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The Bank of Canada's New Quarterly Projection Model (QPM)

Part 4

A Semi-Structural Method to Estimate Potential Output: Combining Economic Theory with a Time-Series Filter

by Leo Butler

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ABSTRACT

The level of potential output plays a central role in the Bank of Canada's new Quarterly Projection Model (QPM). This report, the fourth in a series documenting QPM, describes a general method to measure potential output, as well as its implementation in the QPM system. The report begins with a short history of the measurement of potential output. Building on this experience, a hybrid method of measuring potential output is developed that combines economic structure with a time-series filter. The resulting filter, known as the extended multivariate (EMV) filter, exploits theoretical relationships that are embodied in QPM in an effort to identify demand-side and supply-side influences on output. These various relationships are combined in a filter that imposes a smoothness property on the dynamics of potential output. This report describes the general structure of the EMV filter, the various economic relationships that it uses, and the weights applied to these different pieces of information. The report concludes with an evaluation of the EMV filter and some suggestions for future improvements.

RÉSUMÉ

Le niveau de la production potentielle occupe une place centrale dans le nouveau Modèle trimestriel de prévision (MTP) de la Banque du Canada. La présente étude, la quatrième de la série traitant du MTP, décrit une méthode générale de mesure de la production potentielle ainsi que la manière dont elle est mise en oeuvre dans le modèle. L'auteur rappelle d'abord brièvement la genèse de la mesure de la production potentielle. Il présente ensuite une méthode hybride de mesure de la production potentielle qui est fondée sur les leçons de l'expérience et combine une structure économique et un filtre appliqué à une série chronologique. Le filtre ainsi obtenu, appelé filtre multivarié élargi, tire parti des relations théoriques que renferme le MTP pour déterminer les influences respectives de la demande et de l'offre sur la production. Ces relations sont réunies dans un filtre qui impose une contrainte de lissage à la dynamique de la production potentielle. L'auteur décrit la structure générale du filtre multivarié élargi, les diverses relations économiques que le filtre met à contribution ainsi que les pondérations appliquées à celles-ci. Il conclut en procédant à une évaluation du filtre proposé et suggère quelques pistes en vue de l'améliorer.

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1 INTRODUCTION

In broad terms, potential output is the level of output that the economy can sustain if all resources are used efficiently. A more precise definition, however, requires elaboration. Analysts dealing with fiscal issues are often interested in determining the portion of a government's deficit that is attributable to structural rather than cyclical factors. This requires a concept of potential output that links output to a sustainable fiscal policy. Other analysts might be interested in determining the level of output that would have been produced were social institutions or various government policies different from those that actually prevailed. For the purposes of monetary policy, the relevant concept of potential output is one that is linked directly to the dynamics of wage and price inflation. Potential output is defined in this sense as the level of output that is consistent with an unchanged rate of inflation over the short run. Based on this concept, the deviation of output from potential, which is called the output gap, provides a measure of the inflationary pressure in the economy. This paper describes a method to estimate potential output using this definition of potential.

Over the business cycle, changes in wages and prices are influenced by the relationship between aggregate demand and aggregate supply. For example, if aggregate demand begins to accelerate ahead of aggregate supply, shortages of key types of labour and commodities will begin to appear. Wages and prices will be bid higher by employers and consumers as markets adjust to the shortages, leading to an increase in inflation. Consequently, wage and price inflation will temporarily climb above their trend levels. If monetary policy is directed towards maintaining a stable rate of inflation, the monetary authority will intervene to rein in the acceleration of inflation by reducing aggregate demand. A period in which aggregate demand grows more slowly than aggregate supply will, therefore, be necessary in order to close the gap between the two, and to relieve inflationary pressures. More generally, in order for the monetary authority to control inflation and successfully avert an exaggerated boom-and-bust cycle, policy makers need accurate information about the level of aggregate supply relative to aggregate demand.

Potential output cannot be directly measured, however, and economists must therefore estimate it. To do so they must decide which changes in output result from supply shocks and which result from demand shocks. In the past, supply shocks were treated as rare, easily observed events such as the two oil-price shocks of the 1970s or the unemployment insurance reforms of the early 1970s. Kinks in otherwise linear trends were used to model potential output in this framework. However, this method is unsuitable in a policy setting for a simple reason: a large amount of evidence of a break in the trend growth rate is needed before a break is introduced into the assumed trend growth of output. Meanwhile, a string of same-sided policy errors may be committed.

In order to allow for shifts in trend growth to be more easily incorporated into estimates of the trend in output, academic economists introduced another method to separate fluctuations in output into their trend and cyclical components. The Hodrick-Prescott (H-P) filter was created with the assumption that unobserved shocks to trend output occur all the time. Academic economists viewed the H-P filter as a technique to distinguish output's long-term trend from its short-term business-cycle variation. Applied economists adapted the filter by identifying the long-term trend as potential output. Relative to a time-trend approach, this method of estimating potential output may reduce the number of same-sided policy errors by attributing a portion of every change in output to a change in potential output. Thus, the filter adapts to changes in output by incrementally revising its estimate of potential output as new data arrive, unlike a kinked time-trend method.

Although the H-P filter may reduce errors in comparison with time-trend methods, as a mechanical filter, it makes no attempt to actually identify demand and supply shocks; rather it simply provides a "flexible ruler" that extracts a smoothed version of output. To separate demand-side and supply-side influences on output requires economic structure. Laxton and Tetlow (1992) and Kuttner (1991; 1992; 1994) pursue this insight by using a Phillips-curve relationship together with a filter to measure potential output. The simple idea is that if inflation is rising and this change in inflation cannot be ascribed to other factors such as relative price shocks, then output must be above its potential. Laxton and Tetlow, in Monte Carlo experiments, show that using this information can significantly improve the confidence that a policy maker places on a point estimate of potential output relative to the estimates produced by an H-P filter alone.

Nonetheless, these authors also show that there is considerable uncertainty surrounding their point estimates of potential output, even with the improved methods. Moreover, the uncertainty about the *current* level of potential output is greatest *precisely* when the estimate matters the most to policy makers. The reason for this is straightforward. If an innovation in output is a combination of a temporary demand shock and a permanent supply shock, observations of data subsequent to the innovation will contain additional information about its precise nature. For example, output will tend to remain at its current level if the shock is permanent, but will tend to return to its former level if the shock is temporary. After some time has passed, the nature of a particular shock becomes clearer, although some uncertainty always remains. However, while hindsight is helpful in interpreting history, it is not available to the policy maker, who must make decisions on the basis of current information.

The methodology described in this paper extends Laxton and Tetlow's work by using a model-based decomposition of output to identify the supply and demand shocks. A model-based method of estimating potential output has the advantage that it is capable of using additional information — beyond a Phillips-curve relationship — to identify the supply shocks that occur to the individual components of potential output. The identification of different types of supply shocks in this method is intended to make the estimator a more accurate tool.

Although a general model-based approach to the estimation of potential output has a great deal of merit, the method presented here recognizes that there are inherent limitations to the estimation of the current level of potential output, even though there remains room for improvement. As a result, how fast the estimator arrives at its final estimate of current potential output, and how much it revises its estimate — the “updating properties” of the estimator — are also important dimensions that are considered in this paper.

In addition, the method presented here recognizes that, from time to time, the researcher may have information regarding the level of potential output that is not formally incorporated into the filter. Accordingly, a channel is provided through which such additional information can be incorporated in the final estimates. This judgment may take the form of historical benchmarks or may take account of institutional or policy changes, such as the use of wage and price controls from 1975 to 1978 or the shift from a sales tax to a value-added tax with the introduction of the GST in 1991.

The method of estimating potential output described in this report is quite general in conception. The main idea is that a hybrid method combining economic structure with a time-series filter can build on the strengths of structural and filter-based approaches to estimating potential while avoiding some of their pitfalls. To implement this method requires a well-articulated economic model of the underlying structural relationships for the estimator of potential output to use. The model used to implement this approach is the Bank of Canada’s new Quarterly Projection Model (QPM), which is the main model used by the staff for economic projections and policy analysis. We call the method for estimating potential output the extended multivariate filter (or EMV filter for short).

Reliable and timely estimates of potential output are a key ingredient in any economic projection exercise. In developing the estimator of potential, particular care was taken to ensure that the structural relationships that are exploited to estimate potential output are consistent with the structure of QPM. Indeed, the method developed to estimate potential output was an important ingredient in the overall QPM model development project. As such, this report represents the fourth in a series of reports documenting QPM.[1]

1. Part 1 in the QPM series (Black, Laxton, Rose and Tetlow 1994) describes the steady-state model that pins down long-run outcomes in QPM. Part 2 in the series (Armstrong, Black, Laxton and Rose 1995) documents the algorithm that was developed to simulate the full dynamic model. Part 3 (Coletti, Hunt, Rose and Tetlow 1996) describes the dynamic model. See also Poloz, Rose and Tetlow (1994) for a more general and less technical discussion of the QPM system and its use at the Bank of Canada.

It is worth mentioning at the outset that QPM and the EMV filter are both actual working tools, and as such they evolve as new data and new research shed light on important issues. This report describes the implementation of the EMV filter at a particular point in time; since then there have been some modifications to the filter, based both on the results of new research and on what we have learned from our ongoing experience using QPM for economic projections and policy analysis. With one exception, the changes to date in the implementation of the EMV filter are minor relative to the implementation that is described in this report. The one more important change will be noted at the appropriate point in the text.

A second point worth mentioning is that the method described in this report to estimate potential output is used in the QPM system to produce an estimate of potential output over history — that is, up to the latest quarter for which National Accounts data have been published. When QPM is used for projections, the estimate of potential output used for the projected quarters is determined by the internal structure of the model, together with the starting point estimate for potential output and the assumptions that underlie the projection, such as the projected rates of population and productivity growth.[2]

The remainder of this report comprises three major sections. The first of these — Section 2 in the text — discusses the requirements of a policy-analysis environment and how it influences the selection of an estimation method. Section 3 describes the methodology behind the EMV filter. It documents the relationship between this filter, Laxton and Tetlow's multivariate filter and the Hodrick-Prescott filter. Section 4 discusses the calibration and evaluation of the EMV filter. Section 5 concludes with some suggestions for future work.

2. More specifically, when QPM is used for economic projections, the predicted level of potential output is obtained from the aggregate production function in the model using as inputs an assumed path for the equilibrium level of labour inputs, the capital stock that is consistent with the investment flows predicted by QPM, and an assumption for the path of total factor productivity.

2 A MODEL-CONSISTENT ESTIMATION OF POTENTIAL OUTPUT

Today there are many methods available for estimating the trend component of output. This section provides a non-technical, historical account of some of these methods, starting with Okun's (1962) pioneering paper. Methods of estimating potential output have progressed substantially since Okun's paper, largely in response to the important supply shocks and concomitant policy errors experienced in the interim thirty years. In reviewing the alternatives, the emphasis is on identifying their relative strengths in the context of a policy-simulation setting.

2.1 A short history

The estimation of potential output was pioneered by Okun's (1962) paper. He used two methods to estimate potential output. The first method imputed a level of potential output under the assumption of a constant (4 per cent) full-employment rate of unemployment. The second method estimated potential output by extrapolating a line through two benchmark years. Schiff's (1962) comments on Okun's paper presaged much of the work that has been done subsequently. His comments pointed out the desirability of explaining the participation rate, the length of the average work week, the level of labour productivity, and the size of the total capital stock. Indeed, he also suggested that the path of potential output might depend on the path of realized output and that the composition of aggregate demand could also determine the level of potential output.

Following Okun's paper, a view emerged among the majority of economists that a relatively simple method of estimation could provide a good approximation to the results obtained by a more elaborate method. The relatively constant post-war output growth and rate of unemployment provided evidence that fluctuations around a time trend could indeed be largely interpreted as business-cycle phenomena attributable to nominal disturbances. Schiff's cautionary comments were to be borne out only in the subsequent thirty years.

Although economists acknowledged the importance of major, readily observed supply shocks, such as the 1971 unemployment insurance reforms or the oil-price shocks, the standard method estimating potential output was a regression of the log-level of output on time. Dummy variables were commonly introduced to capture the effects of these major supply shocks. Even in work that employed a relatively sophisticated model of the supply side, such as Helliwell et al. (1971) and Clark (1979), a time trend continued to play the dominant role in representing potential output.

Experience has indicated, however, that the time-trend methods of estimating potential output can cause systematic policy errors for at least two reasons. For example, when a string of significant supply shocks is encountered that permanently lowers the level of potential output, the time trend will tend to overestimate its level for a prolonged period of time, which could lead policy makers to underestimate the degree of excess demand and trigger an increase in inflation. The time-trend method can also mislead policy makers about the uncertainty surrounding forecasts of potential output (Stock and Watson 1988).

Efforts to salvage the time-trend methodology, such as correcting for significant, observed supply shocks, or the introduction of kinks, are also unable to satisfactorily disentangle demand and supply innovations in output.[2] In an aggregate framework, the range of possible variables that can influence supply behaviour is vast — and for the most part unobservable by the economist. For instance, technological diffusion and governmental regulatory actions most certainly influence potential output. But, given the idiosyncratic content of each innovation, and the difficulty of measuring its impact, it is not practical to believe that the economist can incorporate this large amount of detail into an aggregate picture. Instead, it is necessary for the macroeconomist to treat potential output as a stochastic phenomenon.

