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Inflation Targeting under Uncertainty

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ABSTRACT

This paper studies the implications of certain kinds of uncertainty for monetary policy. It first describes the optimum policy rule in a simple model of the transmission mechanism as in Ball and Svensson. It then examines how this rule ought to be modified when there is uncertainty about the parameters, about the time lags, or about the nature of shocks. The paper also discusses the case of a small open economy such as Canada's, with particular attention being given to uncertainty about the weights in a monetary conditions index.

RÉSUMÉ

L'étude examine les implications de certains types d'incertitude pour la conduite de la politique monétaire. Dans un premier temps, l'auteur décrit la règle optimale de politique dans le cadre d'un modèle simple du mécanisme de transmission analogue au modèle élaboré par Ball et Svensson. Puis il étudie de quelles façons il faut modifier cette règle lorsqu'on est incertain des paramètres, de la longueur des décalages ou de la nature des chocs. L'étude traite également du cas des petites économies ouvertes comme celle du Canada et s'attarde tout particulièrement à l'incertitude entourant le poids relatif des composantes d'un indice des conditions monétaires.

[My] proposal to increase the money stock at a fixed rate month-in and month-out is certainly simple. . . . Surely, you will say, it would be better to "lean against the wind" . . . rather than to stand straight upright whichever way the wind is blowing. . . . We seldom in fact know which way the economic wind is blowing until several months after the event, yet to be effective, we need to know which way the wind is going to be blowing when the measures we take now will be effective, itself a variable date that may be a half year or a year or two years from now. Leaning today against next year's wind is hardly an easy task in the present state of meteorology.

(Friedman 1960, 93)

1. INTRODUCTION

There is a consensus among central bankers today that the primary role of monetary policy is to promote price stability while at the same time being concerned for the variability of output. Fulfilling that role, however, is not straightforward. Central bankers' understanding of the workings of the economy in general, and of the monetary transmission mechanism in particular, is far from precise. Indeed, it is fair to say that uncertainty accompanies every step of the process that links the instruments of monetary policy with the variables of interest—from the interpretation of current economic developments to the expected effects of policy actions.[1] A key question confronting central banks, therefore, is how to conduct monetary policy under conditions of uncertainty.

To tackle this problem, it is natural to use the case of certainty (or rather certainty-equivalence) as a benchmark. One first sets up a core representation of the transmission mechanism without uncertainty; from this, one derives a basic rule for monetary policy. Then one asks how this basic rule should be altered in the presence of some type of uncertainty. In this manner, it might be possible to classify the most common types of uncertainty into broad categories and to examine their implications for monetary policy.

The benchmark used in this paper is the closed-economy dynamic model of Ball (1997) and Svensson (1997a) and the associated optimal rule for monetary policy. The dynamic character of this model, which consists of a reduced-form demand equation and a Phillips curve, is intended to capture the fact that it takes time for changes in the interest rate to affect the economy. The optimal rule is derived on the assumption of a standard quadratic loss function. It is similar in form to the rule proposed by Taylor (1993), although the level of the interest rate responses to deviations of current output or inflation will in general be different. Three kinds of uncertainty are then considered: uncertainty about the coefficient of an explanatory variable as in Brainard (1967); uncertainty about the time lag before monetary actions influence the economy; and uncertainty about the nature of a shock. The paper concludes with a discussion of a specific example of coefficient uncertainty that has attracted recent attention—uncertainty about the weights in a monetary conditions index.

1. See Thiessen (1996).

A description of the three main types of uncertainty and their implications follow.

Uncertainty about the coefficient of a variable in the transmission mechanism. This is sometimes referred to as multiplicative uncertainty and can be intrinsic to the economy or due to econometric estimation. In contrast to uncertainty generated by additive shocks, it implies that the larger the change in the variable concerned, the greater the uncertainty about its effects on the economy. It is therefore not surprising to find that this type of uncertainty induces the policy-maker to attempt to minimize the deviations of the variable concerned. For example, if there is uncertainty about the elasticity of demand to the interest rate, then the policy-maker will be reluctant to move interest rates too sharply in response to shocks. This is the classic result obtained by Brainard (1967) and which, in some authors' view, is behind the interest rate smoothing behaviour often attributed to policy-makers.

In general, it is true that uncertainty about the coefficient of a variable in the transmission mechanism will always induce the policy-maker to try to minimize deviations in that variable. However, this does not necessarily translate into caution with regard to movements in the interest rate. For example, uncertainty about the manner in which inflation surprises feed into future inflation would lead the central bank to respond *more* sharply to inflation shocks, not less, in order to minimize deviations in inflation. Being "cautious" in this case means taking stronger action to minimize the potential for inflation to get away from the target.

Consequently, if there is uncertainty about all or most of the coefficients in the model, one cannot conclude a priori what this entails for the interest rate response. In general, whether uncertainty means moving interest rates to a greater or lesser degree depends on the relative uncertainties of the different coefficients and the structure and dynamics of the model.

The specifications of the model used in this paper, basically its linear-quadratic character, imply that the relevant measure of coefficient uncertainty is the standard deviation of the coefficient divided by its average value. Thus one finds that, when the model fits the data reasonably well, uncertainty about a single coefficient in the transmission mechanism is likely to have only a small effect on the benchmark policy rule. However, it is still unclear to what extent uncertainty about *all* the coefficients simultaneously would affect the basic rule.[2]

Uncertainty about the time it takes for one variable to affect another. This means there are random variations that can shift expected effects in the economy from one period to another. These variations can be inherent in the economy's process of adjustment to exogenous shocks or they can be inherent in the shocks themselves. Examples of the latter are variations caused by labour strikes or changes in weather. Without developing a model from first principles, these shifts in effects can a priori take many forms. For now, since the implications of coefficient uncertainty have already been examined, it is assumed that variations in the time lag amount to additive shocks that shift the demand equation or the Phillips curve in one direction in one period and in the opposite direction the next period.

Thus represented, uncertainty about the time it takes for changes in the economy to have an effect in the future has no bearing on current decisions. What matters for current policy is the *expected* time at which the effects will take place. However, the special nature of lag variations has important implications as far as the response to contemporaneous shocks is concerned. Indeed, it is shown in this paper that the central bank ought not to respond to variations in the economy that amount to shifts between periods. These variations will be automatically offset before any action of the central bank can have an effect.

The policy recommendations above are limited because, more often than not, the monetary authorities are uncertain about the nature of the shock. For example, an unanticipated rise in output could be due to lag effects or it could signal the beginning of a new economic expansion. Since different types of shocks require different responses, the policy-maker is usually bound, until more information becomes available, to follow a middle-of-the-road course, one that balances the risks of acting too quickly against those of being too slow.

Uncertainty about the nature of a shock. With this type of uncertainty, the optimal policy is simply to base the response on the expected nature of the shock. For example, if it is unknown how long a shock is going to last, then the optimal policy is to base the response on the *expected* degree of persistence of the shock. With persistence on average

2. See Sack (1998).

positive over history, and expectations based on past experience, this implies that the optimal response to a new shock of unknown persistence will be larger than the response to a shock known to be temporary, but smaller than the response to a shock known to be long lasting. It also implies that the response to a shock of unknown persistence will be readjusted (and perhaps reversed) over time as one learns more about the shock. Because learning about the nature of a shock is a gradual process, ex post it may appear as if the central bank is reluctant to take sharp actions when called for. In fact, what the bank is doing is following the optimal path, given the information available at the time.

