsov and Midrigan (2009). The estimates are also fairly robust across major sectors, which is consistent with our treatment of inventories in the model as those of one homogeneous output good.

3 Simple Model with Inventories

There are five production sectors in ToTEM, for consumption, investment, government, import, and export goods. In this paper, we focus on production sectors that hold a significant amount of inventories and produce and sell domestically. Thus we are left with the consumption and investment sectors, indexed by $j \in \{C, I\}$.[9] Each sector consists of a continuum of monopolistically competitive firms, indexed by i. Fraction γ^j of these firms in sector j produces storable finished goods and accumulates finished-good inventories. This section describes the problem of firms that accumulate inventories.

Let $S_t^j(i)$ denote the demand that firm i in sector j faces in period t. We will assume that the demand function takes the following form:

$$S_t^j(i) = S_t^j \left(\frac{P_t^j(i)}{P_t^j} \right)^{-\theta} \Psi \left(\frac{Z_t^j(i)}{S_t^j} \left(\frac{P_t^j(i)}{P_t^j} \right)^{\theta} ; \sigma \right), \tag{1}$$

where Ψ is a function that characterizes the effect of inventories on final sales given the firm's pre-sale stock of finished-good inventories $Z_t^j(i)$ relative to the average sales in the sector $S_t^j \left(\frac{P_t^j(i)}{P_t^j} \right)^{-\theta}$. We will assume that Ψ's derivative is positive, $\Psi' > 0$; that is, ceteris paribus, higher stock relative to sales is associated with higher sales. In this case, demand specification (1) introduces a motive for firms to hold a non-zero stock of inventories; parameter σ pins down the value of an extra unit of stock. Modelling the inventory-holding motive in this way has two advantages: (i) it nests two widely used motives for holding inventories: the sales accelerator motive as in Bils and Kahn (2000) and the stock-avoidance motive as in Kryvtsov and Midrigan (2009, 2010); and (ii) it has been shown to be successful in replicating the

[9] While the exclusion of the government sector seems reasonable, the exclusion of the foreign trade sector may not be innocuous for the case of Canada. Modelling inventory behaviour in a foreign trade sector would require taking into account the effects of international factors (exchange rate movements, global supply disturbances, etc.) on the domestic supply of goods. We leave this work for future research. Iacoviello, Schiantarelli, and Schuh (2007) make this assumption for the case of the United States.

stylized inventory facts outlined in the previous section.

Let MC_t^j denote firms' nominal marginal costs in sector j in period t. It is assumed that firms buy labour, capital, and material inputs in the competitive markets and use them to produce finished goods according to a constant-returns-to-scale production technology. In this case, firm i's total variable costs equal $MC_t^j Y_t^j(i)$, where $Y_t^j(i)$ is firm i's output level in period t. Denote Ξ_t to be the vector of marginal utilities of consumption across states in period t. The firm's optimization problem consists of choosing sequences of sale prices $\left\{P_t^j(i)\right\}$ and pre-sale stocks $\left\{Z_t^j(i)\right\}$ to maximize the present discounted sum of period profits:

$$E_t \sum_{s=0}^{\infty} \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left[P_{t+s}^j(i) S_{t+s}^j(i) - MC_{t+s}^j Y_{t+s}^j(i)\right],$$

subject to (1), sticky-price constraints on $P_{t+s}^j(i)$, and the law of motion for the pre-sale level of stock:

$$Z_t^j(i) = (1-\delta^j)\left(Z_{t-1}^j(i) - S_{t-1}^j(i)\right) + Y_t^j(i) - \frac{\eta}{2}\left(v_t^j(i) - v_*^j\right)^2 \left(v_*^j\right)^{-1},$$

where $v_t^j(i) = \frac{Z_t^j(i)}{S_t^j}\left(\frac{P_t^j(i)}{P_t^j}\right)^{\theta}$ is the target (pre-sale) inventory-sales ratio for firm i in sector j, v_*^j its mean across time, and δ^j the stock depreciation rate in sector j. The last term represents the quadratic costs of stock adjustment that punish deviations in the inventory-sales ratio from its mean.[10]

This problem yields the following optimality condition for optimal stock holdings:

$$\Psi_t'^j(i) = \frac{MC_t^j(i)\left[1 + \eta\left(\frac{Z_{t+s}^j(i)}{S_{t+s}^j}\left(\frac{P_t^j(i)}{P_{t+s}^j}\right)^{\theta} - v_*^j\right)\right] - (1-\delta^j) E_t \frac{\beta \Xi_{t+1} P_t}{\Xi_t P_{t+1}} MC_{t+1}^j(i)}{P_t^j(i) - (1-\delta^j) E_t \frac{\beta \Xi_{t+1} P_t}{\Xi_t P_{t+1}} MC_{t+1}^j(i)}. \quad (2)$$

The left-hand side is an increasing function of the firm's stock-to-sales ratio. Equation (2) shows that the marginal cost of producing and adjusting an additional unit of inventories, $MC_t^j(i)\left[1 + \eta\left(\frac{Z_{t+s}^j(i)}{S_{t+s}^j}\left(\frac{P_t^j(i)}{P_{t+s}^j}\right)^{\theta} - v_*^j\right)\right] - (1-\delta^j) E_t \frac{\beta \Xi_{t+1} P_t}{\Xi_t P_{t+1}} MC_{t+1}^j(i)$, must equal the marginal

[10] Jung and Yun (2005) show that adding an adjustment cost of this form is crucial for matching countercyclical inventory-sales ratio responses after a nominal demand shock.

benefit: the profits made from an additional unit sold, $P_t^j(i) - (1-\delta^j)E_t \frac{\beta \Xi_{t+1} P_t}{\Xi_t P_{t+1}} MC_{t+1}^j(i)$, multiplied by the number of extra units of stock needed to generate an extra unit of sales.