The response of some academic economists to the supply shocks and the policy errors of the 1970s helped pave the way for this major recon-

2. Gordon (1990) and Braun (1990) are two recent authors who utilize a segmented time trend to estimate potential output.

ceptualization of potential output. The work took the form of two principal and complementary innovations. Nelson and Plosser (1982) demonstrated that most key U.S. macroeconomic time series contain a unit-root component — including output.[3] This finding implies that there is a permanent component in the "typical" output innovation, and a permanent change in output is often interpreted as evidence of a supply shock. Nelson and Kang (1984) showed that regressions of series with a unit root, or an I(1) series, on time can introduce spuriously periodic behaviour in the resulting residuals. This demonstration supported the modelling efforts of a second group of researchers. These economists started from the premise that the observed fluctuations in output can be explained by supply shocks. Researchers such as Kydland and Prescott (1982) and Long and Plosser (1983) demonstrated that a model of the macroeconomy that is Pareto-optimal, in which the sole contributor to output fluctuations is a supply disturbance, is capable of replicating relatively well the second moments of a selected number of real variables from the post-war U.S. economy, although some difficulties remain.

Policy-oriented research did not immediately synthesize these new developments. At the Bank of Canada, a macroeconomic model with a well-articulated supply block was developed (Rose and Selody 1985) and research into the sustained productivity decline was conducted (Stuber 1986). The estimation of potential output lagged behind these developments, however, with the result that it was often revised only in the face of overwhelming evidence of its change. Indeed, it is only recently, with the development of the Bank of Canada's new Quarterly Projection Model (QPM), that macroeconomic model-building and research on potential output has incorporated the insights of those economists in an operational, applied way.

An obvious question arises with respect to the emphasis on the supply side: Why not attempt to estimate potential output by modelling and

3. It is worth noting that this result has not gone unchallenged. Perron (1989) argues that output is better characterized as a stationary process with a single determininistic break in trend in 1973. Kwiatkowski et al. (1992) have also shown that some of Nelson and Plosser's (1982) results are not robust to alternative test procedures.

estimating the entire supply side of the economy? In principle, such an approach is indeed desirable.[4] Endogenous theories of growth, such as those in Romer (1986; 1989) or Lucas (1988) for instance, predict that even in the long run the supply side of the economy can behave in a radically different manner under alternative policy regimes. A summary of this complex set of relationships — potential output — can be very misleading, especially in forecasting situations where the summary can be very poor owing to the omission of important information. On a more practical level, however, research has indicated that modelling the supply side is a very difficult task; that many relations that appear to exist over one sample vanish over another; that institutions, policies and tastes change only gradually; and that a robust approximation is worth more than a fragile estimate that incorporates the entire supply side (see Rose 1988).

There is an alternative to returning to Okun's original position, however. Economists and econometricians have developed methods to estimate potential output that take advantage of the advances in economic theory and econometrics. While the real-business-cycle research program seems incapable of explaining some stylized macroeconomic facts, its emphasis on the supply side has provided a strong theoretical case for the abandonment of time trends. On the other hand, advances in econometric theory have permitted a greater range of statistical tools to be arrayed against the problem of extracting an estimate of potential output. Indeed, today the problem is perhaps a plethora of alternative methods to detrend output or estimate potential output.

Some of these contemporary methods are univariate in nature; that is to say, they use only information from realized output to infer the level of potential output. Methods that explicitly consider the stochastic nature of potential output include the H-P filter (Hodrick and Prescott 1981), Watson's (1986) unobserved-components model and the Beveridge-Nelson decomposition method (Beveridge and Nelson 1981). Univariate methods of estimating potential output, even those that consider its stochastic nature, are open to two criticisms. The first and most compelling from an

4. Rose and Selody (1985) undertake this exercise.

economic perspective is that these methods do not incorporate additional information about the state of potential output. In other words, they are subject to criticisms similar to those made of Okun's original method. The second criticism is that univariate methods are unable to produce an estimate of potential output without an a priori restriction on the correlation between supply and demand shocks.[5] In essence, the univariate methods are able to produce an estimate of potential output only by making an untestable assumption about supply and demand shocks, rather than utilizing additional economic information to identify the shocks. In practice, the identifying assumption is very important for the estimate of potential output (Watson 1986; Canova 1993). This sensitivity makes univariate methods relatively imprecise.[6]

There also exists a large selection of multivariate methods to estimate potential output. Among the most widely used is the Blanchard-Quah decomposition (Blanchard and Quah 1989). This method has been applied to many alternative data sets, but the common characteristic of all these applications is the use of a just-identified vector autoregression (VAR) to estimate the supply and demand components of each innovation.[7] Dea and Ng (1990) estimate, for example, a two-variable and a five-variable VAR in order to gauge the importance of supply shocks to the Canadian business cycle. DeSerres, Guay and St-Amant (1995) use a trivariate VAR to estimate potential output for Mexico. Another method, introduced by Kuttner (1991; 1992; 1994), uses a more structured approach. Kuttner takes Watson's (1986) unobserved-components model and adds a Phillips curve. The merit of this method is that inflation data are used in conjunction with output to disentangle supply and demand shocks. Finally, Laxton and Tetlow (1992) develop estimates of potential output by

5. Although this is readily apparent in the latter two methods (see Watson 1986), it is not always appreciated in the case of the H-P filter. However, Whittle (1983) demonstrates that the optimal smoothing parameter is a function of the variances and covariances of the supply and demand innovations (see also Bell 1984).

6. Additional reasons to reject univariate methods are given below.

7. DeSerres, Guay and St-Amant (1995) provide a more extensive discussion of this literature.

adding a Phillips curve and an Okun's law relationship to an H-P filter. This they called a multivariate filter.

2.2 Potential output and the policy-analysis model

There has been significant progress in economists' understanding of the importance of supply shocks and the need to disentangle supply and demand shocks if costly policy errors are to be avoided. The preferred method of estimating potential output is one that incorporates information from a variety of sources in order to better isolate the two types of shocks.

A method to effectively separate supply and demand shocks is not, however, the end of the story — the method must be able to work within the context of the range of issues that it is being used to address. For example, in the context of cross-country comparisons, Giorno et al. (1995) have stressed that not only must the method produce reliable estimates of potential output, but it must also be applied in a uniform manner across countries. In the context of a policy-analysis framework, the method used to estimate potential output must be consistent both with the simulation model and with the requirements associated with using this model to prepare economic projections.

The specific requirements of a policy analysis framework outlined above, together with the history of estimating potential output, suggest several desirable features for a method to estimate potential output that is to be used in a policy setting. The following properties were judged of prime concern in the context of QPM: consistency with the economic model (QPM); the ability to incorporate additional judgment in a flexible manner; the ability to both reduce and quantify uncertainty about the current level of potential output; and robustness to a variety of specifications of the trend component.

This is a demanding set of criteria for any method to satisfy. Moreover, these criteria are sometimes conflicting, so trade-offs emerge. While these criteria provide a set of "guiding principles," the resulting method to estimate potential output only goes part way towards satisfying each of these criteria. The implementation of the method developed in this report may suggest further improvements. We return to this theme in Sections 4 and 5. First, however, the criteria themselves warrant some discussion.

2.2.1 Model consistency

Constructing an estimator of potential output in a policy-analysis environment is typically approached in a somewhat ad hoc manner. In a typical exercise, potential output is estimated and then fed into a policy-analysis model to conduct counterfactual policy simulations or to make projections of the future under alternative policy scenarios. Rarely does the method used to estimate potential output incorporate the same set of assumptions about potential output as the policy-simulation model. This discrepency between the method used to estimate potential output and the use to which it is put exists in all of the work so far conducted by policy-oriented institutions (Giorno et al. 1995; Torres and Martin 1990; Adams and Coe 1990; Kuttner 1994). In part, this practice reflects the difficulty, discussed above, of modelling the supply side of the macroeconomy. In other words, exogenous measures of potential output are often used because it is hard to derive a suitable, stable specification for supply equations.

Because of this practice, however, the methods used to estimate potential output often overlook one or more of the implications of the policy-analysis model. Insofar as those implications are correct, then, such methods omit valid information and so produce estimates that are less reliable than they could otherwise be. An important case in point is the real wage / marginal product of labour relationship. In neoclassical models with a steady state, these two variables will have a stable, long-run relationship. Because the real wage reacts differently to supply shocks than to demand shocks, these variables will possess valuable information to differentiate the two shocks.

The incorporation of model-consistent criteria into the estimator of potential output raises a further set of questions, however. Typically, the theory on which the policy-analysis model is built is not sufficiently nuanced to avoid some rejection in econometric testing.[8] To return to the example of the real wage / marginal product of labour relationship,

8. French et al. (1994) and Kennedy et al. (1994) offer discussions of this issue.

researchers such as Côté and Hostland (1994) have had difficulty demonstrating the existence or absence of a long-run relationship between these two variables with Canadian data. In some cases, the data do not contain enough information to decide a particular proposition. In the face of such imprecision, one possible response is to introduce additional complexity into the model to allow the proposition to be decided with the available data. However, this means adding complexity to a model whose strengths lie along another dimension.

A policy-analysis model must balance parsimony and realism. Some researchers have begun to stress the importance of internal consistency and a solid microeconomic foundation in formulating policy-simulation models (Masson, Rose and Selody 1980; Rose and Selody 1985; French et al. 1994; Black et al. 1994; Coletti et al. 1996).

The focus on a tight theoretical foundation rests on two complementary observations. Policy-analysis models are most often concerned with medium- to long-term issues, such as the long-term implications of alternative fiscal policies (Macklem, Rose and Tetlow 1994). A short-term forecasting model, whose construction is dominated almost entirely by a concern for disequilibrium dynamics, is necessarily overparameterized. This overparameterization leads to questionable answers when these models are asked questions of a longer-term nature (Masson, Rose and Selody 1980; Rose and Selody 1985). A complementary objection to the use of overparameterized models comes from the Lucas critique (Lucas 1976). Lucas criticizes economic modellers for their practice of fitting models to historical data that are conditioned by a particular set of policies and then asking the model to answer questions about an alternative policy. Overparameterized models will give unsatisfactory answers to these questions because they fail to model the systematic changes in economic behaviour that are attributable to shifts in policy.

The choice to emphasize economic theory, and therefore to utilize certain propositions that may not be irrefutable, flows from the desire to analyse policy questions in a clear, consistent manner.

2.2.2 End of sample

The data that exist subsequent to a shock to output can help determine the permanent and temporary components of that shock. The estimate of current potential output obtained using this information will necessarily be less precise than an estimate of potential output that uses data four, eight or twenty quarters in the future. Adding extra economic information to the estimation process is a partial remedy (Laxton and Tetlow 1992). Nonetheless, the addition of contemporaneous indicators of the level of potential output cannot hope to identify current shocks to potential output as accurately as they can be identified with subsequent data, simply because all the effects of the current shocks will not be observed until several quarters in the future.[9] Laxton and Tetlow (1992) report this increased uncertainty with their multivariate filter, while Kuttner (1992) observes a similar phenomenon with his Kalman filter.

In the absence of precise estimates of the level of potential, a gauge of their uncertainty is desirable. The work of Laxton and Tetlow (1992), Kuttner (1994), and DeSerres, Guay and St-Amant (1995) neatly illustrates the degree of uncertainty in estimates of potential output that provide useful input to policy advice.

2.2.3 Additional judgment

A complementary method to reduce the uncertainty of an estimate of current potential output is to incorporate the informed judgment of economists charged with monitoring current developments. Typically, economists are able to bring some additional information to the estimate because of their detailed knowledge of the current situation. Incorporating additional judgment can be a compromise between explicitly modelling idiosyncratic events and ignoring important information.

The development of an explicit channel for additional judgment is also important in its own right. In a working environment, the estimates of potential output will inevitably be subject to judgmental revision. In order

9. This is a well-known result in the literature on filtering and smoothing and is a characteristic of any useful estimator (Anderson and Moore 1979).

to clearly distinguish between the contributions of the mechanical estimating procedure and the additional judgment, an avenue that allows this kind of judgment to influence the estimate is needed. In the absence of such an avenue, experience indicates that additional judgment tends to override the mechanical estimate of potential output for extended periods of time. Revisions to potential output consequently tend to be too infrequent. By providing an explicit channel through which such additional judgment can enter as a conditioning factor in the estimation process, the method for estimating potential output can continue to exploit useful economic relationships and the desirable updating properties of the estimator.

2.2.4 Robustness and trend misspecification

There is a large volume of research that attempts to document the relative importance of the permanent and temporary components of output. This research is typically directed at estimating a trend-cycle decomposition of output, and although the trend is not typically interpreted as potential output in this literature, this is a useful way to think of it.[10] Unfortunately, there is a fundamental problem with these statistical models that is inherent to a trend-cycle decomposition. Typically, an assumption must be made concerning the relationship between the permanent and cyclical innovations in output in order to estimate the decomposition. Since an identifying restriction is by definition untestable, it is not possible to discriminate on the basis of statistical criteria between reasonable and unreasonable assumptions. Furthermore, Quah (1992) has shown that for any integrated process there is an infinite number of decompositions, and we can make the cyclical component account for an arbitrarily large proportion of the variance in the underlying series. Consequently, the "sensibility" metric is also, in principle, unable to discriminate between reasonable and unreasonable decompositions.

The situation might be salvaged if, over an economically plausible range of specifications, the trend-cycle decomposition were robust to alternative identifying assumptions. Unfortunately, this is not the case.

10. Prescott (1986), for example, cautions that the filter employed in his work is not intended to capture the unconditional means of the economic data that he uses.

Watson's (1986) work considers two alternative decompositions, the first of which supposes that the cyclical innovation is orthogonal to the trend innovation and the second of which corresponds to Beveridge and Nelson's (1981) assumption that the cyclical and trend innovations are perfectly correlated. The first assumption would be the natural assumption to make if one believes that demand shocks drive the business cycle and longer-term real factors drive the growth path of the economy. The second assumption would be a natural assumption to make if one believes that the business cycle is an equilibrium response to the high-frequency innovations in the real factors that also determine the long-run growth path of the economy. Perhaps not surprisingly, the decomposition of output is quite different under the two alternative identifying assumptions. Typically, the former estimates a trend component with relatively little high-frequency variation, while the latter estimates a trend component that is virtually identical to realized output (Watson 1986). Canova (1993) confirms this finding on a wider class of decompositions and data sets.