As an application, the case of Canada is examined briefly, with particular attention paid to the role of the exchange rate in policy formulation. Using a very simple model with constant coefficients, Ball (1999) shows that the optimal policy rule in a small open economy is similar to the Taylor rule. However, unlike the Taylor rule, it is not expressed in terms of the interest rate but in terms of an index that is a weighted average of the interest rate and the exchange rate. Further, Ball shows that, under reasonable conditions, the weights are roughly proportional to the coefficients of the same variables in the IS curve. These results therefore support the Bank of Canada's use of a similar index as a measure of monetary conditions in the economy.

One possible objection to Ball's model is that it omits explanatory variables, such as commodity prices and foreign output, which are key to understanding exchange rate and output developments in Canada. Not surprisingly, one finds that, when the effects of these variables are taken into consideration, monetary policy must respond to their fluctuations as well as to fluctuations in demand and inflation. But as long as the new explanatory variables are given exogenously, nothing changes in the optimal rule aside from adding extra arguments (on "the right hand side"). The weights in the index as well as those on output and inflation shocks are the same.[3]

The introduction of new explanatory variables, while relatively simple, highlights the importance of isolating the underlying causes of observed variations in the economy. In particular, it is important to distinguish between autonomous variations in the exchange rate and variations that arise from more fundamental sources, such as shifts in commodity

3. This is assuming, of course, that the model's calibration is not changed either.

prices in the case of Canada. The optimal policy rule indicates, for example, that the interest rate ought to be increased following an autonomous depreciation of the domestic currency, in order to counteract the ensuing rise in exports. In contrast, the interest rate may not need to be changed and, in fact, may need to be decreased if the depreciation has its source in lower commodity prices. A reduction in the interest rate would be necessary if the negative effect of lower commodity prices on domestic demand more than offset any increase in exports of non-commodities following the depreciation of the currency.

Another possible objection to Ball's results is that the model's coefficients are not known with certainty. In particular, some authors (e.g., Ericsson et al. [1997]) have suggested that the weights in the monetary conditions index (MCI) are sufficiently uncertain that they render calculated MCIs uninformative for monetary policy.

This paper expresses a different point of view. It shows, given the uncertainty about the coefficients, that the optimal policy rule can still be expressed in terms of an index such as the one above. Of course, the higher the uncertainty, the greater the expected costs from error; nevertheless, the rule obtained is the best possible under the circumstances.[4] Moreover, one finds that, if measured by standard *t*-statistics, the degree of uncertainty about the coefficient estimates is likely to cause only small changes in the MCI weights.

The remainder of the paper is organized as follows. Section 2 reviews the closed-economy models of Ball and Svensson in some detail. Sections 3, 4, and 5 examine respectively the implications of parameter uncertainty, lag uncertainty, and uncertainty about the degree of persistence of a shock. Section 6 provides a brief application to a small open economy such as Canada's, and Section 7 concludes.

4. A comparison with alternative choices of policy instruments or targets is outside the scope of this paper.

2. THE BASELINE MODEL

As a benchmark, consider first the following representation of the transmission mechanism in a closed economy:[5]

(1) $\pi_{t+1} = \pi_t + d(y_t - y^*) + \varepsilon_{t+1}$

(2) $y_{t+1} - y^* = b(y_t - y^*) - c(r_t - r^*) + \eta_{t+1}$

where y_t is the log of aggregate output; y^* is the log of potential output (assumed for now to be constant); π_t is the inflation rate; i_t is the instrument of monetary policy (here identified with the one-period nominal interest rate); r_t is the real interest rate, i.e., $r_t \equiv i_t - \pi_{t+1|t}$, where $\pi_{t+1|t}$ is the expected rate of inflation at time $t+1$ conditional on information available at time *t*; r^* is the average real interest rate (assumed for now to be constant); b, c, and d are positive constants, with $b < 1$; and ε_t and η_t are white noise random shocks. Equations (1) and (2) of course stand for an accelerationist Phillips curve and an IS curve respectively.

The main feature of this model is that the instrument of monetary policy acts on inflation through aggregate demand, so that a monetary action can affect inflation only with a two-period lag. This is roughly consistent with the empirical facts in Canada if annual periods are chosen.

2.1 The loss function

Following common practice, the policy-maker is assumed to minimize in each period *t* the discounted sum of expected (weighted) deviations of output and inflation from target,

(3) $E_t \sum_{i=0}^{\infty} \delta^i L(\pi_{t+i}, y_{t+i})$,

where

(4) $L(\pi, y) = \alpha(y - y^*)^2 + (1 - \alpha)(\pi - \pi^*)^2$

5. See Ball (1997) or Svensson (1997a).

$0 \le \alpha \le 1$ and $0 \le \delta \le 1$. The closer the value of δ to 1, the greater the weight placed on long-run costs. At the limit, $\delta = 1$, only the long-run costs matter, in which case expression (4) is identified with the unconditional expectation $EL(\pi_t, y_t)$.[6]

The standard quadratic form of the loss function chosen above allows tractable solutions, but it should be kept in mind that it presumes certain facts that are still controversial. For example, under that specification, the inflation rate π^* is presumed to be strictly better than any other rate and/or there is a benefit to fixing some level of inflation, such as π^*, as a point-target.[7] Also, the form of the cost function supposes that positive output gaps per se are costly, as costly as negative output gaps. This would be of little consequence if the long-run costs are thought to dominate the short-run costs, for the output gap would be expected to converge to zero in the long run under any reasonable specification. However, short-run sacrifices could be relevant, particularly in low-inflation regimes, when it is unclear whether or not the return of inflation to a preselected target has important benefits. In this event, even if there is no long-run trade-off between output and inflation, the short-run trade-off could imply an optimum long-run inflation rate that is higher than π^*.[8]

2.2 The optimal rule

The linear-quadratic optimization problem described above satisfies certainty-equivalence. In other words, minimizing expected deviations as in equation (4) is equivalent to minimizing deviations of expectations as in

$$(5) \qquad \sum_{i=0}^{\infty} \delta^i [\alpha(E_t y_{t+i} - y^*)^2 + (1-\alpha)(E_t \pi_{t+i} - \pi^*)^2].$$

In particular, if the central bank is strictly targeting inflation, i.e., $\alpha = 0$, then only the deviations of expected inflation from the target matter. Because the monetary instrument

6. More precisely, under some regularity conditions, $\lim_{i \to \infty} E_t L(\pi_{t+i}, y_{t+i}) = EL(\pi_t, y_t)$, hence $\lim_{\delta \to 1} (1-\delta)\left(E_t \sum_{i=0}^{\infty} \delta^i L(\pi_{t+i}, y_{t+i})\right) = EL(\pi_t, y_t)$.

7. O'Reilly (1998) provides an extensive survey of the benefits of low inflation.

8. This would be the case for instance if the loss function includes a linear term in output, e.g., $L(\pi, y) = \alpha(y-y^*)^2 - \beta(y-y^*) + (1-\alpha+\beta)(\pi-\pi^*)^2$.

can affect inflation only with a two-period lag, it follows immediately that the optimal policy rule in that case involves setting the instrument each period so that the expected inflation two periods later equals the target:[9]

$$(6) \qquad \pi_{t+2|t} = \pi^* .$$

From equations (1) and (2), $\pi_{t+2|t}$ can be expressed as a function of the state variables and the monetary instrument at time *t*,

$$(7) \qquad \pi_{t+2|t} = \pi_{t+1|t} + db(y_t - y^*) - dc(r_t - r^*) .$$

Therefore, the policy rule summarized by equation (6) can be interpreted unambiguously as follows. If the two-period forecast of inflation, as derived from equation (7), is higher than the target, the instrument will be adjusted upwards so as to constrain next-period output below its potential and to lower inflation the period after. If the two-period forecast of inflation is lower than the target, the instrument will be adjusted downwards.