In the partial-equilibrium model we ignore the effect of inventories on pricing decisions, and so we omit a discussion of the optimal pricing equation.

There are two aggregate equations in the model. The first is for sectoral price P_t^j:

$$P_t^{j\ 1-\theta} = \int_0^1 P_t^{j\ 1-\theta}(i)\, \Phi(x_t^j(i)) di,$$

where $\Phi(x_t^j(i))$ are new weights that take into account the effect of stocks on households' valuations of consumption goods; see the appendix in Kryvtsov and Midrigan (2010).

The second aggregate equation is for sectoral output Y_t^j:

$$Y_t^j = S_t^j + INV_t^j$$

where INV_t^j, is investment in inventories:

$$INV_t^j = \int \left[Z_t^j(i) - S_t^j(i) - (1-\delta)\left(Z_{t-1}^j(i) - S_{t-1}^j(i) \right) \right] di.$$

There are two functional forms of $\Psi_t^j(\cdot)$ corresponding to common motives for inventory holdings: a sales accelerator and stockout avoidance.

3.1 Sales accelerator

In this specification, a positive stock of finished-good inventories increases a firm's ability to generate sales.[11] Specifically, it is assumed that the elasticity of substitution between a unit of inventories and a unit of sales is constant, σ :

$$\Psi(x) = \left[1 + x^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

[11] See Bils and Kahn (2000) and Jung and Yun (2005).

which implies that

$$\Psi'(x) = \left(\frac{x}{\Psi(x)}\right)^{\frac{-1}{\sigma}}.$$

3.2 Stockout avoidance

This specification corresponds to a model in which firms hold inventories to limit the number of stockouts due to unexpectedly high demand.[12] Kryvtsov and Midrigan (2009) show that, in this case,

$$\Psi(x) = \int_{\nu_{\min}}^{\nu_{\max}} \min(v, x) dF(\nu),$$

where ν is a firm-specific demand shock that is i.i.d. across firms and time, and drawn from distribution with a cumulative distribution function of F. When F is log-normal with a mean of unity and a standard deviation of σ (in percentage points),

$$\begin{aligned} \Psi(x) &= \exp\left(\frac{\sigma^2}{2}\right) F\left(x - \sigma^2\right) + x\left(1 - F(x)\right), \\ \Psi'(x) &= 1 - F(x), \end{aligned}$$

where the number of extra units of stock required to generate an extra unit of sale, $\Psi'(x)$, equals the probability of a stockout, $1 - F(x)$.

3.3 Linearization and aggregation

Since we are interested in the aggregate dynamics of inventories, we aggregate the optimality condition (2). For tractability, we restrict the analysis to a linear approximation. First, we normalize nominal variables by an aggregate price: $mc_t^j(i) = \frac{MC_t^j(i)}{P_t}$, $p_t^j(i) = \frac{P_t^j(i)}{P_t}$. The normalized optimality condition becomes

$$\Psi_t'^j(i) = \frac{mc_t^j(i)\left[1 + \eta\left(\frac{Z_{t+s}^j(i)}{S_{t+s}^j} p_t^j(i)^\theta - v_*^*\right)\right] - (1-\delta^j) E_t \frac{\beta \Xi_{t+1}}{\Xi_t} \lambda_{t+1}^j(i)}{p_t^j(i) - (1-\delta^j) E_t \frac{\beta \Xi_{t+1}}{\Xi_t} mc_{t+1}^j(i)}.$$

[12] See Kahn (1987), Wen (2008), and Kryvtsov and Midrigan (2009, 2010).

After log-linearization around steady state, we have

$$
\begin{aligned}
&\frac{\Psi''^j x^j}{\Psi'^j}\left(\widehat{z}_t^j(i) - \widehat{s}_t^j + \theta \widehat{p}_t^j(i)\right) \\
= \quad & \frac{\widehat{mc}_t^j(i) + \eta\left(\widehat{z}_t^j(i) - \widehat{s}_t^j + \theta \widehat{p}_t^j(i)\right) - (1-\delta^j)\beta E_t\left(\widehat{\Xi}_{t+1} - \widehat{\Xi}_t + \widehat{mc}_{t+1}^j(i)\right)}{1-(1-\delta^j)\beta} \\
&-\frac{p^j \widehat{p}_t^j(i) - (1-\delta^j)\beta mc^j E_t\left(\widehat{\Xi}_{t+1} - \widehat{\Xi}_t + \widehat{mc}_{t+1}^j(i)\right)}{p^j - (1-\delta^j)\beta mc^j},
\end{aligned}
$$

where variables without time indexes denote steady-state values.