Perron (1989) has also suggested that the innovation in the trend component of output may be an extremely low-frequency one. His research indicates that the post-war trend in U.S. output may be well represented by a time trend with a break in 1973 due to the OPEC oil-price shock. Perron's identifying assumption might be seen as the assumption that the supply side of the economy evolves along a largely deterministic path with an occasional, major innovation.

Two conclusions emerge from this literature. First, the identifying assumption and its accompanying economic rationale are more important to the estimation of a trend-cycle decomposition than the data themselves. The inevitable conclusion seems to be that economic structure is required in order to pin down an estimate of potential output. Second, given that there will always be uncertainty concerning the appropriate structure, it would be advantageous to have a method of estimating the trend component of output that is robust over some reasonable range of alternative specifications.

This section has presented four broad criteria that are useful metre-sticks against which an estimator of potential output can be assessed. No estimator of potential output can be expected to satisfy all these criteria, but an estimator that goes some way toward satisfying these modelling, statistical and institutional criteria will help produce a more accurate picture of the current economic situation.

3 THE EXTENDED MULTIVARIATE FILTER

Owing to the unique requirements of the policy-analysis framework, the commonly used methods to estimate potential output are largely unsuitable. Simple univariate Hodrick-Prescott (H-P) filters of output provide an improvement over linear trends, but make no attempt to identify demand or supply influences, and as such contain no link to the underlying economic theory. Other methods, such as Blanchard and Quah (1989) decomposition, are based on a single identifying assumption that is consistent with the policy-analysis model, but that ignores many of the model's other implications.

Rather than adapt one of the existing methods to the needs of a policy-analysis environment, a new method was developed that builds on the work of Laxton and Tetlow (1992). This method shares many similarities with recent work by statisticians on smoothing splines (Wahba 1990; Gao 1993). The general idea is that an unobserved variable — such as potential output — may be estimated by making it explain one or more relationships, subject to a basic smoothness constraint. This approach provides a manageable way to incorporate the multitude of implications contained in the policy-analysis model; it provides a transparent mechanism to incorporate additional judgment; and it deals in some manner with issues of trend misspecification. This section outlines the new method, dubbed the extended multivariate (EMV) filter, in an intuitive way after examining its predecessors. Section 4 then outlines the specifics of the filter and its link to QPM in more detail. A more technical treatment of the filter is provided in Appendix 2.

3.1 Background

3.1.1 The Hodrick-Prescott filter

Before explaining the multivariate and extended multivariate filters, it is useful to describe the H-P filter (Hodrick and Prescott 1981; Prescott 1986). In many respects, this filter strongly resembles the two-sided moving-average filter, except that its moving-average coefficients are a complicated function of a parameter that controls the "smoothness" of the trend component (Osborn 1995) (King and Rebelo 1989; 1993).[13]

The advantage of a mechanical filter like the H-P filter is that it allows the researcher to apply a well-known and well-understood method of estimating the trend component of a series. For a given shock to output, the filter will attribute some portion to a shock to the trend component, and some portion to a shock to the temporary component. The precise ratio depends on the degree of smoothness that the researcher chooses on an a priori basis.[14] In general terms, the filter allows the trend component in output to change in response to what it perceives as supply shocks. In a very mechanical way, without the use of information other than that of output itself, the H-P filter therefore provides the researcher with an estimate of potential output that contains a stochastic component.

A number of criticisms have been levelled at the H-P filter since its use became prevalent among applied researchers. In the context of estimating potential output, Laxton and Tetlow (1992) use Monte Carlo evidence to show that an H-P trend applied to output provides a relatively imprecise estimate of potential output. In particular, in their benchmark experiment, they find that the 95 per cent confidence interval around the H-P filter's estimate of the output gap is about 6 percentage points (that is, + or - about 3 percentage points). On the theoretical side, Harvey and Jaeger (1993) illustrate that the H-P filter can introduce spurious features into its estimate of the trend if the true data-generating process differs from the class for which the H-P filter is optimal.[15] Interestingly, Harvey and Jaeger's (1993) empirical estimates show that for U.S. real GNP, the H-P filter esti-

13. Diewert and Wales (1993) document Hodrick and Prescott's rediscovery of a smoothing technique developed in the 1920s by Whittaker (1923). Wahba (1990) discusses the recent history of similar smoothing techniques from a statistician's perspective.

14. The standard setting of the smoothness parameter λ for quarterly data is 1600. Although the common practice in the economics literature is to impose the smoothness parameter, it is also possible to estimate this parameter; see Wahba (1990) and Altman (1987) in a univariate context, and Côté and Hostland (1994) in a multivariate setting.

15. "Optimal" refers to minimum mean-square error. Whittle (1983) discusses the optimal filtering of stationary data and Bell (1984) extends their conclusions to non-stationary data. See King and Rebelo (1989; 1993) for a characterization of the H-P filter's properties.

mates a trend that is virtually identical to that produced by their preferred unobserved-components model. While this suggests that the general theoretical objections may not be significant for U.S. potential output, this result may be somewhat specific to the U.S. case and does not remove the general concern. This concern is reinforced by more recent results of Guay and St-Amant (1996). They take Harvey and Jaeger's (1993) results a step further by pointing out that the class of data-generating processes for which the H-P filter is optimal is not one that is typical of macroeconomic time series; they conclude that the conditions required for the H-P filter to provide a good approximation to the optimal filter are rarely met in practice.[16]

Another criticism of the H-P filter that is very important from the policy perspective is that its accuracy deteriorates near the end of the sample for which data are available. Laxton and Tetlow (1992) show that the uncertainty surrounding the H-P filter's estimate of potential output widens very noticeably in the final twelve quarters. Prior to the final twelve quarters, the 95 per cent confidence interval for potential output in their benchmark Monte Carlo simulations is about 6 percentage points; this widens to about 8 percentage points at the last observation in the sample. The reason for this deterioration in accuracy is straightforward: as the end of sample is approached, the filter becomes one-sided and the contemporaneous data are given a weight that is much greater than in the middle of sample (Buja, Hastie and Tibshirani 1989; Laxton and Tetlow 1992). In effect, contemporaneous data are used by the filter as both current and future information at the end of the sample. Since the contemporaneous level is, at best, an indicator of the future level, increased error occurs. This proxy for future data also tends to alter the co-movement of the trend component with the measured data. If the mid-sample estimate is compared with the

16. A number of criticisms have also been levelled at the H-P filter in the real-business-cycle literature. King and Rebelo (1989), for instance, explain that they became interested in the effects of the H-P filter on the evaluation of real-business-cycle models when they were unable to replicate results with untransformed data. Cogley and Nason (1995b) show that real-business-cycle models contain weak internal propagation mechanisms: the "good fit" of these models disappears with untransformed data. However, because these two articles principally criticize the use of the H-P filter in the real-business-cycle literature, they are not directly relevant to its use to estimate the level of potential output.

end-of-sample estimate of the output gap, it is apparent that the latter, which uses only contemporary and lagged information, tends to lag the former, which uses both lead and lagged information. The use of contemporaneous data as a proxy for future data therefore tends to skew the timing of the end-of-sample estimate of potential output. Figures 1, 2 and 12 show this effect in three different ways. Figure 1 shows that at the end of sample the H-P filter places a very large weight on the final two or three observations, while in the middle of the sample it more evenly weights observations around the period in question. Figure 2 shows the gain and induced phase change of the H-P filter at the end and middle of the sample. The end-of-sample H-P filter has a gain greater than 1 at low frequencies, which indicates that it will tend to favour low-frequency but stationary innovations in its estimate of the trend relative to the middle-of-sample H-P filter. Figure 2 also shows that the middle-of-sample H-P filter introduces no timing change in its estimate, while the end-of-sample H-P filter distorts the timing of its estimate. Figure 12 indicates how the H-P filter works in practice by plotting the middle-of-sample and end-of-sample estimates of potential output. It is apparent that the rolling or end-of-sample estimate tends to lag the middle-of-sample estimate, and that it tends to incorporate a good deal of variation that is judged ex post to be stationary. This phenomenon is especially noticeable in the two most recent business cycles.

3.1.2 Laxton and Tetlow's multivariate filter

While the H-P filter may provide a useful tool for some purposes, its properties at the end of the sample and its univariate nature make it unsuitable as a fully elaborated, model-consistent estimator of potential output. Furthermore, its mechanical method of separating demand and supply shocks means that the H-P filter is incapable of providing the correct interpretation of a pure demand or supply shock.

Laxton and Tetlow (1992) propose using macroeconomic relationships in conjunction with the H-P filter's mechanical judgment to estimate the level of potential output. This suggestion is motivated by a desire to improve the efficiency of the H-P filter and to increase the economic content of its estimates by adding macroeconomic relationships.

Laxton and Tetlow extend the H-P filter in two principal directions. First, the multivariate (MV) filter adds two macroeconomic relationships in order to better identify supply and demand shocks. The MV filter inverts a Phillips-curve relationship by taking inflation and output data and estimating potential output to explain inflation. The filter also uses an “Okun’s law” relationship that links a change in the underlying non-accelerating-inflation rate of unemployment (NAIRU) to a change in the level of potential output.

Second, the MV filter also incorporates an explicit mechanism with which to incorporate a researcher’s additional judgment about the level of potential output. Additional judgment might be used when unmodelled exogenous influences are believed to have disrupted the relationships used in the filter. For example, the introduction of the Goods and Services Tax (GST) resulted in a large change in inflation that was unexplained by the size of the output gap at the time of its introduction. This shock would tend to lower the estimates of potential output if left uncorrected. The MV filter allows the researcher to reduce the impact on the final estimate of potential output of such anomalous observations.

The Monte Carlo experiments conducted by Laxton and Tetlow indicate that the macroeconomic relationships make the MV filter a more reliable estimator of potential output than the univariate H-P filter. In particular, the MV filter is a more efficient estimator of the current level of potential output.

While the MV filter enjoys many advantages over the univariate H-P filter, there are also areas where its performance might be further improved. Several of these areas refer strictly to Laxton and Tetlow’s implementation. For example, expected inflation in the Phillips curve is modelled as a moving average of past observed inflation, although recent research indicates that this may not be a wise choice (Section 3.2.3, below). In addition, the labour-market gap and the NAIRU are left unexplained by the MV filter — Laxton and Tetlow use a largely judgmental estimate of the labour-market gap, a practice that can offset the other advantages of the MV filter.

A final difficulty, which is evident in Laxton and Tetlow's Monte Carlo experiments, is that the confidence interval around the MV filter's estimate of potential output widens noticeably at the end of sample. In part, this increased uncertainty is unavoidable, given the lack of future observations to help disentangle supply and demand shocks. However, their Monte Carlo evidence indicates that further reductions in uncertainty at the end of the sample might be obtained by incorporating additional conditioning relationships into the filter.

3.2 An overview of the extended multivariate filter

The extended multivariate (EMV) filter builds on Laxton and Tetlow's MV filter in four important respects. First, the EMV filter uses more information to condition the estimate of potential output by exploiting more of the structural relationships that are embodied in the policy-analysis model, QPM. To accomplish this, output is first decomposed according to an aggregate production function. This allows information contained in relationships involving potential output's components to be usefully exploited in building up potential output, rather than simply using relationships involving potential output itself, as in the MV filter.

Second, consistent with a range of empirical work and the structure of QPM, the EMV filter incorporates an asymmetric inflation-output relationship. The asymmetry implies that excess demand is more inflationary than an equivalent degree of excess supply is disinflationary.

Third, the modelling of inflation expectations is treated differently in the EMV filter. Fourth and finally, the EMV filter incorporates new restrictions at the end of the sample in order to reduce the likelihood of a spurious estimate.

3.2.1 A decomposition of output

The MV filter is applied directly to actual output. Consequently, it omits several important relationships that economic theory suggests might be useful in distinguishing supply and demand shocks. The only way to incorporate these additional relationships is to decompose output into a set of components for which economic theory provides important predictions. There is no natural or obvious decomposition of potential output that produces the most information, however. One decomposition, commonly used in the "production-function approach," splits potential and actual output into three components: a labour input, a capital input and a total factor productivity input. An estimate of potential is obtained by estimating the short-run equilibrium values of these variables. For instance, the equilibrium labour input might be estimated using an estimate of the NAIRU, the equilibrium capital stock might be simply the actual capital stock, and equilibrium total factor productivity might be estimated with an H-P filter. The OECD uses a similar method (Giorno et al. 1995).

The EMV filter does use a production function to motivate its decomposition of output, but it uses an alternative decomposition. If it is assumed that the aggregate production function is a constant-returns-to-scale Cobb-Douglas function, and output is decomposed into the sum of the marginal product of labour and labour input less the labour-output elasticity.[17] [18] The advantage of this decomposition is twofold.

First, the decomposition omits any direct reference to the capital stock. This helps produce timely estimates because the capital stock is measured with a long lag and only on an annual basis. It is also useful from the perspective of accuracy since the capital stock is subject to significant measurement problems, and since relying on poorly measured data can introduce unnecessary errors.

17. Implicitly, it is also assumed that firms are always on their demand curve for labour and that they operate in perfectly competitive markets.