Alternatively, if the central bank regards output stability as its sole objective, i.e., $\alpha = 1$, then in each period it will set the instrument so that the expected output next period equals potential output. In this case, inflation will follow a random walk:

$$(8) \qquad \pi_{t+2} = \pi_{t+1} + \varepsilon_{t+2} + d\eta_{t+1} .$$

In practice, inflation-targeting countries, such as Canada, are concerned with both output and inflation variability, i.e., for them, $0 < \alpha < 1$. In that case, one can show that the optimal policy rule can be expressed in the form

$$(9) \qquad \pi_{t+2|t} - \pi^* = k(\pi_{t+1|t} - \pi^*)$$

for some constant *k* between 0 and 1 that increases with α.[10] The interpretation is as follows. The Phillips curve implies that there is a short-run trade-off between output and

9. $x_{j|i}$ is short for $E_i x_j$, the expected value at time *i* of the variable *x* at time *j*.

10. To see this, notice that the policy-maker's problem is equivalent to choosing a path for $y_{t+1|t}$ that minimizes the loss function. The solution to this linear-quadratic problem states $y_{t+1|t} - y^*$ as a linear function of $\pi_{t+1|t} - \pi^*$; hence the claim. See Ball (1997).

inflation—bringing inflation down requires a temporary negative output gap. If no weight is placed on output stability ($\alpha = 0$), then the policy-maker will seek to achieve the inflation target as quickly as possible ($k = 0$) but at the cost of large fluctuations in output. If a positive weight is placed on output stability ($\alpha > 0$), then following a shock the policy-maker will bring inflation back to its initial target more slowly ($k > 0$) so as to reduce the fluctuations in output—the greater the weight α on output stability, the larger the coefficient k and the more gradual the adjustment of inflation. At the other extreme where the policy-maker is targeting only output ($\alpha = 1$), inflation follows a random walk ($k = 1$).

From equations (1), (2), and (9) above, one can easily infer the associated instrument rule,

$$(10) \qquad r_t - r^* = B(y_t - y^*) + C(\pi_t - \pi^*)$$

where $B = \dfrac{1 - k + b}{c} > 0$, and $C = \dfrac{1 - k}{cd} > 0$

or, equivalently,

$$(11) \qquad i_t - i^* = B'(y_t - y^*) + C'(\pi_t - \pi^*)$$

where $B' = B + d > 0$, $C' = C + 1 > 1$, and $i^* = \pi^* + r^*$.

The optimal policy rule therefore has the same form as the one proposed by Taylor (1993), except of course that the coefficients may differ. It prescribes increasing (decreasing) the real interest rate when current output or inflation are above (below) their targets.

Notice that, as an immediate consequence of the certainty-equivalence property, the uncertainty in the transmission mechanism introduced by the shocks ε_t and η_t has no bearing on monetary policy. The reason is that the shocks enter *additively* and are *serially uncorrelated* over time. This means that the magnitude of current shocks gives no information regarding future shocks, and monetary actions cannot affect the uncertainty introduced by the shocks. In this sense, this type of uncertainty plays a passive role in policy formulation. In the sections to follow, the condition of additiveness and that of serial uncorrelation are relaxed alternatively.

3. PARAMETER UNCERTAINTY[11]

The baseline model assumes that the coefficients of the explanatory variables are known with certainty. It is more realistic, however, to assume that the coefficients are known with some degree of uncertainty, whether this is due to measurement errors or inherent variability of the effects that the explanatory variables have on the economy. Accordingly, suppose that the model has the form,

$$\pi_{t+1} - \pi^* = e_t(\pi_t - \pi^*) + d_t(y_t - y^*) + \varepsilon_{t+1} \qquad (12)$$

$$y_{t+1} - y^* = b_{t+1}(y_t - y^*) - c_{t+1}(r_t - r^*) + \eta_{t+1} \qquad (13)$$

where b_t, c_t, d_t, and e_t are mutually uncorrelated and i.i.d. random variables with mean b, c, d, and 1 respectively.[12]

The expected periodic loss $E_t L(\pi_{t+i}, y_{t+i})$ in period $t+i$ can be written as the sum of two parts: one part due to the variances of output and inflation at time $t+i$, conditional on information at time t; and another part due to the deviation of their (conditional) means from the fixed targets:[13]

$$E_t L(\pi_{t+i}, y_{t+i}) = \qquad (14)$$
$$\alpha var_t(y_{t+i}) + \alpha(y_{t+i|t} - y^*)^2 + (1-\alpha)var_t(\pi_{t+i}) + (1-\alpha)(\pi_{t+i|t} - \pi^*)^2$$

In other words, the policy-maker has an incentive both to target the forecasts of output and inflation and to dampen their volatilities.

Uncertainty about the coefficient of a variable in the transmission mechanism implies that the larger a change in that variable, the larger the uncertainty about its effects (i.e., the larger the variance of its effects) on the economy. This type of uncertainty therefore induces the policy-maker to attempt to minimize the deviations of the variable concerned. For example, uncertainty about the elasticity of demand to the interest rate (i.e.,

11. The implications of parameter uncertainty for policy were first studied by Brainard (1967), and a number of other authors since then—see, for instance, Estrella and Mishkin (1998), Sack (1998), and Svensson (1997b). A substantial literature examines parameter uncertainty in relation to learning; see Wieland (1996) for a review. This paper abstracts, however, from learning by assuming all distributions to be known. This paper's presentation follows that of Svensson (1997b).

12. Notice that the uncertainty is attached to the coefficients of variable deviations from steady state rather than their absolute levels.

13. Recall the identity $E(x - x^*)^2 = var(x) + (Ex - x^*)^2$.

the coefficient *c*) leads the policy-maker to move the interest rate less in response to shocks. This is the classic result obtained by Brainard (1967). It is also, in some authors' view, behind the interest rate smoothing behaviour often attributed to policy-makers. To see this heuristically, suppose for simplicity that the policy-maker strictly targets inflation ($\alpha = 0$), only c_t is random, $\eta_t = 0$, and at time *t* the policy-maker seeks to minimize $E_t L(\pi_{t+2}, y_{t+2})$ rather than the full discounted sum.[14] Elementary calculus then shows that the optimal level of the instrument is

$$(15) \qquad r_t - r^* = B'(y_t - y^*) + C'(\pi_t - \pi^*)$$

where $B' = \dfrac{B}{(1 + s_c^2)}$, $C' = \dfrac{C}{(1 + s_c^2)}$,

B and *C* are the response coefficients found earlier when all parameters are assumed known with certainty, and s_c is the "relative uncertainty" of *c*, i.e., the ratio of its standard deviation to its mean. As expected, the larger the relative uncertainty about *c*, the smaller the interest rate responses, B', C', to inflation and output.

However, uncertainty about the coefficients of variables other than the interest rate may cause the opposite behaviour. For example, uncertainty about the effects of inflation surprises on future inflation (i.e., uncertainty about the coefficient *e*) would lead the central bank to respond *more* forcefully, not less, to inflation shocks in order to minimize deviations in inflation, hence the variance of its effects. Being "cautious" in this case means taking stronger action to minimize the potential for inflation to get away from the target.