These firm-level equations can be easily aggregated across firms within sector j. Terms $\int \widehat{p}_t^j(i)di$ are eliminated with the help of a linearized aggregate price equation; see the appendix in Kryvtsov and Midrigan (2010). The resulting log-linearized equation for optimal stock holdings is

$$
\begin{aligned}
&\left(C_*^j B_*^j - \frac{\Psi''^j x^j}{\Psi'^j} - \frac{\eta}{1-(1-\delta^j)\beta}\right)\left(\widehat{z}_t^j - \widehat{s}_t^j\right) = \frac{1}{\mu_*^j - (1-\delta^j)\beta}\widehat{mc}_t^j \\
&+\frac{\mu_*^j - 1}{\left(1-(1-\delta^j)\beta\right)\left(\mu_*^j - (1-\delta^j)\beta\right)}\left[\widehat{mc}_t^j - (1-\delta^j)\beta E_t\left(-\widehat{r}_t + \widehat{mc}_{t+1}^j\right)\right], \qquad (3)
\end{aligned}
$$

where

$$
\begin{aligned}
B_* &= \frac{\frac{\Phi'(x_*^j)x_*^j}{\Phi(x_*^j)}}{(1-\theta) + \theta\frac{\Phi'(x_*^j)x_*^j}{\Phi(x_*^j)}} \\
C_* &= -\frac{\Psi''^j x^j}{\Psi'^j}\theta - \frac{\mu_*^j}{\mu_*^j - (1-\delta)\beta}.
\end{aligned}
$$

It can be seen that the dynamics of the inventories in sector j characterized by the last two equations are identical in sales-accelerator and stockout-avoidance models as long as $\frac{\Psi''^j}{\Psi'^j}$ and $\frac{\Phi'(x_*^j)}{\Phi(x_*^j)}$ are the same. We therefore use a stockout-avoidance model as our benchmark, since it is more tractable and is tied to collaborating evidence on the incidence of stockouts. In that model, $\frac{\Psi''^j}{\Psi'^j}(x) = -\frac{F'(x)}{1-F(x)}$.

Finally, denoting the end-of-period stock of inventories by $inv_t^j = z_t^j - s_t^j$, we can write the log-linearized law of motion for inventories:

$$inv^j \; \widehat{inv}_t^j = z^j \widehat{z}_t^j - s^j \widehat{s}_t^j - \left(1-\delta^j\right)\left(z^j \widehat{z}_{t-1}^j - s^j \widehat{s}_{t-1}^j\right), \tag{4}$$

$$\frac{s^j}{Y^j}\widehat{s}_t^j + \frac{inv^j}{Y^j}\widehat{inv}_t^j = \widehat{Y}_t^j. \tag{5}$$

The system of equations (3) to (5) characterizes the dynamics of $\widehat{z}_t^j$, $\widehat{inv}_t^j$, $\widehat{mc}_t^j$, $\widehat{s}_t^j$, and the given time series for $\widehat{Y}_t^j$, $\widehat{mc}_t^j$, and $\widehat{r}_t$. The respective steady-state values for z^j/s^j, p^j, inv^j, mc^j, and Y^j are given by the following system of equations:

$$\begin{aligned}
1 - F\left(\frac{z^j}{s^j}\left(p^j\right)^\theta\right) &= \frac{1-(1-\delta^j)\beta}{p^j/mc^j - (1-\delta^j)\beta}, \\
inv^j &= \delta^j\left(z^j - s^j\right), \\
Y^j &= s^j + inv^j, \\
mc^j &= \frac{\theta-1}{\theta}\frac{p^j}{\left(\frac{\frac{z^j}{s^j}(p^j)^\theta}{\Psi\left(\frac{z^j}{s^j}(p^j)^\theta\right)} + \left(1 - \frac{\frac{z^j}{s^j}(p^j)^\theta}{\Psi\left(\frac{z^j}{s^j}(p^j)^\theta\right)}\right)(1-\delta_z^j)\beta\right)}, \\
\left(p^j\right)^{1-\theta}\Phi\left(\frac{z^j}{s^j}\left(p^j\right)^\theta\right) &= 1.
\end{aligned}$$

3.4 Discussion of the model

There are several advantages and disadvantages to our methodology for modelling inventories for Canada. The main advantage is simplicity: model-based predictions of the behaviour of inventories are based on the dynamics generated by ToTEM, and so we do not need to go through the challenging task of solving ToTEM from scratch. Another advantage is tractability: it is easy to pin down and interpret factors that affect inventory dynamics (demand, average markup, real marginal costs, and the real interest rate). The main disadvantage stems from the fact that our partial-equilibrium model is silent about some interactions between inventory dynamics and the behaviour of prices and quantities in ToTEM. In particular, the model does not take into account the effect of inventory accumulation on firms' pricing decisions. Aguirregabiria (1999) shows that, at the micro level, inventories explain around half of the variation in retail price markups. The second disadvantage is the effect of adjustment costs on price markups: Kryvtsov and Midrigan (2009, 2010) show that, in general equilibrium, large stock adjustment costs lead to more elastic fluctuations of firms' optimal prices, leading to counterfactually transient inflation fluctuations.