18. All variables are assumed to be in logarithmic levels, unless stated otherwise.

Second, there are more substantive economic predictions about the short-run equilibrium marginal product of labour than about equilibrium total factor productivity. For example, neoclassical theory suggests that the gap between the marginal product of labour and its short-run equilibrium value should be useful in explaining movements in unemployment and wages, and the equilibrium marginal product of labour and producer real wage should enjoy a stable long-run relationship. On the other hand, while endogenous models of economic growth have recently attracted considerable attention in macroeconomics, generally accepted economic relationships that might be used to pin down the evolution of total factor productivity remain elusive.

The second component in the EMV filter's decomposition of output is the labour input. There are many possible ways to estimate the equilibrium labour input. The method chosen here is to estimate the short-run equilibrium aggregate participation rate and the NAIRU using reduced-form regressions. The deviations from these labour-market equilibrium conditions then provide one of the many bits of information that are used to condition the estimate of potential output. The resulting measure of potential output does not, therefore, vary in lock-step with the deviation of actual unemployment from the estimated NAIRU, although on the margin this labour-market gap will influence the EMV filter's assessment of the output gap.

The third component in the EMV filter's decomposition of output is labour-output elasticity, or labour's share of income as it is sometimes called. This is estimated by fitting a very "stiff" H-P trend (with a smoothness parameter λ of 10 000) to the quarterly labour's share series. The result is an estimate of the labour-output elasticity that changes slightly over the sample period.

An alternative and indeed simpler and more obvious method of determining the labour-output elasticity would be to restrict it to a constant equal to its average value over the historical sample. The decision to allow labour-output elasticity to change very gradually over history reflects the fact that, with a constant elasticity, the evidence in favour of a long-run relationship between the marginal product of labour and the producer real wage is only about as strong as the evidence against it (Côté and Hostland 1996). However, in QPM there is a well-defined steady state that is consistent with neoclassical growth theory (see Black et al. 1994). Thus, in QPM, competitive forces ensure that workers are paid their marginal product in the long run.

When tension arises between the relatively simple theory embodied in the model and the real-world data it is designed to explain, one approach is to extend the theoretical framework to encompass some of the additional features left out of the simpler model. The cost, of course, is increased complexity, which makes the model a less effective tool for explaining the main forces at work in the economy. As a result, in some cases it may be preferable to maintain the simpler theoretical structure while transforming the data to make it more consistent with the structure of the model. This approach is taken with labour-output elasticity. Using a very stiff H-P filter to estimate labour-output elasticity has the effect of removing the low-frequency variation in the real wage / marginal product of labour gap so that the business-cycle frequency variation in this gap can be effectively used to identify supply and demand shocks.

3.2.2 Asymmetric Phillips curve

The second major feature of the EMV filter is that it incorporates an asymmetric relationship between inflation and the output gap. As documented in Coletti et al. (1996), asymmetry in the short-run relationship between the output gap and inflation is an important feature of QPM. Accordingly, this feature is also incorporated into the EMV filter so that the measurement of potential output is itself consistent with the model.

The asymmetry that is built into QPM and the EMV filter implies that inflation changes to a greater degree and more quickly in response to excess demand than to an equivalent degree of excess supply. This feature is based on several empirical studies of the inflation process in Canada and elsewhere. In particular, based on their estimated Phillips curves for Canada, Laxton, Rose and Tetlow (1993b) conclude that inflation rises more when the economy is in excess demand than it falls when there is an equivalent amount of excess supply. Laxton, Rose and Tetlow (1993a) and Laxton, Shoom and Tetlow (1992) also make the case, on the basis of Monte Carlo experiments, that several competing specifications of the Phillips curve appear to fit the data when the true data-generating process is an asymmetric Phillips curve. Subsequent cross-country work by Laxton, Meredith and Rose (1994), Clark, Laxton and Rose (1995) and Turner (1995) indicate that a non-linear Phillips curve also fits the data of a larger sample of countries reasonably well.

3.2.3 Inflation expectations

Perron (1994), Laxton, Ricketts and Rose (1994), Ricketts and Rose (1995) and Hostland (1995) provide evidence of significant non-linearities in the inflation process in Canada. In particular, they all find that the time-series behaviour of inflation, such as its persistence, have changed markedly over time. These results imply that simply burying expectations of inflation in lags of inflation in an estimated Phillips curve may produce seriously misleading results.

To address this problem, the EMV filter puts some weight on a survey of professional Canadian forecasters in the construction of its proxy for expected inflation. Although in the past these survey measures have been treated with suspicion, the modelling efforts of Laxton, Ricketts and Rose (1994) provide a persuasive explanation for the apparent “irrationality” of these forecasters. They estimate a Markov-switching model of the inflation process to allow for changes in the inflation process, and associate the estimated changes in the persistence of inflation with shifts in the monetary-policy regime. An implication of their model is that when there is uncertainty about the true policy regime, rational agents can make forecasting errors that are correlated ex post.[19]

3.2.4 End of sample and the role of additional judgment

The MV filter shows that there are efficiency gains to be made by introducing additional conditioning relationships, and that these gains appear at

19. Taylor (1975) also shows that learning can introduce serially correlated expectational errors, but his model, unlike that of Laxton, Ricketts and Rose, does not imply that inflation is a non-linear process.

the end of the sample as well as in mid-sample. Therefore, it can be expected that the EMV filter's decomposition of output reduces uncertainty about the current level of potential output relative to the MV filter. On the other hand, there is a limit to the gains that can be made from these additional relationships simply because future data will always contain additional information about the precise composition of shocks that have affected the macroeconomy.

One implication of this is that as new data arrive, the assessment of potential output of the past several quarters may change as the demand and supply components of past shocks become more apparent. But at each point in time, the best that can be hoped for is that the available information is used efficiently.

Another implication is that it may be useful to augment the relationships in the filter with additional judgment, particularly at the end of the sample. Such judgment in the EMV filter can be implemented in one of three ways. First, the researcher can "benchmark" the unobserved series by requiring that it pass through a particular set of points. This is useful if in some periods the researcher has other sources (such as firm-level surveys) that provide information on the level of potential output at particular moments in time. Second, the researcher is able to reduce the importance of those historical episodes where a structural relationship is believed to have broken down. For example, the spike in inflation in 1991Q1 reflects the introduction of the GST and is not indicative of the underlying relationship between inflation and the output gap. Accordingly, this period can be unweighted so that the filter does not interpret the rise in inflation in this quarter as evidence of excess demand. Finally, the researcher is also able to impose strong judgment on the growth rate of the unobserved series. The latter channel for judgment is very useful because it can reduce the dependence of the filter on the final set of data points when it estimates the current level of the unobserved variable. Independent research on the factors that determine the short-run growth of potential output may also be incorporated through judgment.

4 SPECIFICATION, CALIBRATION AND EVALUATION

The previous sections have explained the general approach that is taken in the EMV filter. As with any exercise in quantitative economics, there remains the problem of selecting the appropriate functional forms and parameters for implementing this general approach. The choice of functional forms is a twofold exercise. First, the relationships that condition the EMV filter's estimate of potential output need to be embodied in an equation of some form. For the purposes of computational convenience, the equations were specified to be linear in the unobserved variable. Second, these conditioning terms must be somehow combined to produce an estimate of potential output. The method chosen here, as with the H-P and MV filters, is to specify an objective function that contains the (weighted) average of the squared errors from each of the conditioning terms. The value of the unobserved variable that best explains the conditioning terms or minimizes the squared errors, subject to the restrictions imposed by the researcher, is selected as the EMV filter's estimate of that unobserved variable. The combination of linear conditioning terms and a quadratic objective function implies that the "best" (in the sense of minimum mean-squared error) estimate is a linear function of the data in the conditioning terms (see Appendix 2 for details).

An example may be instructive. One input into the construction of potential output is the equilibrium rate at which the working-age population participates in the labour force. The unobserved variable here is thus the equilibrium participation rate. One source of information — or conditioning term — that can be used to estimate the equilibrium participation rate is the trend participation rate, estimated from a reduced-form model. The deviation of the equilibrium participation rate from its estimated trend is squared, and this squared "error" is assigned a weight in the estimate of the equilibrium participation rate. More specifically, the estimated equilibrium participation rate is chosen to minimize the squared errors from this conditioning term and squared gaps between the actual and equilibrium participation rate series, subject to a smoothness constraint imposed by the researcher. Thus, in setting up the filter, one must specify functional forms to determine the errors coming from the conditioning terms, as well as the method of combining the conditioning information with the filter.

The choice of the EMV filter's parameters, or its calibration, also involves two distinct types of choice. First, the equations that express the conditioning relationships require parameters that determine the influence of the unobserved variable on the observed data. In the context of the participation rate example above, parameters must be attached to the various factors that are thought to influence the trend participation rate. Second, the EMV filter also requires weights or parameters conditioning the adherence of an unobserved variable to a particular conditioning relationship. So, for example, in determining the equilibrium participation rate, some weight must be placed on the deviations of the equilibrium participation rate from the estimated trend participation rate that is obtained from reduced-form estimates.

The parameters in the conditioning relationships are calibrated according to evidence from relevant empirical studies similar in spirit to the real-business-cycle school's calibration approach. The parameters that determine the sensitivity of the unobserved variable to a specific conditioning term, or the weight parameters, are more difficult to select. The first difficulty is that, in the absence of judgment, the weight parameters should reflect the variability and persistence of each of the conditioning terms. The second difficulty is that, for some of the conditioning terms, the weights are used to impose the researcher's judgment and are therefore determined a priori (see Appendix 2). To deal with these difficulties, the weight parameters in the current EMV filter were selected with the use of benchmark settings, Monte Carlo experiments and judgment. In addition, in order to restrict the dimension of the parameter selection problem, the weights do not model or correct the persistence properties of the individual residuals.

No method of specifying the EMV filter's functional forms and choosing its parameters can be considered infallible. For that reason, the properties of the EMV filter under alternative weight settings are examined. The EMV filter's updating properties are also examined and contrasted with the properties of the H-P filter. Finally, the properties of the residuals from the filter are examined in order to determine to what extent information is lost by ignoring the covariance structure of the residuals.

4.1 Specification and calibration

The EMV filter's estimate of potential output is built up from the decomposition of output into the marginal product of labour, the labour input, and the labour-output elasticity. This decomposition is obtained by simple manipulation of the Cobb-Douglas production function that is at the heart of the supply side of QPM. The aggregate Cobb-Douglas production function is

$$Y = (TFP) \cdot N^{a} K^{1-a} \quad , \tag{4.1.0a}$$

where Y is output, N is labour input, K is the aggregate capital stock, TFP is the level of total factor productivity, and a is the labour-output elasticity (or labour's share of income). The marginal product of labour is therefore

$$\frac{\partial Y}{\partial N} = a\frac{Y}{N}. \tag{4.1.0b}$$

Thus, output can be written as

$$Y = \frac{\partial Y}{\partial N}\left(\frac{N}{a}\right). \tag{4.1.0c}$$

So, taking logs of both sides we have

$$y = n + \mu - \alpha \ , \tag{4.1.0d}$$

where y, μ, n and α are the logs of output (Y), the marginal product of labour $\left(\frac{\partial Y}{\partial N}\right)$, labour inputs ($N$), and the labour-output elasticity (a) respectively.[20]

In the present application, the labour input is measured as total persons employed, and the labour-output elasticity is estimated using the

20. Although this report is about estimating potential output over history with a formalized production function in QPM, it is useful to outline how potential output is treated when QPM is used for economic projections. As noted in the Introduction, the EMV filter is used up to the latest quarter for which National Accounts data have been published.

(Continued on next page.)

share of labour and unincorporated farm and business income in total national income.[21] Because economists only observe output, employment and labour's share of income, equation 4.1.0b is used to measure the marginal product of labour. The economic assumptions underlying this particular decomposition of output are that the production technology is Cobb-Douglas in labour and in all other inputs and that markets are perfectly competitive. Were these assumptions invalid, the variable (μ) might be interpreted as a scaled average product of labour, but it would not be correct to interpret it as the marginal product of labour.

The EMV filter computes an equilibrium level of employment ($\hat{n}$), the marginal product of labour ($\hat{\mu}$) and labour-output elasticity ($\hat{\alpha}$) in order to estimate potential output ($\hat{y}$). To estimate equilibrium employment, we use the identity that total employment is simply the total working-age population *(POP)* multiplied by the participation rate *(p)* multiplied by the employment rate (*e*), or $\exp(n) = POPpe = POPp(1\text{-}u)$, where *u* is the unemployment rate. The EMV filter estimates the equilibrium level of employment by estimating the equilibrium participation rate ($\hat{p}$) and the NAIRU ($\hat{u}$). The equilibrium level of employment is then calculated as $\exp(\hat{n}) = POP\hat{p}(1-\hat{u})$ or $\hat{n} = \log[POP\hat{p}(1-\hat{u})]$.

20. (Continued) When QPM is used for projections, the estimate of potential output is built up from the production function (4.1.0a), with total factor productivity and labour inputs set to their trend or equilibrium levels. More specifically, in logs, potential output (y^*) is the solution to

$$y^* = tfp^* + a^*n^* + (1-a^*)k \quad ,$$

where lower-case letters are the logs of the upper-case counterparts in (4.1.0a), and variables with "*" are set to their trend or equilibrium level. Operationalization requires paths for *k*, *tfp** and *n**. The capital stock (*k*) is simply cumulated from the investment flows predicted by the model, given the actual capital stock at the start of the projection. The standard assumption for trend total factor productivity is that in the medium to long term, it is determined by the historical rate of convergence of Canadian productivity towards the level of productivity of the world productivity leader — the United States. A short-run path for *tfp** then links the EMV filter's estimate of *tfp** at the start of the projection to this medium-term path for *tfp**. The short-run behaviour of *tfp** is based on its typical cyclical behaviour. The equilibrium employment rate in the model is determined by assumptions regarding the rate of population growth, the trend participation rate, and the NAIRU. Finally, the equilibrium labour-output elasticity is set to a constant equal to the average labour share of income over history.