For example, consider a positive demand shock at time *t*, which leads to a rise in inflation at time $t + 1$. When all parameters are known with certainty, monetary actions can affect only the expected deviations of output and inflation from their targets. Policy-makers respond to the shock by raising interest rates at time *t*, causing a contraction at time $t + 1$. This in turn lowers inflation at time $t + 2$. Subsequently, the interest rate is gradually returned to its long-run equilibrium, while output rises to its potential and inflation declines back towards its target. When the coefficient *e* is uncertain, then monetary actions can also affect the volatility of output and inflation. In this case, there is an incen-

14. Alternatively, one can assume there are only two periods.

tive to lower inflation more quickly towards its target, for the lower the inflation, the lower the volatility of inflation in subsequent periods. Consequently, when the coefficient e is uncertain, the interest rate is raised more at time t (and, typically, is returned more slowly to its long-run equilibrium) than when e is known with certainty.

More formally, suppose for simplicity that only e_t is random and the policy-maker seeks to minimize $E_t L(\pi_{t+1}, y_{t+1}) + E_t L(\pi_{t+2}, y_{t+2}) + E_t L(\pi_{t+3}, y_{t+3})$ rather than the full discounted sum.[15] Then, the optimal level of the instrument is:[16]

$$(16) \qquad r_t - r^* = B''(y_t - y^*) + C''(\pi_t - \pi^*),$$

where $B'' = \dfrac{\alpha b + (1+b)d^2 X}{c(\alpha + d^2 X)}$, $C'' = \dfrac{dX}{c(\alpha + d^2 X)}$, $X = 1 + \sigma_e^2 + \dfrac{\alpha}{\alpha + d^2}$.

As expected, one can verify that the larger the uncertainty, σ_e, about e, the larger the interest rate responses, B'', C'', to inflation and output.

As more parameters are involved, the effects of uncertainty on monetary policy can add to or offset each other, depending on the trade-offs that exist between the deviations of the variables involved. If greater stability in one variable implies more variability in another, then the effects are likely to offset each other. If, on the contrary, it reinforces the stability of another variable, then the effects are likely to be cumulative. For example, combining uncertainty on the two parameters c and e considered separately above would yield offsetting effects. On the other hand, if the parameters c and d are random (and uncorrelated), then one can show, under the same heuristic conditions assumed above when c alone is uncertain, that the effects of uncertainty add up. Specifically, the optimal policy rule is again expressed as in equation (17) but with response coefficients,

$$B = \frac{1 + b(1 + \sigma_d^2/d^2)}{c(1 + \sigma_c^2/c^2 + \sigma_d^2/d^2)}, \quad C = \frac{1}{cd(1 + \sigma_c^2/c^2 + \sigma_d^2/d^2)},$$

15. Alternatively, one can assume there are only three periods. At least three periods are needed in the example because monetary actions affect uncertainty only three periods later. Also, a positive weight α on output stability is needed to ensure that the expected inflation two periods later is not zero.

16. See Appendix.

which typically are smaller, the larger the variances of c and d.[17]

The above results imply that the optimal response to uncertainty about how interest rate changes will affect the dynamic path of output (i.e., uncertainty about the parameters b and c) and to uncertainty about how changes in output will subsequently affect inflation (i.e., uncertainty about the parameter d) is to move interest rates less, relative to the case of certainty. On the other hand, uncertainty about how much inflation surprises will feed into ongoing inflation (i.e., uncertainty about the parameter e) leads to larger interest rate responses. However, except when uncertainty is solely about the direct effect of interest rate changes on output (i.e., uncertainty about the parameter c), similar claims may not hold in more complex models with additional lagged variables or correlated parameters. In general, whether parameter uncertainty means moving interest rates more or less depends on the relative uncertainties of the different coefficients and the structure and dynamics of the model. It is therefore an empirical issue.

In principle, parameter uncertainty may be extensive enough to call for a neutral policy. For example, if the relative uncertainty, σ_c / c, about the effect that a change in the instrument has on future output is very high, then the best policy practically is a neutral policy: $r_t - r^* \approx 0$.[18]

However, at first sight, rough estimations of the model seem to suggest that the relative uncertainties of the coefficients, as measured by estimated t-statistics, are not large enough to warrant a neutral policy. (However, this does not necessarily mean that the gain in welfare from following an optimal policy over a neutral one is significant.) Indeed, as shown in Section 6, it would seem that parameter uncertainty (at least, on certain parameters) does not substantially alter the benchmark rule obtained under certainty-equivalence.

17. Clearly, C declines with σ_c and σ_d, and B declines with σ_c. However, B declines with σ_d if and only if $b(\sigma_c^2 / c^2)$ is less than 1.

18. More precisely, the best policy practically is to accommodate *small* deviations of output and inflation from target. However, the same cannot be said regarding very large deviations.

    If money is the instrument of policy rather than the interest rate, one should be able to justify Friedman's proposal to keep money growth constant in the same manner as above, on the grounds that the effects of changes in the money supply are highly unstable. In any case, Friedman's proposal is much more sensible than keeping the real interest rate constant because a constant money growth acts as an automatic stabilizer of inflation.

4. LAG UNCERTAINTY

Uncertainty about the length of time it takes for one variable to affect another arises from random variations that can shift expected effects in the economy from one period to another. Such variations can be inherent in the economy's adjustment process or they can be due to exogenous shocks, such as weather changes and labour strikes. Without developing a model from first principles, these variations can a priori take many forms and, in some cases, are perhaps best expressed as variations about the coefficients of explanatory variables. However, for simplicity, and to isolate the implications of lag uncertainty from parameter uncertainty, variations in the time lag are assumed to amount to additive shocks that shift the demand or the Phillips curve in one direction one period and in the opposite direction the next period. Specifically considered is the model,

$$(17) \qquad \pi_{t+1} = \pi_t + d(y_t - y^*) + \theta_{t+1} - \kappa\theta_t + \varepsilon_{t+1}$$

$$(18) \qquad y_{t+1} - y^* = b(y_t - y^* - \gamma\lambda_t) - c(r_t - r^*) + \lambda_{t+1} - \lambda_t + \eta_{t+1}$$

where θ_t and λ_t are white noise shocks representing variations between periods t and $t+1$; γ is a constant between 0 and 1 that expresses the fact that the shock λ_t is not necessarily transmitted to the future at the same rate as other shocks; and κ is a constant between 0 and 1 that expresses the fact that the shock θ_t is not necessarily fully reversed in the future.[19]

For example, a positive shock λ_t raises demand at time t by λ_t above potential and lowers it the next period by the same amount below potential. The rationale is this: If the process of output adjustment is slower than usual in one period, it will then be quicker in the next period as those agents who have not yet adjusted join the regular cohort of agents adjusting at that time. However, a fraction $(1-\gamma)\lambda_t$ of the increase in output at time t

19. Perhaps a more realistic scenario can be described as follows:

$$(19) \qquad \pi_{t+1} = \pi_t + d(y_t - y^*) + \varepsilon_{t+1}$$

$$(20) \qquad y_{t+1} - y^* = b(y_t - y^*) - (c + \lambda_{t+1} - \lambda_t)(r_t - r^*) + \eta_{t+1}$$

This model incorporates uncertainty specifically about the time lag between a change in the interest rate and its effect on inflation. If λ_{t+1} turns out to be negative, then monetary action taken at time t (say, a 100-basis-point increase in the interest rate) does not have its full effect until time $t+2$: Demand at $t+1$ is higher by λ_{t+1} per cent than expected, while if the increase in the instrument is maintained, demand at $t+2$ is lower by λ_{t+1} per cent than usual.

feeds through to the next period (at a ratio b). If $\gamma = 1$, then the shifts in demand between the two periods offset each other exactly. In this case, the return of inflation to the initial target is just delayed one period (other things being kept equal). If $\gamma < 1$, then part of the increase in demand feeds through to time $t + 1$, leading to a higher inflation at time $t + 2$.[20]

Similarly, a positive shock θ_t raises inflation at time t by θ_t and lowers it next period by the same amount. The rationale is that prices may be higher than usual in one period because, for example, of a variation in the adjustment process or because of a transitory shock such as a one-time tax. They adjust only a period later (taking the price level in the previous period as given). However, a fraction $(1 - \kappa)\theta_t$ of the inflation increase at time t feeds into inflation at time $t + 1$. If $\kappa < 1$, inflation remains above the target at time $t + 1$; if $\kappa = 1$, then the shifts in inflation between the two periods offset each other exactly, and inflation returns to the target at time $t + 1$.