In a companion paper (Kryvtsov and Zhang 2010), we add inventories as a sales accelerator to a full version of ToTEM and solve it in general equilibrium. Our partial-equilibrium model of stockout behaviour complements the work in the companion paper by extending the analysis to modelling inventories as a buffer against stockouts. It can also be used as a satellite model of inventories in ToTEM backing up projections produced by ToTEM-II.

4 Parameterization and Experiments

4.1 Parameterization

There are eight parameters in the model: β, θ, δ^j, σ^j, η^j, where $j \in \{C, I\}$. Table 4 provides parameter values. The data are in quarters. Since we do not have time series for inventories for each sector separately, we assume that $\delta^j = \delta$, $\sigma^j = \sigma$, and $\eta^j = \eta$. Discount factor β is parameterized as in ToTEM at 0.992, corresponding to a 3.3 per cent annual real interest rate. The elasticity of good substitution θ is 5, which is lower than 21 in ToTEM. Assuming a high demand elasticity implies a very low markup, leading to low average inventory stocks in the model and very frequent stockouts. We interpret the intermediate value of demand elasticity to be a compromise between high elasticities typically assumed in the macro literature (between 8 and 20) and low elasticities in the industrial organization literature (between 1.5 and 3). The depreciation rate, δ, is 1.05 per cent, to match the ratio of the mean change in the end-of-period inventories to output for manufacturing and trade, which is 0.48 per cent. The standard deviation of demand shocks, $\sigma^j = 27.5$ per cent, is chosen to match the aggregate inventory-to-sales ratio in manufacturing and trade at 0.47, and the adjustment cost parameter η is 0.743, to fit the standard deviation of the aggregate inventory-to-sales ratio in manufacturing and trade, at 1.98 per cent.

We simulate the dynamics in the model imposing historical data for $\widehat{Y}_t^j$ and $\widehat{r}_t$ for the inflation-targeting period, as well as for the corresponding estimated real marginal cost $\widehat{mc}_t^j$ and expected real marginal cost $\widehat{mc}_t^j$ from ToTEM.[13] Table 5 provides the unconditional

[13]The original ToTEM includes inventories as part of the consumption sector. To obtain the estimated real marginal cost, we exclude inventories from ToTEM.

moments for aggregate inventory behaviour predicted by our model. The model is successful in capturing the main facts of inventory behaviour, summarized in section 2. Inventory investment (expressed as a fraction of output) displays large but transient fluctuations around its small mean, with a standard deviation of 0.8 per cent (vs. 0.7 per cent in the data) and a serial correlation of -0.03 (0.01 in the data). Inventories are procyclical, having a 0.74 correlation with output, which is slightly higher than 0.56 in the data. Output volatility in the model is 2.02 per cent, closely matching the 2.08 per cent in the data. Sales and output fluctuations in the investment sector are more than three times more volatile than in the consumption sector, implying more volatile inventory behaviour. The standard deviation of inventory investment in the investment (consumption) sector is 1.65 per cent (0.75 per cent).

As noted in section 2, inventory stock is procyclical, but less so than sales; therefore, the inventory-sales ratio is counter-cyclical. The correlation of the aggregate inventory-sales ratio with sales (output) in the model is -0.22 (-0.13), which is somewhat weaker than found in the data: -0.59 (-0.54). Accordingly, elasticities of the inventory-sales ratio with respect to sales (output) in the model are lower in absolute value than in the data: -0.29 (-0.12) in the model, and -0.61 (-0.51) in the data. Weaker negative correlations of the inventory-sales ratio in the model are somewhat due to the dynamics of inventories in the investment sector, where the inventory-sales ratio is actually slightly procyclical (e.g., the correlation with output is 0.17).

Figure 1 compares the time series for the estimated aggregate inventory-sales ratio in the model to that in the data during the inflation-targeting period. The model captures the majority of the larger swings in the inventory-sales ratio: out of six peaks and five troughs in the data, the model captures five peaks and four troughs, missing on the first peak and the first trough at the beginning of the sample. One explanation for the poorer fit of the model to the data in 1993–94 is that the economy was still on the transition path after the introduction of inflation targeting in 1991–92. In section 4.2, we discuss the main determinants of inventory behaviour in the model.

4.2 Determinants of inventory behaviour in the model

To understand the sources of inventory-sales (I/S) fluctuations, we use the optimality condition for pre-sale inventory-sales ratio deviations (3) to write the end-of-period inventory-sales ratio as a function of average markups (the inverse of marginal cost in the model), the expected growth of marginal cost, and the real interest rate:

$$\widehat{is}_t^j = \varepsilon_{mkp}^j \left(-\widehat{\lambda}_t^j\right) + \varepsilon_{dmc}^j \left[\widehat{mc}_t^j - (1-\delta^j)\beta E_t\left(\widehat{mc}_{t+1}^j\right)\right] + \varepsilon_r^j \widehat{r}_t \ ,$$

where $\widehat{is}_t^j = \frac{z^j/s^j}{z^j/s^j-1}\left(\widehat{z}_t^j - \widehat{s}_t^j\right)$ is the log-linear deviations of sector j end-of-period inventory-sales ratios, and ε_{mkp}^j, ε_{dmc}^j, ε_r^j are their respective elasticities with respect to the average markup, the expected growth of real marginal cost, and the real interest rate. Table 6 summarizes the dynamics of each of these factors and their importance in shaping the dynamics of inventories in the model.