21. A richer measure of labour inputs would also include hours worked per worker, but in this implementation the measure of labour inputs is simply the number of workers.

4.1.1 *Equilibrium participation rate*

The equilibrium participation rate $(\hat{p})$ is estimated by fitting it to the observed participation rate and a specialist's estimate of the trend participation rate, subject to a smoothness constraint. The estimate is obtained formally as

$$\hat{p} = \max_{\hat{p}} \quad -(p-\hat{p})'W_p(p-\hat{p}) - (\hat{p}-s)'W_s(\hat{p}-s) - \lambda\hat{p}'D'D\hat{p}, \tag{4.1.1a}$$

where p is the observed participation rate, s is an estimate of the trend participation rate based on evidence from reduced-form estimates and informed judgment, W_p is the weight matrix on the participation-rate gap, W_s is the weight matrix on the structural estimate of the trend participation rate, λ is the smoothness parameter, and the matrix D second differences the series in question.

The smoothness parameter is set to 16 000 (or ten times Hodrick and Prescott's preferred setting for output) in order to obtain a very smooth estimate of the equilibrium participation rate. The two weight matrices are set equal to the identity matrix.[22]

4.1.2 *NAIRU*

As discussed in surveys of the empirical literature on the NAIRU by Rose (1988) and Setterfield et al. (1992), robust structural estimates of the NAIRU have proven elusive. In particular, both studies find that estimates vary considerably depending on the methodology used, the variables in

22. As noted in the Introduction, the EMV filter is designed as a flexible tool that can be updated through time as new research suggests changes. This report documents the implementation of the filter at one point in time, and since that point further research examining participation rates across age and gender categories has suggested that the aggregate trend participation rate may be lower than previously estimated. The main factors that contribute to the lower participation rate are the flattening out of the participation rate of women 25 years and over, the continued rise in school enrolment rates for people 15 to 25, and the increase in early retirements. The new research uses reduced-form participation rate equations for the various age / gender categories to build up an estimate of the aggregate trend participation rate. In the current implementation of the filter, this reduced-form estimate is given all the weight so the filtered participation rate $\hat{p}$ is simply set equal to the trend participation rate s produced by the reduced-form model. In other words, the matrix W_p is set to the zero matrix, and the smoothness parameter is also set to zero. For a discussion of the resulting estimate of the trend participation rate, see Technical Box 2 in Bank of Canada (1996, 9).

the estimation, and the sample period. Since the NAIRU is an important input into the measurement of potential output, this fragility of structural estimates poses a problem — uncertainty about the NAIRU translates into uncertainty about potential output. The use of the EMV filter allows a variety of evidence to be combined, thereby diversifying across different types of evidence. The hope is that this will produce more robust estimates of the NAIRU. Nonetheless, it must be recognized that considerable uncertainty remains.

In principle, a wide range of evidence could be included in the filter. In the implementation documented here, the NAIRU is estimated by drawing on a structural estimate of the trend unemployment rate (c) that is based on the work of Côté and Hostland (1996); a price-unemployment rate Phillips curve (ε_π) based on the work of Laxton, Rose and Tetlow (1993b); the previous quarter's estimate of the NAIRU ($\hat{u}_{pr}$); a growth-rate restriction that is applied in the final fifteen quarters of the sample; and a smoothness constraint. Formally it is

$$\hat{u} = \max_{\hat{u}} \quad -(u-\hat{u})'W_u(u-\hat{u}) - (\hat{u}_{pr}-\hat{u})'W_{pr}(\hat{u}_{pr}-\hat{u}) - \hat{u}'P'W_gP\hat{u} \\ -(c-\hat{u})'W_c(c-\hat{u}) - \varepsilon_\pi{}'W_\pi\varepsilon_\pi \\ -\lambda\hat{u}'D'D\hat{u}. \tag{4.1.2a}$$

The weights on the unemployment-rate gap (W_u), and the gap between the structural estimate of the trend unemployment rate and the filter's estimate of the NAIRU (W_c) are set to the identity matrix. The weight on the revision (W_{pr}) is unity in all periods except the final, when it is zero. The weight on the Phillips-curve residual (W_π) is set to unity in all periods except 1991Q1, the quarter in which the Goods and Services Tax (GST) was introduced. In this quarter it is set to zero. The weight on the steady-state growth restriction (W_g) is set to 64 for the last 15 quarters at the end of sample and is 0 otherwise. The matrix P first differences the series in question, so the "growth-rate" restriction has the effect of penalizing any changes in the unemployment rate near the end of the sample. The smoothness parameter (λ) is set to the standard setting of 1600.

There are several items that merit attention. Changes in the estimate of the NAIRU are penalized very strongly at the end of sample. This has the effect of reducing the importance of the last two observations for the current estimate of the NAIRU (Figure 1). Although the selected weight (64) is somewhat arbitrary, Monte Carlo tests indicate that this value tends to reduce uncertainty surrounding the current NAIRU without any offsetting losses elsewhere. Figure 3 shows the standard errors of the H-P filter augmented with a growth-rate restriction, when it is applied to a data-generating process with a random-walk trend and a very persistent cyclical component. The weights 32, 64 and 128 clearly reduce the uncertainty at the end of sample relative to the H-P filter. Weights less than 32 or greater than 128 tend to under- or overweight the growth-rate restriction and thereby decrease the accuracy of the filter.

The incorporation of the previous quarter's estimate of the NAIRU ($\hat{u}_{pr}$) is intended to reduce the incidence of large revisions to the level of the NAIRU due to the arrival of small amounts of new data. This constraint is complementary to the growth-rate constraint because it prevents substantial cumulative revisions to the NAIRU near the end of sample, while the growth-rate restriction tends to prevent large, quarter-over-quarter movements in the estimate of the NAIRU. The smoothness constraint can be viewed as a means to operationalize the belief that the NAIRU does not change sharply, and the changes that do occur are spread out over time.

The structural estimate of the trend unemployment rate serves a dual purpose. It provides a convenient means to insert new research on the trend unemployment rate without necessitating a complete overhaul of the method used to estimate potential output. It also permits a degree of judgment to be exercised at the end of sample. The structural estimate that is currently used in the EMV filter is based on the research of Côté and Hostland (1996), which uses cointegration theory to explain the stochastic trend in the unemployment rate.

Finally, ε_{π} is the residual from a Phillips curve. The Phillips curve is consistent with its counterpart in QPM and with the research of Laxton, Rose and Tetlow (1993b) on asymmetries in the inflation-output relationship. The asymmetry implies that a negative gap between actual unemployment and the NAIRU is more inflationary than an equivalent positive gap is disinflationary.

The expected inflation measure, (π^e), is constructed as an eight-period one-sided moving average of observed inflation prior to 1975; after 1975 it is a weighted average of the Conference Board of Canada's survey of commercial forecasters' inflation forecasts (π^econf) — interpolated from annual to quarterly frequency — and an eight-period one-sided moving average of observed inflation:

$$\pi_t^e = \frac{\left(\sum_{i=1}^{8} \pi_{t-i}\right)}{8}(s_t) + (1 - s_t)\pi_{conf,t}^e , \qquad (4.1.2b)$$

where $s_t = 1$ if $t < 1975Q2$ and 0.339 otherwise. The asymmetric Phillips curve is a piece-wise linear function of the unemployment rate gap (u-$\hat{u}$)

$$\pi_t = \pi_t^e + \sum_{i=0}^{8} (\alpha_i + \delta_t\beta_i)(u_{t-i} - \hat{u}_{t-i}) + \varepsilon_{\pi,t} . \qquad (4.1.2c)$$

The variable δ_t is an index of the state of the economy. It is set to 1 if the economy is judged to be in a state of excess demand, and 0 otherwise.[23] The parameters α_i determine the response of inflation in conditions of excess supply. They are -0.168/8 for i=1,...,8 and 0 otherwise. The parameters β_i summarize the response of inflation in conditions of excess demand. They are -1.023/4 for i=1,...,4 and 0 otherwise. This parameterization is

23. The economy is judged to have been in excess demand over the following periods: 1963Q3-1967Q2, 1971Q4-1974Q3, 1978Q3-1981Q4, and 1985Q2-1989Q3. The dates are broadly consistent with those obtained from the EMV filter. Related work with a non-linear estimation method that determines this state endogenously has also produced results that are consistent with these dates.

based on the econometric work of Laxton, Rose and Tetlow (1993b, Table 1).

4.1.3 *Labour-output elasticity*

Labour's share of income contains a large amount of high-frequency noise, in addition to a possible non-stationary component.[24] In an economy with a Cobb-Douglas aggregate production function and perfect competition, labour's share of income is equal to the labour-output elasticity. Departures from the perfectly competitive levels of wages and employment in an economy with nominal wage rigidities will induce business-cycle frequency variation in labour's share of income. The H-P filter is therefore applied to the measured labour's share of income with a large smoothing parameter (10 000) to remove the high-frequency variation. The smoothed component is preserved as the labour-output elasticity, while the business-cycle variation in labour's share of income is transferred to the marginal product of labour.

4.1.4 *Equilibrium marginal product of labour*

The short-run equilibrium marginal product of labour is estimated with information from: the previous quarter's estimate of the equilibrium marginal product ($\hat{\mu}_{pr}$); a growth-rate restriction that is applied in the final 15 quarters of the sample; the real producer wage (w); an inflation-marginal product of labour relationship (ε_{π}); a modified Okun's law relationship that relates the current change in the unemployment-rate gap to the lagged change in the marginal-product gap; and a smoothness constraint. Formally, it is

24. The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test of stationarity is 0.365, less than the 95 per cent critical value of 0.463 but greater than the 90 per cent critical value of 0.347. The augmented Dickey-Fuller (ADF) test statistic is -2.14, which is greater than the 90 per cent critical value of -2.58, and the Phillips-Perron (PP) test statistic is -11.351, which is greater than the 90 per cent critical value of -11.30. The first test fails to reject the null of stationarity while the latter two tests fail to reject their null hypotheses of non-stationarity. The ADF lag length was selected at 1, and the PP lag length was set to 12 (see Table 1 for further details).

$$\hat{\mu} = \max_{\hat{\mu}} \; -(\mu-\hat{\mu})'W_{mpl}(\mu-\hat{\mu}) - (\hat{\mu}_{pr}-\hat{\mu})'W_{pr}(\hat{\mu}_{pr}-\hat{\mu}) - (\hat{\mu}'P'-g')W_g(\hat{\mu}P-g) - (w-\hat{\mu})'W_w(w-\hat{\mu}) - \tilde{\varepsilon}_\pi{}'W_\pi\tilde{\varepsilon}_\pi - \varepsilon_o{}'W_o\varepsilon_o - \lambda\hat{\mu}'D'D\hat{\mu}. \tag{4.1.4a}$$

The weights on the marginal product gap (W_{mpl}), the real wage (W_w), and the modified Okun's law relationship (W_o) are set to the identity matrix. The weight on the previous quarter's estimate of the equilibrium marginal product (W_{pr}) is unity in all periods except the final, when it is zero. The weight on the marginal product/inflation relationship (W_π) is set to unity in all periods except 1991Q1 (the quarter in which the GST was introduced), in which it is zero. The weight on the growth-rate restriction is set to 64 over the final 15 quarters. The smoothing parameter (λ) is set to the standard 1600.

Several points deserve elaboration. First, the steady-state growth rate of the marginal product of labour *(g)* is set to 1.2 per cent per annum, slightly less than the mean rate of growth of the marginal product of labour over the historical period.

Second, $\tilde{\varepsilon}_\pi$ is the residual from an inflation/marginal product of labour relationship. Specifically, $\tilde{\varepsilon}_\pi$ is obtained by replacing $(u-\hat{u})$ in 4.1.3c with $(\mu-\hat{\mu})$. The presence of $\tilde{\varepsilon}_\pi$ in the filter is motivated by the idea that the deviation of the marginal product of labour from its short-run equilibrium level provides an alternative index of excess demand pressures. Given short-run labour hoarding on the part of firms, a fall in aggregate demand will cause labour productivity to fall below its short-run equilibrium (i.e., $\mu-\hat{\mu}<0$), and the resulting excess supply will be associated with falling inflation. Similarly, a positive supply shock will also be associated with falling inflation. The positive supply shock (such as a rise in total factor productivity) will lead to a rise in the equilibrium level of the marginal product of labour. Owing to adjustment costs and recognition lags, the actual marginal product of labour will typically not rise as quickly, so $\mu - \hat{\mu} < 0$. Again this is indicative of more slack in the economy and reduced price pressures.

Third and finally, the modified Okun's law relationship is intended to capture the quantity adjustment process that firms undertake when their marginal product deviates from its short-run equilibrium value. For example, if the marginal product of labour is above its equilibrium value, firms hire additional workers and the unemployment rate declines relative to its short-run equilibrium value. This relationship is captured in the dynamic equation

$$u_t - \hat{u}_t = \theta(\mu_{t-1} - \hat{\mu}_{t-1}) + 0.9(u_{t-i} - \hat{u}_{t-i}) , \tag{4.1.4b}$$

where u and $\hat{u}$ are the observed and filtered unemployment rates respectively (Section 4.1.2), and θ is "Okun's coefficient," which is set to -0.15.[25]

4.2 Evaluation

The EMV filter is designed as a tool that can estimate potential output in a timely and flexible manner, readily incorporate new research and the researcher's judgment, and remain consistent with an associated policy-analysis model.