Thus introduced in terms of additive shocks, uncertainty about the time it takes for changes in the economy to have an effect in the future has no bearing on current decisions. What matters for current policy is the expected time at which the effects will take place. However, the special nature of lag variations has important implications as far as the response to contemporaneous shocks is concerned.

More formally, suppose that the policy-maker targets inflation strictly, so the instrument in each period t is chosen so that $\pi_{t+2|t} = \pi^*$, i.e., $y_{t+1|t} - y^* = -\frac{1}{d}(\pi_{t+1|t} - \pi^*)$. Then, the optimal level of the interest rate is

$$(21) \qquad r_t - r^* = \frac{1+b}{c}(y_t - y^* - \lambda_t) + \frac{1}{cd}(\pi_t - \pi^* - \theta_t) + \frac{(1-\gamma)b}{c}\lambda_t + \frac{(1-\kappa)}{cd}\theta_t .$$

Notice that $y_t - y^* - \lambda_t$ and $\pi_t - \pi^* - \theta_t$ denote respectively the output gap and the deviation of inflation at time t excluding the lag effects. As already suggested, lag uncertainty about the future is immaterial for current policy. Moreover, if $\gamma = 1$ and $\kappa = 1$, then the

20. Notice that, for simplicity (except for the usual route by which changes in output are transmitted to the future), the model abstracts from the effects of the shock λ_t on demand beyond $t + 1$. As it stands, following a positive λ_t, demand at time $t + 2$ will be below potential, and inflation at time $t + 3$ will not return to its target without further adjustment.

central bank should pursue a neutral policy with respect to lag variations, i.e., the real interest rate at time *t* should not respond to lag variations at that time, for these variations will be automatically offset before any action taken by the central bank can have an effect.[21] If $\gamma < 1$, then the real interest rate should respond by an amount $\frac{(1-\gamma)b}{c}\lambda_t$ necessary to offset the fraction of the demand shock that fed through to period $t+1$. Nevertheless, the response is smaller than the response $\frac{1+b}{c}\eta_t$ that would be necessary following a transitory shock η_t (of equal magnitude). The case $\kappa < 1$ is similar.

5. UNCERTAINTY ABOUT THE NATURE OF A SHOCK

A critical assumption in the previous scenario is that the policy-maker can distinguish between unanticipated variations in output due to lag effects and variations due to other shocks. The consequences can be significant if that is not the case, as the textbook example suggests. If it is mistakenly believed that an unexpectedly high output at time t is due to an exogenous rise in demand, while in fact it is due to past monetary policy taking more time than usual to affect the market, then the policy-maker is apt to raise interest rates further, thus compounding contractionary effects on output in the future.

More generally, once any particular shock is observed, several important questions arise. For example, in the case of an unexpected change in output or unemployment, does the shock signal a structural change or a cyclical effect? In the case of fluctuations in the exchange rate, is the effect due to portfolio adjustments or variations in commodity prices? Will the shock persist? Is it a shift between periods so that it will be offset in the future by movements in the other direction, or is it transitory? In brief, what is the nature of the shock? Clearly, this will determine the direction and the level of action the central bank needs to take. For example, the longer the shock is suspected to persist, the stronger the current actions may need to be.

However, more often than not, initially it is uncertain what the nature of the shock is, and one learns about it only gradually. Under these circumstances, the policy-maker is bound to follow a middle-of-the-road course, one that balances the risks of acting too

21. Of course, keeping the real interest rate constant requires the central bank to accommodate the expected inflation rate, $\pi_{t+1|t}$, since the latter is affected by the shock.

quickly against those of being too slow. To illustrate this point, consider the following model, which incorporates uncertainty about the degree of persistence of a shock:

$$\pi_{t+1} = \pi_t + d(y_t - y^*) + \varepsilon_{t+1} \tag{22}$$

$$y_{t+1} - y^* = b(y_t - y^*) - c(r_t - r^*) + \Delta_{t+1} + \theta_t \rho \nu_{2t} + \eta_{t+1} \tag{23}$$

$$\Delta_{t+1} = \theta_{t+1} \nu_{2,\,t+1} + (1 - \theta_{t+1}) \nu_{1,\,t+1} \tag{24}$$

where: the shocks ε_t, η_t, ν_{1t}, ν_{2t} and θ_t are i.i.d. and mutually uncorrelated; ε_t, η_t, ν_{1t}, ν_{2t} are white noise, while θ_t takes the value 0 or 1 and Δ_t equals ν_{1t} or ν_{2t} according to whether θ_t equals 0 or 1; ρ is a constant, positive or negative, which can incorporate both the degree of persistence of the shock ν_{2t} as well as the degree to which that shock is transmitted to future demand through the usual channel (i.e., at a ratio *b*).

This is essentially the benchmark model, except that now there are two types of demand shocks: one, ν_{1t}, whose effects last only one period and the other, ν_{2t}, whose effects last for two periods. If at time *t*, θ_t takes the value 0, then Δ_t equals ν_{1t} and the demand shock is transitory. If instead θ_t takes the value 1, then Δ_t equals ν_{2t} and the shock shifts demand by ν_{2t} at time *t* and by $\rho\nu_{2t}$ at time $t+1$. Moreover, only the magnitude Δ_t of the demand shock is assumed to be observed at time *t*; the values of η_t, ν_{1t}, ν_{2t}, and θ_t, hence the type of the demand shock, are not revealed until one period later.

Suppose again for simplicity that the policy-maker targets inflation strictly; hence monetary policy is set so that $\pi_{t+2|t} = \pi^*$, or equivalently,

$$y_{t+1|t} - y^* = -\frac{1}{d}(\pi_{t+1|t} - \pi^*).$$

From the demand equation (24), it follows that

$$r_t - r^* = \frac{(1+b)}{c}(y_t - y^*) + \frac{1}{cd}(\pi_{t+1|t} - \pi^*) + \frac{1}{c}\theta_{t|t}\rho\Delta_t \tag{25}$$

where $\theta_{t|t}$ is the probability that the shock is ν_{2t} conditional on information at time *t*.

If θ_t equals 0 with certainty (hence $\theta_{t|t} = 0$), one recognizes the rule found earlier in the basic scenario when shocks are i.i.d. If $\theta_t = 1$ with certainty (hence $\theta_{t|t} = 1$), equation (26) describes the optimal rule under persistent shocks. As expected, the more persistent the shock, i.e., the larger the value of ρ, the higher the response should be. In general, $\theta_{t|t}$ takes a value between 0 and 1, and the optimal rule recommends a level of the instrument that is an average of the two extremes just mentioned. For example, if the degree of persistence ρ is positive, then the instrument response at time *t* is higher than the response to a transitory shock, but lower than the response to a persistent shock. Accordingly, at time $t + 1$, the monetary authority will have to reverse its earlier actions if it turns out that the shock was transitory, or to increase its action if it was persistent.