The real interest rate exhibits mildly procyclical but small fluctuations around the trend, with a standard deviation of 0.08 percentage points. Therefore, even though the elasticity of the inventory-sales ratio with respect to the real interest rate is the highest of the three factors, the real interest rate explains only 8.4 per cent (6.2 per cent) of the I/S variance in the consumption (investment) sector. Fluctuations in the average markup are the largest, 0.67 per cent and 1.66 per cent in the C and I sectors, respectively, but the elasticity of I/S with respect to the average markup is the smallest, leading to an even smaller fraction of the explained I/S variance than for the real interest rate: 3.7 per cent and 6.2 per cent. Finally, the expected growth of marginal cost is procyclical and half as volatile as the average markup. That, combined with the high elasticity of the inventory-sales ratio with respect to the expected growth of real marginal cost, leads to the high fraction of I/S variance, around 88 per cent in each sector. Hence, the expected growth of real marginal cost is the most dominant determinant of inventory behaviour in our model. Figures 2a and b plot the time series of the end-of-period inventory-sales ratio in the C and I sectors, respectively, and decompose them into three components.

The second key factor for matching the observed behaviour of the I/S ratio is the adjustment cost. Indeed, without the cost of adjustment, the inventory-sales ratio would be strongly procyclical, positively correlating with output at 0.63. Kryvtsov and Midrigan (2010) emphasize that this is a feature of the New Keynesian models of business cycles such as Smets and Wouters (2007) or ToTEM, stemming from their prediction of the procyclicality of the real marginal cost growth and the high sensitivity of inventory investment with respect to the change in its shadow replacement cost.

5 Conclusions

In this paper, we develop a partial-equilibrium structural model of inventory adjustment that is based on real marginal cost and its expected growth estimated by ToTEM. The model is an adaptation of the models in Bils and Kahn (2000), Wen (2008), and Kryvtsov and Midrigan (2010), where firms hold stocks of inventory to buffer against stockouts. In booms, firms boost their inventories to avoid stocking out due to the rise in demand. The model predicts that firms should hold more stock (relative to sales) when the markup is high, real marginal cost is expected to grow, or the real interest rate is low. The model is able to capture all of the salient business cycle facts for inventory behaviour in Canada during the inflation-targeting period. The main determinant of inventory behaviour in the model is the expected growth of real marginal cost combined with the large cost of inventory adjustment. This paper complements the analysis in a companion paper (Kryvtsov and Zhang 2010) by extending it to modelling inventories as a buffer against stockouts. The model can also be used as a satellite model of inventories in ToTEM, backing up projections produced by ToTEM-II.

References

Aguirregabiria, V. 1999. "The Dynamics of Markups and Inventories in Retailing Firms." *Review of Economic Studies* 66 (2): 275–308.

Bils, M. and J. A. Kahn. 2000. "What Inventory Behavior Tells Us About Business Cycles." *American Economic Review* 90 (3): 458–81.

Blinder, A. S. and L. J. Maccini. 1991. "Taking Stock: A Critical Assessment of Recent Research on Inventories." *Journal of Economic Perspectives* 5 (1): 73–96.

Caplin, A. 1985. "The Variability of Aggregate Demand with (S,s) Inventory Policies." *Econometrica* 53 (6): 1395–1409.

Chacra, M. and M. Kichian. 2004. "A Forecasting Model for Inventory Investments in Canada." Bank of Canada Working Paper No. 2004-39.

Holt, C. C., F. Modigliani, J. F. Muth, and H. Simon. 1960. "Planning Production, Inventories and Work Force." Englewood Cliffs: Prentice-Hall.

Iacoviello, M., F. Schiantarelli, and S. Schuh. 2007. "Input and Output Inventories in General Equilibrium." Federal Reserve Bank of Boston Working Paper No. 07-16.

Jung, Y. and T. Yun. 2005. "Monetary Policy Shocks, Inventory Dynamics, and Price Setting Behavior." Federal Reserve Bank of San Francisco Working Paper No. 2006-02.

Kahn, J. A. 1987. "Inventories and the Volatility of Production." *American Economic Review* 77 (4): 667–79.

——. 1992. "Why is Production More Volatile than Sales? Theory and Evidence on the Stockout-Avoidance Motive for Inventory-Holding." *Quarterly Journal of Economics* 107 (2): 481–510.

Khan, A. and J. Thomas. 2007a. "Inventories and the Business Cycle: An Equilibrium Analysis of (S,s) Policies." *American Economic Review* 97 (4): 1165–88.

——. 2007b. "Explaining Inventories: A Business Cycle Assessment of the Stockout Avoidance and (S,s) Motives." *Macroeconomic Dynamics* 11 (5): 638–64.

Kryvtsov, O. and V. Midrigan. 2009. "Inventories, Markups, and Real Rigidities in Menu Cost Models."Bank of Canada Working Paper No. 2009-6.

Kryvtsov, O. and V. Midrigan. 2010. "Inventories and Real Rigidities in New Keynesian Business Cycle Models." *Journal of the Japanese and International Economies* 24 (2): 259–81

Kryvtsov, O. and Y. Zhang. 2010. "Inventories in ToTEM." Forthcoming Bank of Canada Discussion Paper.

Kydland, F. E. and E. C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50 (6): 1345–70.