Overall, the EMV filter does quite well. The decomposition of potential output into four components permits a good deal of economic structure to be applied to disentangle supply and demand shocks. This includes information from nominal dynamics, as well as structural relationships, such as the link between real wages and labour productivity. The view of nominal dynamics — in particular, the asymmetric Phillips curve — and the longer-run structural relationships in the filter are both designed and calibrated to be consistent with the associated policy-analysis model, QPM (Coletti et al. 1996). In addition, the EMV filter allows a researcher to carefully track the importance of his or her additional judgment, a feature that is important in a context where users may be con-

25. In this report, the relationships between the marginal product of labour and prices and the marginal product of labour and unemployment have been treated separately. In future work it may be beneficial to examine these relationships jointly.

cerned about both the “additional-judgment-based” and “pure EMV filter” estimates of potential output.

On the other hand, the structure of the filter does not match that of QPM in every respect. The principal difference is the incorporation of a time-varying labour-output elasticity in the EMV filter, whereas QPM uses a constant elasticity. There are two reasons for this inconsistency. First, a labour-output elasticity that is constant over a fixed historical period — such as the mean labour share — will be time-varying as new data are added (unless these data are ignored). Second, a constant labour-output elasticity tends to incorporate an excessive amount of low-frequency variation in the marginal product/real wage gap and therefore reduces the information in that relationship.[26]

In addition, there are many aspects of the EMV filter that are less easily evaluated. One major deficiency is the inability to compute confidence intervals around the point estimates of potential output. Such confidence intervals would provide useful, additional information for policy makers about the reliability of any particular estimate of the output gap.[27] A second related question is the robustness of the EMV filter to alternative ways of characterizing potential output’s data-generating process. In order to incorporate the full richness of the dynamic economic structure underlying the EMV filter, however, it is necessary to perform stochastic simulations on dynamic, non-linear, forward-looking rational expectations models. Such work would merit a technical report in its own right.

Evaluating the EMV filter in either of these two areas is beyond the scope of this paper. We propose a more modest goal. We examine the filter from the perspective of the choice of alternative weights on the structural relationships. Broadly speaking, our finding is that the output gaps pro-

26. In informal tests, it is found that over fixed historical periods the two alternatives produce very similar output gaps, although the marginal products of labour do differ more significantly. In a real-time setting, the differences would be more pronounced because of the additional, potentially spurious, low-frequency variation in the mean labour-share case.

27. Wahba (1990) provides one avenue for exploring this question.

duced under reasonable alternative methods of choosing the weights are very similar.

A second important question involves the updating properties of the EMV filter. We examine this question by running the filter recursively and comparing its estimate of the current output gap in each period with its full-sample estimate. This exercise tests the degree to which the EMV filter's estimate of the current output gap with only current information accurately reflects the estimate that incorporates future data.

Finally, we examine the information content of the structural residuals produced by the EMV filter. The residuals are tested for stationarity, and their auto- and cross-correlations are computed. In general, the residuals display a high degree of persistence and cross-correlation. This suggests that further research may also be warranted on the choice of the off-diagonal elements of the weight matrices.[28]

A less formal way of assessing the EMV filter is to compare the EMV filter's estimate of potential output with the estimate of potential output that is produced by applying a simple H-P trend to real output. Since the EMV filter uses a multivariate approach that combines H-P filters with economic information and alternative updating features, the difference between the EMV and H-P filters' estimates of potential output can provide an informal idea of the importance of the additional information in the EMV filter. The top panel of Figure 4 shows actual output alongside the H-P and EMV filter estimates of potential output; the bottom panel of Figure 4 plots the associated output gaps.

As shown, over certain periods, the output gaps suggested by the EMV filter differ considerably from those defined by the straight H-P filter. In broad terms, the two measures of potential output do tend to move together, but the size of the output gap, and thus the monetary policy implications, can be quite different. For example, through the first half of the 1970s, the EMV filter's estimate of potential output remains below the

28. Altman (1987) considers the related problem of simultaneously selecting a smoothing parameter and covariance matrix in a smoothing-spline problem with correlated residuals.

H-P filter's estimate, partly reflecting the acceleration of inflation through much of this period. With inflation rising, the EMV filter ascribes less of the observed output growth through this period to growth in potential output than does a simple H-P filter.

Near the end of the sample, the differences between the H-P and EMV filters' estimates of potential output often become more marked. In Figure 4, the sample ends in 1991Q4, and through 1991, the H-P filter estimates the average output gap at 1.8 per cent, while the EMV filter puts it at 3.4 per cent. This large difference near the end of the sample reflects the fact that over this range, the updating features of the EMV filter (such as the "stiffness" and "growth-rate" restrictions discussed in the previous section) become important, in addition to the economic structure embodied in the filter.

4.2.1 Determination of the weights on the structural relationships

There are several ways to determine the weight matrices for each structural relationship. In analogy with a generalized least-squares (GLS) estimator, the weight matrices might be estimated based on the covariance structure of the residuals. This procedure would be appropriate if the researcher were concerned with obtaining a least-squares estimate of the level of potential output that "whitens" the equations' residuals. Economists tend to be suspicious of such techniques, however. Although it might be a worthwhile enterprise to correct for the coarsest correlations, the covariances themselves are functions of policy parameters and should not be expected to be constant over time. An example of this would be the covariance of the marginal product of the labour/real wage gap. Policies that affect the speed of labour-market adjustment to shocks will alter the covariance of that gap. Although the size of these effects over history is an empirical question, it is less clear that the same evidence can be used at the end of sample if large policy changes have occurred whose effects remain to be observed.

Rather than attempt to estimate the covariance properties of the structural residuals, a more parsimonious method was used that weights the relationships by their unconditional variance.[29] First, potential output was estimated with the weights on all the relationships set to zero, so that only the smoothness parameters were non-zero. The unconditional variance of the residual from each relationship was computed, and then potential output was recomputed using the inverse of the variances as weights. Second, the procedure was repeated, this time starting with all weights on the structural residuals set to unity. Figures 5 to 7 show the results of the second calibration experiment in terms of potential output, the equilibrium marginal product of labour and the NAIRU. There are a few things to note about these figures. The most striking is that the estimated level of potential output with the alternative weighting methods deviates from the unit-weight case by no more than 2/3 of 1 per cent — and the maximum difference over the past fifteen years is not more than 1/10 of 1 per cent (Figure 5).

On the whole, there is not much evidence that a weight setting of unity for all structural relationships is grossly at odds with either method. As a result, the weights on the structural relationships in all the evaluation work that follows are set to unity.

4.2.2 Updating properties

One of the most important aspects of the EMV filter is its ability to estimate the current level of potential output using only current and historical data. There are two aspects to this task. The filter should ideally be able to provide an accurate estimate of potential output, and it should not alter the timing between potential and realized output. Figures 8 to 11 illustrate that the EMV filter generally does a good job in both departments compared with the H-P filter (Figure 12). A comparison of Figures 8 and 12 reveals that the EMV filter's current estimate of current potential output (the "rolling estimate" in Figure 8) is revised much less than the H-P filter's comparable estimate (the "rolling estimate" in Figure 12). In particular, the EMV filter's rolling estimate in the two most recent business cycles does not dis-

29. In the following discussion, the modified Okun's law relationship has been omitted because initial results were completely unreasonable. In effect, the relationship is underweighted on the basis of its variance. However, in light of evidence that this relationship is somewhat unstable, underweighting the relationship is probably appropriate.

play the pronounced out-of-phase property that is evident in the H-P filter's rolling estimate.

The reasons for the better end-of-sample properties of the EMV filter relative to the H-P filter lie in its decomposition of output. Figure 11 illustrates that the rolling estimate of the equilibrium participation rate generally does a good job predicting the full-sample estimate. Figure 9 shows that the current estimate of the NAIRU does a very good job predicting the full-sample estimate. Although some errors are committed, such as in the early 1970s, the additional information from the Phillips curve and the end-of-sample growth constraint clearly allow the EMV filter to predict its final estimate fairly consistently. The high contemporaneous correlation between the two unemployment-rate gaps and inflation should also be noted.

Of course, the EMV filter does produce some large errors when it uses only current information. An example occurs during the episode of wage and price controls, which were in place from 1975Q4 to 1978Q3, and in the years immediately following the removal of these controls (see especially Figures 8 to 10). The rising marginal product of labour through 1976-77 is consistent either with an economy experiencing a positive supply shock that raises the equilibrium level of productivity, or with an economy in excess demand that pushes output and productivity above a sustainable level. A positive productivity shock would be associated with slowing inflation, while an increase in excess demand would be associated with rising inflation. With the absence of a significant uptick in inflation over the 1976-77 period (due to the presence of wage and price controls), the EMV filter initially interprets the rise in output as containing an important supply component, so the estimate of potential output over this period is initially relatively high (Figure 8). However, when the sample is extended to include the acceleration of inflation — a symptom of excess demand — following the removal of wage and price controls, the estimate of potential output is revised downward in the earlier period accordingly. This episode indicates that while no estimate of the current level of potential output can eliminate errors, the EMV filter can reduce them. This episode also provides a good example of a period when there is a clear role for some informed judgment as an input to the filter as well as the incoming data. The implications of wage and price controls were predictable at least in qualitative terms when they were implemented, and given that the filter cannot take account of this institutional change, putting some weight on an informed judgment of the impact of wage and price controls helps reduce the error.

4.2.3 Residual dynamics

The aim of this section is to evaluate the importance of the omission of information on the covariance properties of the residuals. This question is examined from two complementary perspectives. The first asks if the filter's residuals are properly characterized as stationary processes. In other words, is it statistically appropriate to impose the structure that exists in the EMV filter? The second asks to what extent the residuals are autocorrelated and cross-correlated with their respective gap terms. Answers to these two questions should provide guidance for the future development of the EMV filter.

The residuals are tested for stationarity using three conventional tests: the augmented Dickey-Fuller (ADF), the Phillips-Perron (PP) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests (Dickey and Fuller 1979; Phillips and Perron 1988; Kwiatkowski et al. 1992).[30] The first two tests have as their null hypothesis that the series is non-stationary or I(1), while the latter test has the null hypothesis of stationarity. All tests are reported excluding a time trend but including a drift term. The Box-Ljung autocorrelation test is also reported for the residuals of the estimated equations.

Table 1 shows that the ADF and PP tests are able to reject the null hypothesis at the 10 per cent level for all structural residuals. In addition, the KPSS tests are uniformly unable to reject the null hypotheses that the series are stationary. The tests, therefore, indicate consistently that all the structural residuals are stationary and support the inclusion of those structural relationships in the EMV filter.

30. Amano and van Norden (1992) advocate this joint testing procedure on the basis of their Monte Carlo tests that indicate that joint tests reduce false acceptances of the non-stationarity hypothesis.

The marginal product, unemployment rate and output gaps are all safely characterized as I(0) processes, for the same reasons (Table 1).

The tests involving the first and second difference(s) of the NAIRU and short-run equilibrium marginal product of labour are somewhat more problematic. The first problem stems from the work of Ghysels and Perron (1993), which shows that unit-root tests have diminished power on inappropriately smoothed data. On the other hand, Harvey and Jaeger (1993) show that researchers who use the standard unit-root tests can be misled into believing that a series is I(1) when it is actually I(2). In any event, the unit-root tests do show that the second difference of both the NAIRU and the equilibrium marginal product of labour are stationary (Table 1). The first difference of the equilibrium marginal product of labour appears to be non-stationary according to the same tests. Both the ADF and PP tests are unable to reject their null hypothesis of non-stationarity at even the 5 per cent significance level and the KPSS test rejects its null hypothesis of stationarity at the 95 per cent confidence level. On the other hand, the unit-root tests are contradictory when applied to the growth rate of the NAIRU. Both sets of results provide qualified support for a non-stationary rate of growth of each series. In the case of the equilibrium marginal product of labour, this behaviour might be explained by the convergence hypothesis.[31]

The stationarity tests indicate that each residual in the EMV filter is autocorrelated to some degree. Table 2 shows this fact by estimating each residual as a nine-lag autoregressive process. Figures 13 to 15 show the same thing with the autocorrelogram of each residual. In the case of the output and unemployment-rate gaps, a reasonably high degree of autocorrelation is desired. Indeed, both of these residuals appear to be well approximated by a four- or five-lag autoregressive or moving-average

31. Over the post-World War II period, Canada's average growth rate of real output has exceeded that of the United States, but has been converging towards the trend U.S. growth rate. This is consistent with the convergence hypothesis, which predicts that growth in Canada will exceed that of the country with the highest level of productivity in the world — the United States — since Canada can benefit from technological catch-up with the world leader. However, as the technology gap closes, average growth in Canada will decline to the trend rate in the United States.

process. Table 2 suggests the former, while the autocorrelogram (Figure 13) suggests the latter. Each result indicates that the EMV filter's estimate of the output gap contains more dynamics than other estimates.[32] On the other hand, the marginal-product gap shows a relatively low degree of persistence: its autocorrelation at lag 1 is 0.68 (versus the two former gaps' 0.90), and the autocorrelogram suggests a two- or three-lag moving-average process (Figure 13). It appears that the marginal product of labour tends to fluctuate around its equilibrium value at a relatively high frequency. The low persistence of the marginal-product gap is inconsistent with other research that shows labour demand often deviating from its equilibrium level for sustained periods of time (Amano 1995; Amano and Butler 1995). It should also be noted that the marginal-product gap also shows signs of seasonality with the lag 4 coefficient significant at the 90 per cent level and the lag 8 coefficient significant at the 85 per cent level (Table 2).