As already suggested, the results described above follow from certainty-equivalence and extend without difficulty to a general context whereby it is uncertain whether a shock is of one type or another. The optimal response under such conditions is then the “average” of the respective optimal responses associated with each type of shock. Expressed differently, the optimal response under such uncertainty is the same as the one that would obtain under certainty, but where the shock is an average of the two types of primary shocks weighted by their respective likelihood of realization.

6. APPLICATION TO A SMALL OPEN ECONOMY

This section examines briefly particular instances of coefficient uncertainty and uncertainty about the nature of a shock in the context of a small open economy such as Canada’s. Recall Ball’s (1999) representation of a small open economy:

$$\pi_{t+1} = \pi_t + dy_t - f(e_t - e_{t-1}) + \eta_{t+1} \tag{26}$$

$$y_{t+1} = by_t - cr_t - ge_t + \varepsilon_{t+1} \tag{27}$$

$$e_t = hr_t + \nu_t \tag{28}$$

where e_t is the log of the real exchange rate (a greater *e* means appreciation), and ε, η, ν are white noise shocks. To simplify notation, all variables are now measured as deviations from their average values (e.g., π_t measures the deviation of inflation from the target π^*) and all parameters are positive $(h \geq 1)$.

This is essentially the baseline (closed-economy) model with the exchange rate added as a new explanatory variable. The exchange rate affects future demand through exports, while the change in the exchange rate affects future inflation through import prices, e.g., foreign firms desire constant real prices in their home currencies, but domestic prices are adjusted with a one-period lag. The rationale for equation (29) linking the interest rate to the exchange rate is that a rise in the interest rate makes domestic assets more attractive. The shock ν captures other influences on the exchange rate such as shifts in expectations and investor confidence.

Assuming the unconditional version of the loss function (3) (i.e., $\delta = 1$), Ball shows that the optimal rule for monetary policy has the following form,

$$wr_t + (1-w)e_t = Ay_t + B(\pi_t + fe_{t-1}) \tag{29}$$

for some positive constants w, A, and B ($0<w<1$). The term $wr + (1-w)e$ can be viewed as a monetary conditions index (MCI), and $\pi + fe_{t-1}$ as a measure of inflation that excludes the direct, but temporary, effects of exchange rate movements. The optimal rule therefore prescribes tightening monetary conditions, as summarized by the MCI, in the event of a rise in the output gap or (modified) inflation, and keeping monetary conditions constant in the event of a contemporaneous change in the exchange rate.

Ball also shows that, under plausible calibration of the model roughly consistent with Canadian data,[22] the relative weight on the interest rate and exchange rate in the MCI is approximately the same as (to be precise, slightly smaller than) in the IS equation, i.e., a ratio of 3 to 1. The intuition for this result is that output and price stabilization is achieved mainly by controlling demand. Although monetary policy could affect inflation more directly through the exchange rate, this would necessitate substantial variations in the interest rate and consequently provoke large fluctuations in output.

6.1 Additional explanatory variables

The results above rely on the assumption that the shocks, as specified, are white noise. But this is not true in a small economy like Canada’s where variables such as commodity prices or foreign output, not represented in the previous model, are known to significantly affect both the exchange rate and demand. The extended model is therefore considered:

22. For example, $c = 0.6, g = 0.2, b = 0.8, d = 0.4, f = 0.2, h = 2$, and a weight on output variance relative to inflation variance close to or greater than 1 in the objective function.

$$(30) \qquad \pi_{t+1} = \pi_t + dy_t - f(e_t - e_{t-1}) + \Psi X_t + \eta_{t+1}$$

$$(31) \qquad y_{t+1} = by_t - cr_t - ge_t + \Phi X_t + \varepsilon_{t+1}$$

$$(32) \qquad e_t = hr_t + \Omega X_t + \nu_t$$

where X is a vector of new exogenous explanatory variables (measured as deviations from average levels), Φ, Ψ, Ω are coefficient vectors, and $X, \varepsilon, \eta, \nu$ are white noise. X_t is known at the beginning of period t, before any monetary action is taken.

Then, one can show that the optimal rule takes the form:

$$(33) \qquad wr_t + (1-w)e_t = Ay_t + B(\pi_t + fe_{t-1}) + CX_t$$

where the coefficients w, A, B are identical to those found earlier, and C is a constant vector that depends on the parameters of the model, including Φ, Ψ, but not on Ω. Rather, the effect of Ω on the MCI is subsumed in the value of the exchange rate. (The formal proof can be found in the Appendix.)

Thus, the MCI's optimal response to innovations in output or (modified) inflation is unaffected by the introduction of the new explanatory variables X. After all, the effects of such innovations on future output and inflation have not changed. What is new of course is that the MCI must also respond to innovations in X, since these do affect the future path of output and inflation.

If Ψ is assumed to equal 0, then it can be shown that the weights on the interest rate and X in the optimal rule are unambiguously proportional to those present in the IS curve (e.g., $\frac{C}{w} = \frac{\Phi}{c}$). This is not too surprising, for if $\Psi = 0$, then the effects of changes in the interest rate or X are witnessed only in the IS equation and the exchange rate equation. However, their effects in the exchange rate equation are already subsumed in the level of the exchange rate incorporated in the MCI.

In particular, if X stands for (non-oil) real commodity prices, then, for a net commodity exporter such as Canada, Φ is positive and Ψ is close to 0,[23] in which case C will be positive. Under these conditions, an autonomous rise in the exchange rate, due to a positive ν, requires constant monetary conditions, whereas a rise in the exchange rate that is

23. The results below would also apply if Ψ is positive.

due to an increase in real commodity prices requires tighter monetary conditions. The reason is that, in contrast to movements in ν, increases in commodity prices cause an extra demand expansion. However, since an increase in real commodity prices automatically raises the MCI through the exchange rate, the direction in which the interest rate needs to be adjusted in this event, if at all, depends on the relative magnitudes of Φ and Ω (as well as the other parameters of the model): a unit increase in *X* immediately raises the exchange rate by Ω, hence the MCI by $(1-w)\Omega$, whereas the optimal increase desired is *C*. Thus the MCI should be further increased if and only if $(1-w)\Omega < \frac{w}{c}\Phi$, if one approximates $\frac{w}{C}$ by $\frac{c}{\Phi}$ (see previous paragraph), i.e., if and only if $g\Omega < \Phi$, if one approximates $\frac{w}{1-w}$ by $\frac{c}{g}$. In other words, under the approximations made above, the interest rate should be raised following an increase in commodity prices if and only if the direct expansionary effect of the commodity price change on demand outweighs the offsetting effect caused by the ensuing rise in the exchange rate.

6.2 Parameter uncertainty

So far, the model's parameters are assumed to be known and constant. On that basis, it is shown that the optimal policy rule can be expressed in terms of an MCI whose weights on the interest rate and the exchange rate, in the case of Canada, are roughly proportional to the corresponding coefficients in the IS curve. Empirical estimates of these coefficients, however, exhibit a certain degree of statistical uncertainty, so that a wide range of possible values for the ratio $\frac{c}{g}$ typically cannot be rejected with reasonable confidence. One might then be tempted to infer that there is an equally wide range of possible values for the MCI and that, therefore, calculated MCIs are uninformative for monetary policy. Indeed, this is the conclusion of Ericsson et al. (1997).

This conclusion, however, is not altogether correct. No doubt the response coefficients and any deduced weights in the optimal rule must be adjusted to take into consideration the uncertainty about the parameters *c* and *g*. But uncertainty about *c* and *g* need not suggest that using an MCI is inappropriate. Indeed, it is shown below that, when there is uncertainty about the parameters *c* and *g*, the optimal policy continues to have the MCI form. Uncertainty about c and g does affect the weights in the MCI, but provided these parameters are statistically significant at conventional levels, the impact is very small.