McConnell, M. M. and G. Perez-Quiros. 2000. "Output Fluctuations in the United States: What Has Changed Since the Early 1980s?" *American Economic Review* 90 (5): 1464–76.

Ramey, V. A. and K. D. West. 1999. "Inventories." In *Handbook of Macroeconomics* 1st edition, 863-923, edited by J. B. Taylor and M. Woodford. New York: Elsevier.

Smets, F. and R. Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97 (3): 586–606.

Wen, Y. 2008. "Input and Output Inventory Dynamics." Federal Reserve Bank of St. Louis Working Paper No. 2008-008.

Table 1. Moments for aggregate sales, output, and change in inventories

Sector or Stage of fabrication	Mean(INV_t/Y_t) %	Std(INV_t/Y_t) %	Ser.corr(INV_t/Y_t)	Std(S_t) %	Std(Y_t) %	Corr($INV_t/Y_t, Y_t$)	% of Var(Y_t) explained
		Quarterly					
Manufacturing and trade	0.48	0.70	0.01	1.82	2.08	0.56	27.5
Manufacturing	0.40	0.76	-0.05	2.18	2.44	0.47	20.7
Wholesale	0.50	0.93	-0.06	2.62	2.90	0.44	22.4
Retail	0.59	1.24	-0.05	2.52	2.97	0.57	29.2
		Monthly					
Manufacturing	0.46	1.40	0.01	3.37	3.75	0.43	17.2
Finished goods	0.15	0.59	0.03	3.37	3.36	0.06	1.4
Work-in-progress	0.13	0.57	-0.08	3.37	3.48	0.27	6.8
Raw materials	0.19	0.71	-0.09	3.37	3.53	0.32	9.0
Durables	0.49	1.91	0.02	4.49	4.93	0.39	15.6
Non-durables	0.38	1.34	-0.01	2.89	3.31	0.50	30.6

Note: Quarterly data are from the Quarterly Survey of Financial Statistics for Enterprises, 1993Q1–2008Q1, and monthly data are from the Monthly Survey of Manufacturing, 1993Q1–2008Q4. Output is the sum of sales (shipments, in the monthly data) and the change in the end-of-period inventory stock in the sector. For inventories by stage of fabrication, output and sales are totals for manufacturing. The inventory-to-sales ratio is defined as the ratio of the end-of-period inventory stock to sales in that period. Nominal variables for manufacturing and trade are deflated by the GDP deflator, for trade by core CPI, and for manufacturing by industrial PPI. All data are seasonally adjusted and HP-filtered. Output, sales, and inventory-sales ratios are defined in % deviations from respective HP trends. Inventory investment is defined as a fraction of output, and we report it in percentage point deviations from its HP trend. The last column provides the regression coefficient of INV_t on Y_t (and a constant).

Table 2. Moments for aggregate inventory-to-sales ratio

Sector or Stage of fabrication	Mean	Std	Ser corr	Correlation with	
				output	sales
		Quarterly			
Manufacturing and trade	0.47	1.98	0.56	-0.54	-0.59
Manufacturing	0.44	2.97	0.61	-0.66	-0.65
Wholesale	0.46	2.95	0.64	-0.48	-0.53
Retail	0.53	3.65	0.57	-0.44	-0.60
		Monthly			
Manufacturing	1.41	3.34	0.39	-0.71	-0.78
Finished goods	0.46	4.02	0.53	-0.74	-0.76
Work-in-progress	0.33	4.09	0.55	-0.58	-0.61
Raw materials	0.62	3.29	0.39	-0.68	-0.70
Durables	1.59	4.51	0.43	-0.71	-0.81
Non-durables	1.19	2.62	0.48	-0.43	-0.49

Note: Quarterly data are from the Quarterly Survey of Financial Statistics for Enterprises, 1993Q1–2008Q1, and monthly data are from the Monthly Survey of Manufacturing, 1993Q1–2008Q4. Output is the sum of sales (shipments, in the monthly data) and the change in the end-of-period inventory stock. For inventories by stage of fabrication, output and sales are totals for manufacturing. The inventory-to-sales ratio is defined as the ratio of the end-of-period inventory stock to sales in that period. Nominal variables for manufacturing and trade are deflated by the GDP deflator, for trade by core CPI, and for manufacturing by industrial PPI. All data are seasonally adjusted and HP-filtered. Output, sales, and inventory-sales ratios are defined in % deviations from respective HP trends. Inventory investment is defined as a fraction of output, and we report it in percentage point deviations from its HP trend.

Table 3. Elasticities of aggregate inventory-to-sales ratio

Sector or Stage of fabrication	Output elasticity	Sales elasticity
	Quarterly	
Manufacturing and trade	-0.51	-0.61
Manufacturing	-0.81	-0.88
Wholesale	-0.49	-0.61
Retail	-0.54	-0.87
	Monthly	
Manufacturing	-0.64	-0.77
Finished goods	-0.89	-0.90
Work-in-progress	-0.68	-0.74
Raw materials	-0.64	-0.69
Durables	-0.65	-0.81
Non-durables	-0.34	-0.44

Note: Quarterly data are from the Quarterly Survey of Financial Statistics for Enterprises, 1993Q1–2008Q1, and monthly data are from the Monthly Survey of Manufacturing, 1993Q1–2008Q4. Output is the sum of sales (shipments, in the monthly data) and the change in the end-of-period inventory stock. For inventories by stage of fabrication, output and sales are totals for manufacturing. The inventory-to-sales ratio is defined as the ratio of the end-of-period inventory stock to sales in that period. Nominal variables for manufacturing and trade are deflated by the GDP deflator, for trade by core CPI, and for manufacturing by industrial PPI. All data are seasonally adjusted and HP-filtered. Output, sales, and inventory-sales ratios are defined in % deviations from respective HP trends.