The residuals from the two inflation relationships display a somewhat surprising degree of autocorrelation; the unemployment-rate residual has an autocorrelation at lag 1 of 0.7 and the marginal-product residual has an autocorrelation at lag 1 of 0.5 (Figures 14 and 15). Since these conditioning terms are based on the estimation work of Laxton, Rose and Tetlow (1993b) one would expect the residuals to display somewhat less serial correlation — although Figures 14 and 15 do indicate that the correlation is significant for the first couple of lags only. This autocorrelation most likely arises from the conversion of the annual results to quarterly data. Other possible explanations include measurement error in the inflation expectations proxy, an omitted explanatory variable and an inappropriate smoothness constraint that creates serially correlated measurement errors.

The relatively small difference between the unemployment-rate and marginal-product-of-labour inflation relationships' residuals, which was noted above in terms of persistence, disappears when it comes to cross-correlations. The residual from the unemployment-rate Phillips curve is rather

32. Laxton and Tetlow (1992) calibrate their output gap as an AR(2) process; Watson (1986) and Kuttner (1994) estimate a model with an AR(2) cyclical component.

highly correlated with the unemployment-rate gap (0.6 at lag 0, Figure 15). This outcome augments the concern raised above about misspecification in the Phillips curve. Here, there is evidence that the residuals are correlated with an explanatory variable. On the other hand, the marginal-product residuals are much better behaved; they display no significant correlation with the marginal-product gap at any lag.

On the whole, it appears that there is a good deal of unmodelled covariation among the residuals in the EMV filter. There may be some benefit in correcting for the coarsest aspects of this covariation in subsequent work. The results of Section 4.2.1 suggest that the overall properties of the EMV filter ought to remain unchanged but that the end-of-sample estimate might be further improved by correcting for this covariation.

5 CONCLUDING COMMENTS

This paper has presented a tool to estimate potential output that combines a set of economic relationships with a mechanical filter. This semi-structural method, dubbed the extended multivariate filter, also incorporates features that allow the researcher to incorporate additional judgment *and* account for its influence. Special measures have also been taken to ensure that the end-of-sample or updating properties of this filter make it suitable for real-time policy analysis and forecasting purposes. The extended multivariate filter is a flexible tool that incorporates a number of implications of the policy-analysis model QPM, allows the researcher to insert judgment where appropriate, and improves upon the the end-of-sample or updating properties of the multivariate filter.

As noted in Section 4.2, there remain a number of issues. Section 4.2.3 indicates that a certain degree of information is lost by omitting the covariances of the residuals. On the other hand, Section 4.2.1 indicates that when the residuals are weighted by their unconditional variances, the resulting estimate of potential is relatively invariant to the method used to select the weights.

A second area that deserves additional attention is a more formal study of an optimal trade-off between the excess sensitivity and the timeliness of the filter at the end of sample. The evidence presented in this paper shows that the trade-off in the EMV filter strikes a good balance between these two aims. It would be useful to study this question in an artificial economy in order to confirm these conclusions.

A third area for future work will be to consider further the implications of using a filter like the H-P filter as a flexible tool to impose additional smoothness on the measure of potential output. The properties of alternative filters and their effectiveness in separating trends and cycles under different circumstances remain an active area of research in economics and statistics. Looking ahead, it will be important to continue to evaluate the implications of this growing literature for the EMV filter approach.

Perhaps the most important area for future work is quantifying the uncertainty that surrounds estimates of potential output. In evaluating the multivariate filter, Laxton and Tetlow (1992) use Monte Carlo experiments to generate confidence intervals for the estimate of potential output. They find that the confidence intervals for the multivariate filter's estimate are noticeably narrower than those for the H-P filter, suggesting that the additional economic information that is incorporated in the multivariate filter is serving a useful purpose. In particular, their estimate of the 95 per cent confidence interval for the H-P filter's estimate of the output gap is about plus or minus 3 per cent, while that for the multivariate filter is plus or minus 1.5 to 2.0 per cent. The EMV filter incorporates more information than Laxton and Tetlow's original MV filter. Thus, provided this information is useful, the 95 per cent confidence interval for the EMV filter estimate should lie within that of the MV filter.

Quantifying this gain requires performing stochastic simulations with a dynamic, forward-looking, non-linear model. This is a large task and is left for future work. Moreover, Laxton and Tetlow's approach does not consider all the relevant sources of uncertainty. In particular, their analysis does not capture the uncertainty associated with the economic structure conditioning the filter, the parameter values in these structural relationships, and the weights assigned to the various conditioning terms in the filter. The total uncertainty surrounding the output gap is therefore likely to be considerably larger than the 1.5 to 2.0 per cent range obtained by Laxton and Tetlow.

Moreover, the above discussion relates to estimates of the uncertainty that are associated with the output gap over history. As stressed by Laxton and Tetlow, the uncertainty surrounding the current output gap is larger than that for past output gaps, since estimates of the current gap can rely only on current and lagged information. As new data arrive, the estimate of the output gap over the most recent quarters typically changes slightly, since incoming data include new information for identifying past movements in output as responses to demand or supply disturbances. But when estimating the current output gap, policy makers do not have the benefit of hindsight.

This uncertainty suggests that estimates of the output gap should be only one piece of information among many used by policy makers for decisions regarding instrument settings. Estimates of the output gap must be balanced against information from detailed assessments of labour, goods and financial markets, from the growth rates of the monetary aggregates, from developments in credit markets, and from more informal evidence on the decisions, intentions and perceptions of consumers and businesses.

The EMV filter is designed as a flexible tool that can be easily adapted to exploit new sources of information. As such, the EMV filter is likely to evolve over time as future research sheds new light on the many relationships that influence and are influenced by potential output. However, no matter how good the estimator, some uncertainty will always remain, and this uncertainty will generally be greatest for the current output gap, precisely when it matters most to policy makers.

In closing, it is worth stressing that, while there will always be uncertainty, an important accomplishment of the EMV filter approach is that it takes uncertainty seriously. Uncertainty is reduced by using multiple sources of information as inputs, by using a flexible filter that will adapt to shifts in underlying structural relationships, and by co-ordinating the development of the economic model with the measurement of potential output. This last feature of the EMV filter methodology bears highlighting.

The output gap is best thought of not as an absolute concept, but as an indicator of excess demand or supply pressures in the economy that can be evaluated only in the context of the model within which it is used. From the perspective of contemporary monetary policy, the implications for the likely future course of inflation are most important. The method of measuring potential output developed here, therefore, places considerable weight on co-ordinating the development of the model and the measurement of potential output. What emerges is a measure of potential output that is adapted to a particular model. The evaluation of the methodology then turns not on whether the measure of the output gap is good, based on some independent criterion, but rather on whether the methodology and the model together produce reliable advice for monetary policy makers. Making this assessment will be an important part of future work on this topic at the Bank.

APPENDIX 1: DATA AND DEFINITIONS

The EMV filter uses data from the following sources:

Total Employment: the Bank of Canada's ETSLABOUR data series lfsa201. The data from 1953 to 1975 are quarterly, seasonally adjusted data that have been linked by the Bank of Canada to data derived from monthly Statistics Canada series. From 1976 to present the data are published by Statistics Canada (D767608, matrix 2075.133.11.3) as monthly series and log-averaged to quarterly frequency.

Total Labour Force: ETSLABOUR data series lfsa101. The data from 1953 to 1975 are quarterly, seasonally adjusted data that have been linked by the Bank of Canada to data derived from monthly Statistics Canada series, unpublished data. From 1976 to present the data are published by Statistics Canada (D767606, matrix 2075.133.11.2) as monthly series and log-averaged to quarterly frequency.

Unemployment Rate: 1 minus Total Employment divided by Total Labour Force.

Population: Total Non-Institutionalized Population 15 and Over, ETSLABOUR data series lfsu1. The data from 1953 to 1965 are unpublished Bank of Canada estimates. From 1966 to present the data are published by Statistics Canada (D767284, matrix 2074.133.11.1) as monthly series and log-averaged to quarterly frequency.

Participation Rate: Total Employment divided by Population.

Real GDP: Statistics Canada's seasonally adjusted, quarterly estimate of gross domestic product at 1986 prices (13-001), D20463.

Nominal GDP: Statistics Canada's seasonally adjusted, quarterly estimate of gross domestic product at market prices (13-001), D20011.

GDP Price Deflator: Nominal GDP divided by Real GDP.

Nominal Labour Income: ETSCITS data series d20088 - d20091 + d20005 + d20006. Statistics Canada's seasonally adjusted, quarterly estimate of wages, salaries and supplementary income less military pay and allowances plus net income of farm operators plus net income of unincorporated businesses including rent (13-001).

Indirect Taxes Net of Subsidies: ETSCITS data series d20008. Statistics Canada's seasonally adjusted, quarterly estimate in current dollars (13-001).

Nominal GDP at Factor Cost: Nominal GDP less Indirect Taxes Net of Subsidies.

GDP at Factor Cost Price Deflator: Nominal GDP at Factor Cost divided by Real GDP.

Consumer Price Index Net of Food and Energy: the Bank of Canada's ETSCPI data series b820555. Unpublished, seasonally adjusted monthly Bank of Canada estimate from January 1952 to October 1978, Statistics Canada's seasonally adjusted monthly estimate from November 1978 to present. Log-averaged to quarterly frequency.

Labour's Share of National Income: Nominal Labour Income divided by Nominal GDP at Factor Cost.

Nominal Wage: Nominal Labour Income divided by Total Employment.

Real Wage: Nominal Wage divided by GDP at Factor Cost Price Deflator.

Inflation: 0.35 multiplied by the change in log level of GDP Price Deflator plus 0.65 times change in log level of Consumer Price Index Net of Food and Energy.

Marginal Product of Labour: Real GDP times Labour-Output Elasticity divided by Total Employment.

APPENDIX 2: TECHNICAL DETAILS

The specification of the EMV filter as the solution to a quadratic maximization can be justified by the following statistical model. Let τ represent a Tx1 variable. Although τ might be thought of as a single variable such as potential output from period 1 to T, it might also be a vector quantity. Suppose that $u_k = B_k\tau - A_kX_k \sim N(0, W_k^{-1})$ for $k=1,...,n$.[1] The coefficient matrices, A_k and B_k, are TxMT and TxT matrices respectively that might represent structural economic relationships; for example, W_k^{-1} is a TxT, positive semi-definite covariance matrix; and X_k is a MTx1 matrix that can be thought of as measured economic data. The coefficients might, for instance, be obtained from a calibration exercise or from a related empirical piece of research. Given the distributional hypothesis for each u_k, the joint probability density function of the vector $u = [u_1, \ldots, u_n]'$ is

$$p(u|\tau) = \prod_{k=1}^{n} (2\pi|W_k^{-1}|)^{-1/2}\exp(-u_k'W_ku_k) \quad . \tag{A.2a}$$

The log-likelihood function of τ is, up to a constant term,

$$lf(\tau|\Psi) = -\sum_{k=1}^{n} (B_k\tau - A_kX_k)'W_k(B_k\tau - A_kX_k) \;, \tag{A.2b}$$

where $\Psi = [A_k, B_k, X_k, W_k]_{k=1,\ldots,n}$. The maximum-likelihood estimate, $\hat{\tau}$, can then be obtained by maximizing $lf(\tau|\Psi)$ over τ.

The specification of potential output as the series that best explains a set of structural economic relationships (in a quadratic norm sense) therefore coincides with the maximum-likelihood estimator of potential output under certain hypotheses; in particular, (A.2a) assumes that the parameter values [Ak, Bk, Wk] are known with certainty.

From a purely computational standpoint, this specification results in a solution, $\hat{\tau}$, that is easily calculated. It is the solution to the linear equation $B'WB\tau = B'WAX$, where $B' = [B_1', \ldots, B_n']$, $W = diag(W_1, \ldots, W_n)$, $A = diag(A_1, \ldots, A_n)$ and $X' = [X_1', \ldots, X_n']$. If $B'WB$ is non-singular, it can be factored using the Choleski decomposition (Golub and Van Loan 1989). In addition, if the coefficient and covariance matrices, B_k and W_k,

1. The Moore-Penrose pseudo-inverse replaces the inverse when W_k is singular.

are banded matrices, then $B'WB$ will also be banded, with bandwidth equal to the maximum bandwidth of the products $W_k B_k$. In the case where each W_k is diagonal, for example, the bandwidth of $B'WB$ is simply the maximum bandwidth of the coefficient matrices. This fact allows the Choleski decomposition to be optimized so that the solution can be computed in O(T) operations.

A penalized log-likelihood function can be formed from (A.2b) by imposing additional constraints on the first, second and possibly higher differences of τ. For example, the penalized log-likelihood function that limits the curvature or second difference of τ is

$$lfp(\tau|\Psi, \lambda) = -\left(\sum_{k=1}^{n} (B_k\tau - A_k X_k)' W_k (B_k\tau - A_k X_k) + \lambda\tau' D' D\tau\right) . \tag{A.2c}$$

D is the TxT matrix that second differences τ, and λ penalizes the curvature in τ. Additional penalty terms can also be introduced into (A.2c) in order to reflect additional a priori information that a researcher may have concerning the unobserved variable in question.

An alternative way to characterize equations of the form of (A.2c) is to interpret the additional term as an additional hypothesis about the behaviour of τ. In this case, it is that the growth rate of τ is a random walk, possibly with drift $D\tau \sim N(0, \lambda^{-1} I)$.