To examine this issue more formally, assume the coefficients c and g of the interest rate and the exchange rate respectively are i.i.d. random variables with mean values $\bar{c}$, $\bar{g}$, and standard deviations σ_c, σ_g. The latter are identified with their empirical estimates (whatever econometric technique is used to estimate the model), so that it is intuitive to think of $\frac{\bar{c}}{\sigma_c}$ and $\frac{\bar{g}}{\sigma_g}$ as standard t-statistics. To simplify the analysis, assume further that $f = 0$, that the policy-maker strictly targets inflation, and there are only two periods.[24] In other words, at time t, the policy-maker seeks to minimize $E_t(\pi_{t+2})^2$. Then, one can show that the optimal rule has the form,

$$(\sigma_c^2 + m)r_t + (h\sigma_g^2 + n)e_t = py_t + q\pi_t + FX_t \tag{34}$$

for some positive constants m, n, p, q, and F that are independent of the variances σ_c and σ_g. In other words, except for a constant of proportion, m, n, p, q, and F are identical to those found under constant parameters.

If the "estimate" $\bar{c}$ of c is highly significant in the sense that $\frac{\sigma_c}{\bar{c}}$ is negligible, then uncertainty in the parameter c can be shown to have only a minor effect on the optimal rule, $\sigma_c^2 + m \approx m$. The case is similar for g (in fact, $m = \bar{c}^2 + \bar{c}\bar{g}h$ and $n = h\bar{g}^2 + \bar{c}\bar{g}$).

In general, one can divide both sides of equation (35) by $s = \sigma_c^2 + m + h\sigma_g^2 + n$ so as to derive the normal MCI form:

$$wr_t + (1-w)e_t = Ay_t + B\pi_t + CX_t\,. \tag{35}$$

One sees, then, that the effect of uncertainty in c and g is first to lower the response coefficients A, B, and C by the same constant of proportion s, and second to raise the weight on the exchange rate in the MCI if the uncertainty on g is roughly greater than on c:[25] the

24. Work on the general case is now in progress, but it is conjectured that the results are robust.

25. More precisely, if $\left(\frac{\bar{g}}{\sigma_g}\right)^2 < \frac{h\bar{g}}{\bar{c}}\left(\frac{\bar{c}}{\sigma_c}\right)^2$.

greater the uncertainty on g, the greater the weight on the exchange rate. These results are consistent with those obtained in Section 3. Deviations of the interest rate or the exchange rate from their steady state magnify the uncertainty introduced by the parameters c and g in the IS curve; hence the variance of output, which in turn raises the variance of inflation. The policy-maker must therefore respond more cautiously to inflationary shocks by lowering the response coefficients A, B, and C. If the uncertainty regarding the effect of the exchange rate on output is greater than that of the interest rate, then the policy-maker must respond more vigorously to fluctuations in the exchange rate, by attaching greater weight to it in the MCI.

The table below gives some numerical examples using Ball's calibrated parameters for Canada (see footnote 22).

TABLE 1. Effects of uncertainty on the optimal rule

$\bar{c}/\sigma_c$	$\bar{g}/\sigma_g$	**w/(1–w)**	**s**
∞	∞	3	.8
∞	1	2.42	.88
2	2	3.13	.91
2	1	2.46	.97
2	.25	.46	2.17
1	1	3.42	1.24
1	.25	.65	2.44

The first line in the table corresponds to the case where the parameters are constant: The ratio $w/(1-w)$ then equals 3 and the factor of proportion s equals 0.8. The third line shows that, if the parameter estimates are significant in the sense that their "*t*-statistics" are no less than 2—in the table they are set equal to 2—then the presence of uncertainty has only a minor effect on the optimal rule.[26] But even when the estimates are only marginally significant in the sense that their *t*-statistics are close to 1, one finds that the optimal rule is

26. If one assumes (like Ball) that a 100-basis-point increase in the interest rate lowers output by 1 per cent, i.e., $\bar{c} + \bar{g}h = 1$, then $\frac{w}{1-w} = \frac{c}{g}\frac{(1 + c/t_c^2)}{(1 + g/t_g^2)}$, where $t_c = \frac{c}{\sigma_c}$ and $t_g = \frac{g}{\sigma_g}$.

not markedly affected, as is apparent from line 6. Empirically estimated *t*-statistics (including those employed by Ericsson et al.) are typically found to be close or greater than 2 for c and greater than 1 for g.[27]

6.3 Uncertainty about the nature of a shock

Another type of uncertainty that is particularly important in practice in open economies is uncertainty about why the exchange rate has changed. The previous sections have examined the baseline case where the transmission mechanism is fully determined and the nature of the shocks is known. It was noted in particular that policy responses ought to be different according to whether the movements in the exchange rate are due to commodity price changes or other reasons.

It is more likely, however, that the source of fluctuations in the exchange rate is unclear. This would be the case, for example, if the effect of commodity price changes on the exchange rate is not known with certainty,

$$e_t = hr_t + \Omega_t X_t + \nu_t$$

where X_t denotes real commodity prices at time t and Ω_t is a random variable, unobserved at time t and with mean Ω. In this context, given the values of r_t and X_t, an unexpected change in the exchange rate may be due to either an autonomous shock ν_t or to an unexpected change in the parameter Ω_t.

However, as long as the random variable Ω_t is serially uncorrelated (and independent from the other shocks), this type of uncertainty simply amounts to adding another white noise shock to the model. The baseline optimal rule derived earlier in Section 6.2 is therefore unaffected. Under these circumstances, fluctuations in the exchange rate beyond those expected, following current commodity price changes, should be treated as autonomous shocks.

27. See Duguay (1994).

7. CONCLUSION

This paper examines the implications that certain types of uncertainty have for monetary policy. To that end, the closed-economy model of Ball and Svensson is used as well as the associated optimal policy rule as a benchmark. This basic rule is then studied to determine how it ought to be modified in the presence of uncertainty. This methodology is particularly well suited to designing a framework for the conduct of monetary policy based on inflation targeting. Indeed, one can define such a framework as consisting of a core rule together with a set of guidelines advising how to deviate from the rule under diverse circumstances.

One finds that, when there is uncertainty about the coefficient of an explanatory variable, the policy-maker ought to minimize the deviations of the variable concerned in order to lower the volatility of its effects. This requires weaker or sharper movements in the interest rate depending on the variables involved. However, if uncertainty affects only a few variables and is estimated by standard *t*-statistics, it seems to lead to only small changes to the basic rule. Lag uncertainty also introduces volatility in the effects of a variable, but its distinctive feature is that it shifts the effects of the variable between periods. In this respect, it is shown that, to some extent, the policy-maker ought to ignore lag effects because, by their very nature, they will automatically be offset in the future. Finally, when uncertain about the nature of a shock, the policy-maker ought to follow a middle-of-the-road course, one that balances the risks of acting too quickly against those of being too slow, until more information becomes available. In practice,[28] however, policy-makers might be reluctant to respond to the expected nature of the shock, because of the difficulty in explaining and justifying such an action on the basis of inherently uncertain forecasts. Instead, they respond only to the perceived shocks at the time. Seen in this light, uncertainty about the nature of shocks may provide another explanation why central banks appear to be smoothing their actions over time.