Table 4. Parameter values

Parameter		Value
Discount factor	β	0.992
Demand elasticity	θ	5
Stock depreciation rate, %	δ^j	1.05
Demand shocks std, %	σ^j	27.5
Adjustment cost	η^j	0.743

Table 5. Inventories in the model

Moment	Data	Model		
		Total	C	I
		Inventory investment		
mean(INV/Y), %	0.48	0.48*	0.48	0.48
std(INV/Y), %	0.70	0.80	0.75	1.65
ser.corr(INV/Y)	0.01	-0.03	-0.07	0.18
std(S)/std(Y)	0.88	0.75	0.73	0.84
std(Y), %	2.08	2.02	1.71	5.60
corr(INV/Y,S)	0.20	0.46	0.43	0.41
corr(INV/Y,Y)	0.56	0.74	0.75	0.64
corr(S,Y)	0.94	0.94	0.92	0.96
		Inventory-sales ratio		
mean(I/S)	0.47	0.47*	0.47	0.47
std(I/S), %	1.98	1.98*	1.90	3.55
ser.corr(I/S)	0.56	0.55	0.52	0.52
corr(I/S,S)	-0.59	-0.22	-0.39	0.21
corr(I/S,Y)	-0.54	-0.13	-0.21	0.17
corr(I/S,I/S data)	1.00	0.28	0.26	0.26
		Elasticities of inventory-sales ratio		
w.r. to S	-0.61	-0.29	-0.59	0.16
w.r. to Y	-0.51	-0.12	-0.24	0.11

* moments are matched by calibration.

Table 6. Determinants of inventory-sales ratio in the model

	Real interest rate	Markup	Real MC growth
Correlation with Y			
C	0.17	-0.05	0.29
I	0.36	-0.92	0.00
Standard deviation			
C	0.08	0.67	0.31
I	0.08	1.66	0.57
Elasticity of I/S			
C	8.80	-0.77	-6.01
I	22.75	-0.73	-6.07
Fraction of var(I/S) explained, %			
C	8.39	3.70	87.91
I	6.24	6.23	87.53