A small open economy is also examined, with particular attention paid to the role of the exchange rate in policy formulation. Ball (1999) shows that, in the context of a small open economy, the optimal rule can be expressed in terms of a monetary conditions

28. This argument is due to Freedman (1998).

index analogous to the one employed at the Bank of Canada. But his model assumes constant coefficients and omits certain key explanatory variables such as commodity prices and foreign output.

It is shown that the optimal policy rule is unaffected when Ball's model is augmented by additional exogenous variables, except of course that it must also respond to shocks to the new variables. One important implication, however, is that the policy-maker ought to distinguish between autonomous variations in the exchange rate and variations arising, say, from changes in commodity prices. In the former case, the MCI ought to be kept constant, whereas in the latter case the MCI ought to be allowed to move with commodity prices. Whether it needs to be adjusted further, and in what direction, depends on the model's parameters.

Finally, uncertainty about the coefficients of the interest rate and the exchange rate in the IS curve is shown to cause monetary policy to respond more cautiously to inflationary shocks. It also typically raises the relative weight on the exchange rate in the MCI because the exchange rate has the more uncertain effects. However, under reasonable calibration, both these changes are small.

The paper leaves many questions for future investigation. Of most immediate concern is that strong simplifications are used in the paper to allow tractable analysis. The results need, therefore, to be confirmed by other means, perhaps with the help of numerical methods and in the context of more structural models. Numerical methods are also needed to evaluate the welfare implications of different rules under uncertainty.

On a different note, it may be worth documenting explicitly, both historically and in current developments, examples of different types of uncertainty. Some exploration regarding the most common types of uncertainty encountered in practice might be useful.

APPENDIX

In this section, all variables are measured as deviations from their average values.

1. Formal derivation of the optimal rule (16) in Section 3

The optimal rule is derived by backward induction. First, one solves the problem $V(\pi_{t+2}, y_{t+2}) = min(E_{t+2}L(\pi_{t+3}, y_{t+3}))$, given π_{t+2} and y_{t+2}. This amounts to minimizing $E_{t+2}y^2_{t+3}$, which requires $E_{t+2}y_{t+3} = 0$, hence $r_{t+2} = \frac{b}{c}y_{t+2}$. An expression for $V(\pi_{t+2}, y_{t+2})$ as a function of π_{t+2} and y_{t+2} can then be easily deduced.

Next, one solves $V(\pi_{t+1}, y_{t+1}) = minE_{t+1}[L(\pi_{t+2}, y_{t+2}) + V(\pi_{t+2}, y_{t+2})]$, given π_{t+1} and y_{t+1}. Using the results above, this amounts to minimizing the expression $E_{t+1}[(2 + \sigma^2_e)\pi^2_{t+2} + (\alpha + d^2)y^2_{t+2} + 2d\pi_{t+2}y_{t+2}]$, given π_{t+1} and y_{t+1}. Elementary calculus then shows that the optimal interest rate at time $t+1$ is $r_{t+1} = \frac{\alpha b + bd^2 + d^2}{c}y_{t+1} + \frac{d}{c}\pi_{t+1}$.

Finally, one can solve for

$min(E_tL(\pi_{t+1}, y_{t+1}) + E_tL(\pi_{t+2}, y_{t+2}) + E_tL(\pi_{t+3}, y_{t+3}))$, or equivalently

$minE_t[L(\pi_{t+1}, y_{t+1}) + V(\pi_{t+1}, y_{t+1})]$. Substituting the expression of $V(\pi_{t+1}, y_{t+1})$, this amounts to minimizing $E_t\left[\alpha y^2_{t+1} + \left(2 + \sigma^2_e - \frac{d^2}{\alpha + d^2}\right)(\pi_{t+1} + dy_{t+1})^2\right]$, whose solution then provides the optimal rule:

$$(16) \qquad r_t - r^* = B''(y_t - y^*) + C''(\pi_t - \pi^*)$$

$$B'' = \frac{\alpha b + (1+b)d^2X}{c(\alpha + d^2X)},\ C'' = \frac{dX}{c(\alpha + d^2X)},\ X = 1 + \sigma^2_e + \frac{\alpha}{\alpha + d^2}.$$

2. Formal derivation of the optimal rule in Section 6

Use equation 32 to substitute for r in equation 31:[1]

$$(31') \qquad y_{t+1} = -\left(\frac{c}{h} + g\right)e_t + by_t + \left(\Phi + \frac{c}{h}\Omega\right)X_t + \frac{c}{h}\nu_t + \varepsilon_{t+1} .$$

A. Assume first X, ε, η, ν are white noise. From equations $31'$ and 30, it follows that $by_t + \left(\Phi + \frac{c}{h}\Omega\right)X_t + \frac{c}{h}\nu_t$ and $\pi_t + dy_t + fe_{t-1} + \Psi X_t$ can be defined as state variables.

The optimal rule can therefore be written as:

$$e_t = m\left[by_t + \left(\Phi + \frac{c}{h}\Omega\right)X_t + \frac{c}{h}\nu_t\right] + n[\pi_t + dy_t + fe_{t-1} + \Psi X_t]$$

where m and n are positive constants independent of the parameters Φ, Ψ, Ω.

Use equation 32 again to substitute $\frac{c}{h}(e_t - hr_t)$ for $\frac{c}{h}(\Omega X_t + \nu_t)$ and algebra:

$$(33) \qquad wr_t + (1-w)e_t = ky_t + l(\pi_t + fe_{t-1}) + CX_t$$

where $w = \dfrac{mch}{h - mc + mch}$ $\qquad k = \dfrac{h(mb + nd)}{h - mc + mch}$ $\qquad l = \dfrac{nh}{h - mc + mch}$

$$C = \frac{h}{h - mc + mch}(m\Phi + n\Psi) .$$

Clearly, if Φ and Ψ are positive, then so is C (recall $h \geq 1$), and $\dfrac{w}{C} = \dfrac{c}{\Phi}$ when $\Psi = 0$.

1. The subscript t below is omitted.

B. Suppose now that c and g are i.i.d., $f = 0$, and, at time *t*, the policy-maker seeks to minimize $E_t(\pi_{t+2})^2$. The latter can also be written,

$$(36) \qquad E_t(\pi_{t+2})^2 = (E_t\pi_{t+2})^2 + var_t(\pi_{t+2}),$$

where $var_t(\pi_{t+2})$ is the variance of π_{t+2} conditional on information at time *t*.

From equations (30) and (31), one easily derives:

$$\begin{aligned}\pi_{t+2} = \pi_t + dy_t + \Psi X_t + \eta_{t+1} + d(-cr_t - ge_t + by_t + \Phi X_t + \varepsilon_{t+1}) \\ + \Psi X_{t+1} + \eta_{t+2}\end{aligned}.$$

It follows

$$E_t\pi_{t+2} = \pi_t + dy_t + \Psi X_t + d(-\bar{c}r_t - \bar{g}e_t + by_t + \Phi X_t)$$

and

$$var_t(\pi_{t+2}) = (dr_t)^2\sigma_c^2 + (de_t)^2\sigma_g^2 + \Sigma,$$

Σ a constant. From equations (32), (34), and simple calculus, one then deduces the optimal rule,

$$(34) \qquad (\sigma_c^2 + m)r_t + (h\sigma_g^2 + n)e_t = py_t + q\pi_t + FX_t,$$

where

$$m = \bar{c}^2 + \bar{c}\bar{g}h \qquad n = h\bar{g}^2 + \bar{c}\bar{g} \qquad p = \frac{\bar{c} + \bar{g}h}{d}$$

$$q = (\bar{c} + \bar{g}h)(1 + b) \qquad F = \frac{\bar{c} + \bar{g}h}{b}(\Psi + d\Phi)$$

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