Figure 1. Aggregate inventory−sales ratio

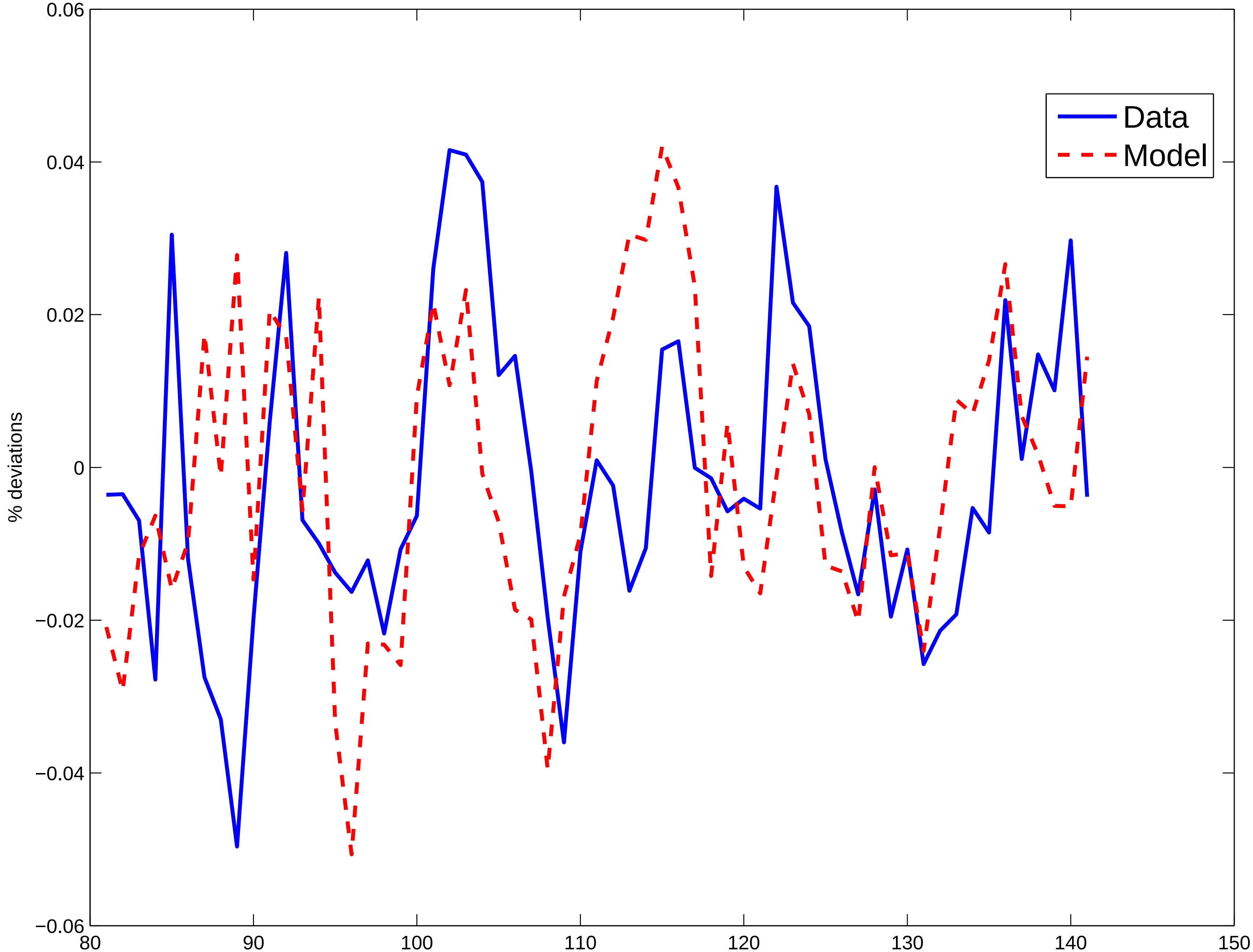

Figure 2a. Decomposition of I/S in the model: C sector

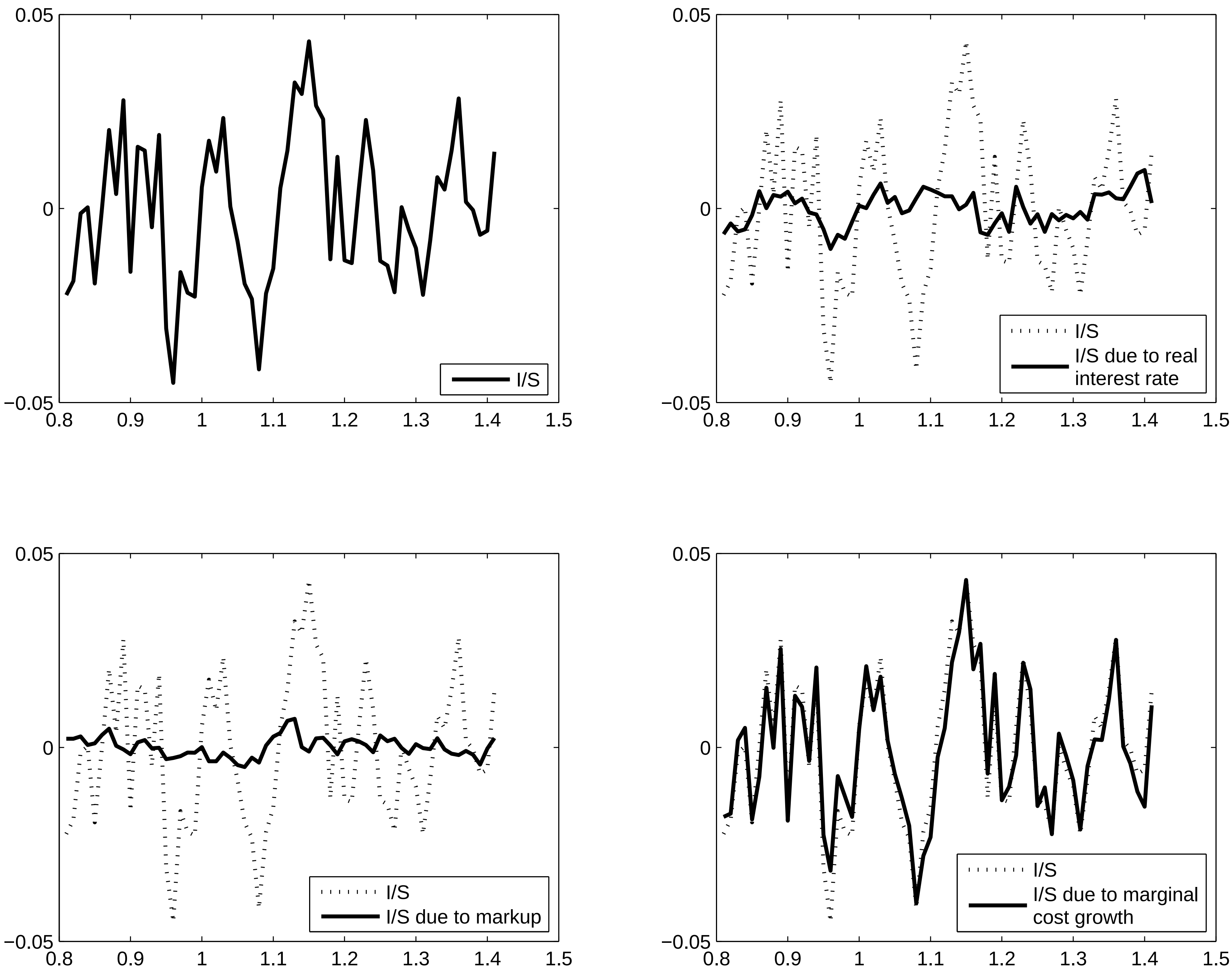

Figure 2b. Decomposition of I/S in the model: I sector