TABLES

Table 1

Stationarity tests

Sample 1954Q4 to 1994Q3

Residual	Lags in ADF / PP tests[a]	ADF[b]	PP[c]	KPSS[d]	Q[e]
Output gap	10 / 12	-3.48	-20.92	0.19	23.04
Unemployment rate gap	9 / 12	-3.67	-15.22	0.13	35.5
Unemployment rate gap	12 / 12	-2.69*	-15.22	0.13	27.53
Marginal-product gap	7 / 12	-5.04	-52.51	0.12	24.96#
Unemployment rate Phillips-curve residual	1 / 12	-4.83	-88.02	0.15	34.34
Marginal-product Phillips-curve residual	1 / 12	-6.26	-120.64	0.24	34.25
Trend unemployment rate residual	8 / 12	-3.43	-14.03	0.09	43.67
Real wage/marginal product residual	7 / 12	-5.88	-36.24	0.06	27.67
Growth rate of NAIRU residual	12 / 12	-1.37**	-13.26**	0.13	17.40##
Growth rate of equilibrium marginal-product residual	10 / 12	-1.54**	-7.95**	0.54**	25.51#
Second difference of NAIRU residual	12 / 12	-2.98	-19.02	0.06	14.44##
Second diference of equilibrium marginal-product residual	9 / 12	-3.88	-15.75	0.07	25.30#
Okun's law residual	7 / 12	-5.15	-54.02	0.10	20.66##

a. Lag length for the Augmented Dickey-Fuller test is selected by Ng and Perron's (1995) procedure. Lag length for the Phillips-Perron test was selected on the basis of a $N^{1/2}$ rule, where *N* is the sample size.
b. The 5 and 10 per cent critical values for 100 observations are -2.89 and -2.58, respectively (Fuller 1976, Table 8.5.2).
c. The 5 and 10 per cent critical values for 100 observations are -13.7 and -11.0, respectively (Fuller 1976, Table 8.5.1).
d. The 5 and 10 per cent critical values for 100 observations are 0.347 and 0.463, respectively (Kwiatkowski et. al. 1992)
e. Chi-squared distributed with 36 degrees of freedom under the null hypothesis.

Note: All test statistics estimated with constant, less time trend terms. Henceforth "*" and "**" indicate a failure to reject the null hypothesis of non-stationarity at the 5 and 10 per cent critical levels, respectively; "^", and "^^" indicate a rejection of the null hypothesis of stationarity at the same critical values; "#" and "##" indicate a rejection of the null hypothesis of no autocorrelation in the estimated equation.

Table 2

Autoregressive properties of the residuals

$$\varepsilon_t = c + \alpha_1 \varepsilon_{t-1} + \ldots + \alpha_9 \varepsilon_{t-9} + \eta_t$$

Residual name	C (t-value)	Coefficients on lagged residuals (t-value)								
		1	2	3	4	5	6	7	8	9
Output	-0.00	1.08	-0.19	0.18	-0.34	0.21	-0.09	0.10	-0.17	0.08
	-1.17	12.95	-1.53	1.45	-2.77	1.64	-0.72	0.83	-1.42	0.93
		R-bar-adj = 0.83		RSE = 0.01		Box-Pierce = 17.30				
Unemployment	-0.00	1.45	-0.52	0.06	-0.28	0.23	-0.02	0.04	-0.09	0.02
	-1.07	17.32	-3.53	0.40	-1.81	1.50	-0.14	0.26	-0.57	0.27
		R-bar-adj = 0.92		RSE = 0.00		Box-Pierce = 19.56				
Marginal product of labour	-0.00	0.63	0.02	0.11	-0.16	0.08	-0.03	0.03	-0.13	-0.03
	-1.44	7.47	0.24	1.10	-1.57	0.80	-0.31	0.31	-1.38	-0.43
		R-bar-adj = 0.46		RSE =0.01		Box-Pierce = 16.44				
Unemployment-gap Phillips curve	0.03	0.46	0.13	0.04	0.07	-0.01	-0.09	0.11	-0.07	-0.04
	0.22	5.50	1.40	0.43	0.79	-0.10	-0.96	1.16	-0.77	-0.52
		R-bar-adj = 0.31		RSE = 1.52		Box-Pierce = 12.64				
Marginal-product-gap Phillips curve	-0.04	0.33	0.06	0.02	0.05	-0.00	-0.07	0.11	-0.04	-0.02
	-0.41	3.92	0.73	0.19	0.63	-0.02	-0.81	1.29	-0.50	-0.27
		R-bar-adj = 0.10		RSE = 1.34		Box-Pierce = 15.79				
Trend unemployment gap	-0.00	1.61	-0.59	-0.21	0.15	-0.10	0.24	-0.18	0.17	-0.15
	-0.48	19.37	-3.77	-1.28	0.92	-0.58	1.47	-1.11	1.05	-1.77
		R-bar-adj = 0.97		RSE = 0.10		Box-Pierce = 21.29				
Real wage/ marginal-product residual	0.06	0.66	0.10	-0.04	-0.09	0.02	-0.04	0.03	-0.17	-0.02
	1.02	7.86	1.03	-0.43	-0.96	0.21	-0.36	0.28	-1.80	-0.28
		R-bar-adj = 0.55		RSE = 0.71		Box-Pierce = 14.26				
Okun's law residual	0.00	0.58	0.02	0.06	-0.14	0.05	-0.03	0.11	-0.18	-0.05
	1.46	6.88	0.22	0.60	-1.49	0.56	-0.30	1.14	-1.91	-0.63
		R-bar-adj = 0.38		RSE = 0.01		Box-Pierce = 12.68				

(Continued on next page)

Table 2 (Continued)

Residual name	C (t-value)	Coefficients on lagged residuals (t-value)								
		1	2	3	4	5	6	7	8	9
Growth of NAIRU	0.00	3.03	-2.98	0.24	1.68	-1.32	-0.05	1.08	-0.99	0.31
	0.66	36.52	-11.18	0.64	4.49	-3.32	-0.13	2.83	-3.32	3.20
		R-bar-adj = 1.00		RSE = 0.00		Box-Pierce = 18.83				
Growth of equilibrium marginal product	0.00	3.41	-4.30	2.37	-0.63	0.38	-0.35	0.17	-0.05	0.01
	0.74	40.60	-14.41	5.07	-1.25	0.74	-0.69	0.36	-0.17	0.12
		R-bar-adj = 1.00		RSE = 0.00		Box-Pierce = 22.10				
Second difference of NAIRU	-0.00	2.09	-1.05	-0.65	1.00	-0.48	-0.31	0.80	-0.53	0.09
	-0.39	23.81	-5.12	-2.89	4.49	-1.87	-1.32	3.47	-2.25	0.88
		R-bar-adj = 1.00		RSE = 0.00		Box-Pierce = 20.34				
Second difference of equilibrium marginal product	-0.00	2.43	-1.93	0.50	-0.15	0.23	-0.12	0.00	0.05	-0.03
	-0.25	28.88	-8.73	1.83	-0.56	0.83	-0.43	0.01	0.24	-0.40
		R-bar-adj = 1.00		RSE = 0.00		Box-Pierce = 22.67				

Note: The residual terms are each normalized by the asymptotic standard deviation of the corresponding gap term. The Box-Pierce statistic is chi-squared distributed with 20 degrees of freedom under the null hypothesis of no serial correlation in the regression residuals.

FIGURES

Figure 1: Moving-average coefficients of H-P filter

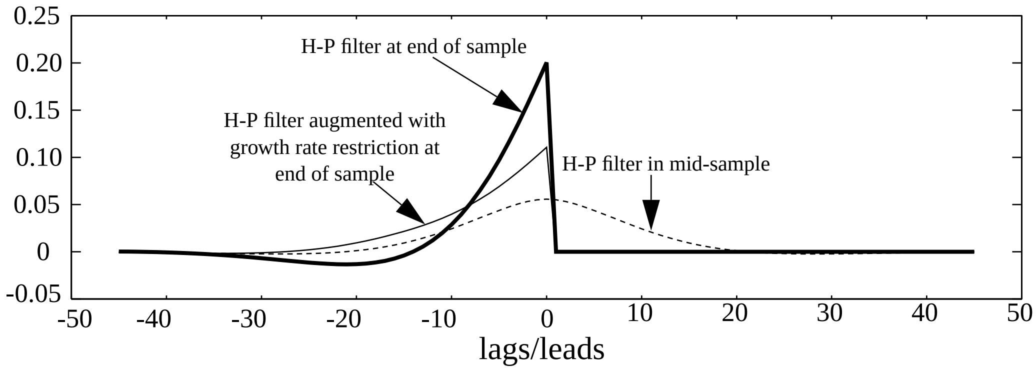


Figure 2: Gain and induced phase change of H-P filter

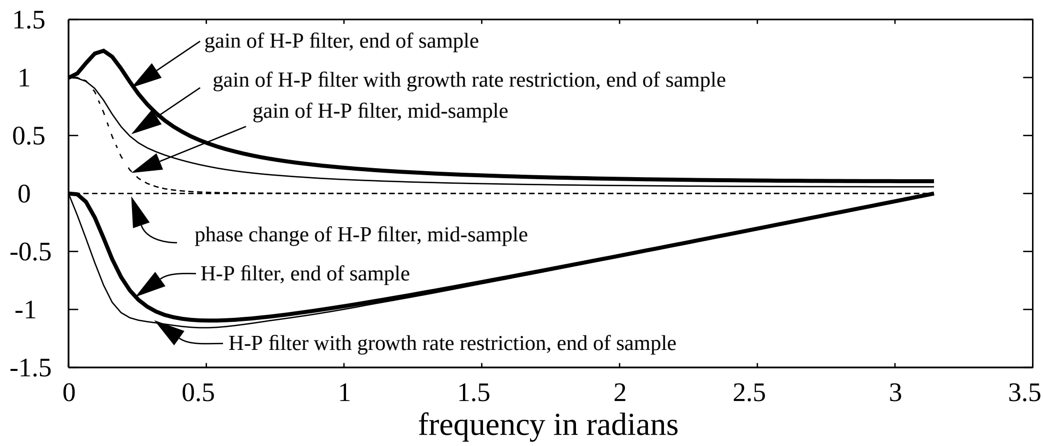


Figure 3: Standard errors of H-P filter

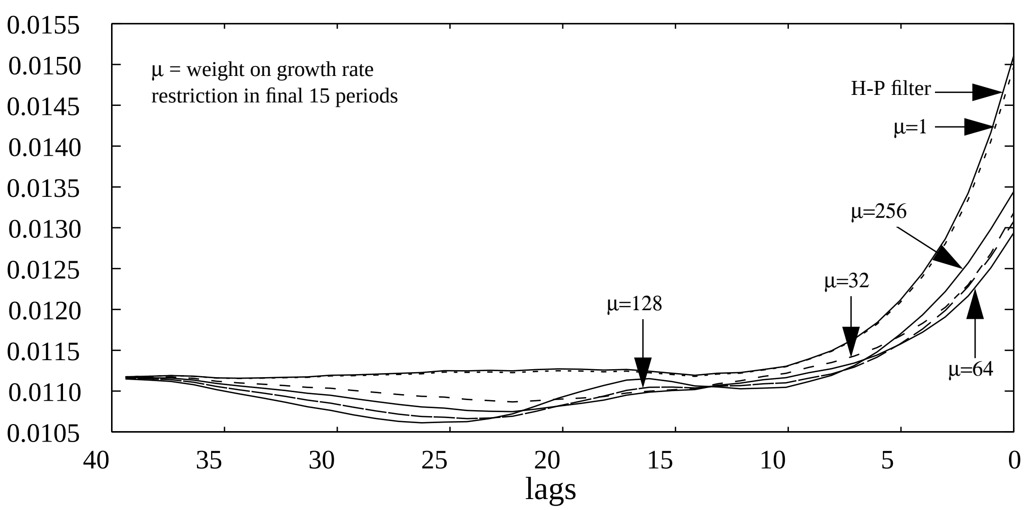
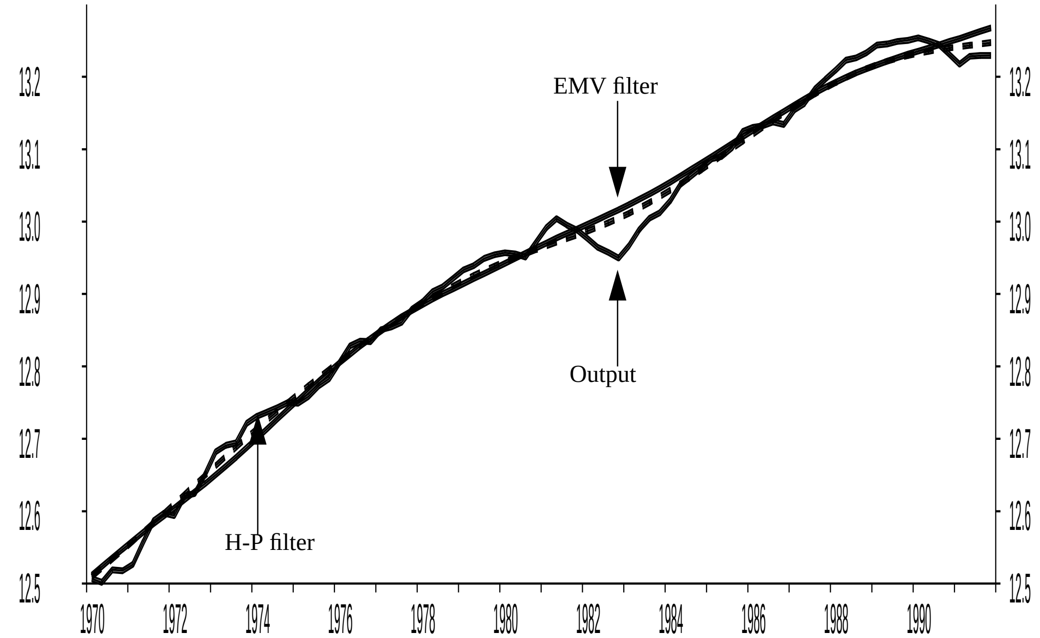


Figure 4: Estimate of potential output by EMV and H-P filters

Log levels of real GDP in 1986$



Log output gaps

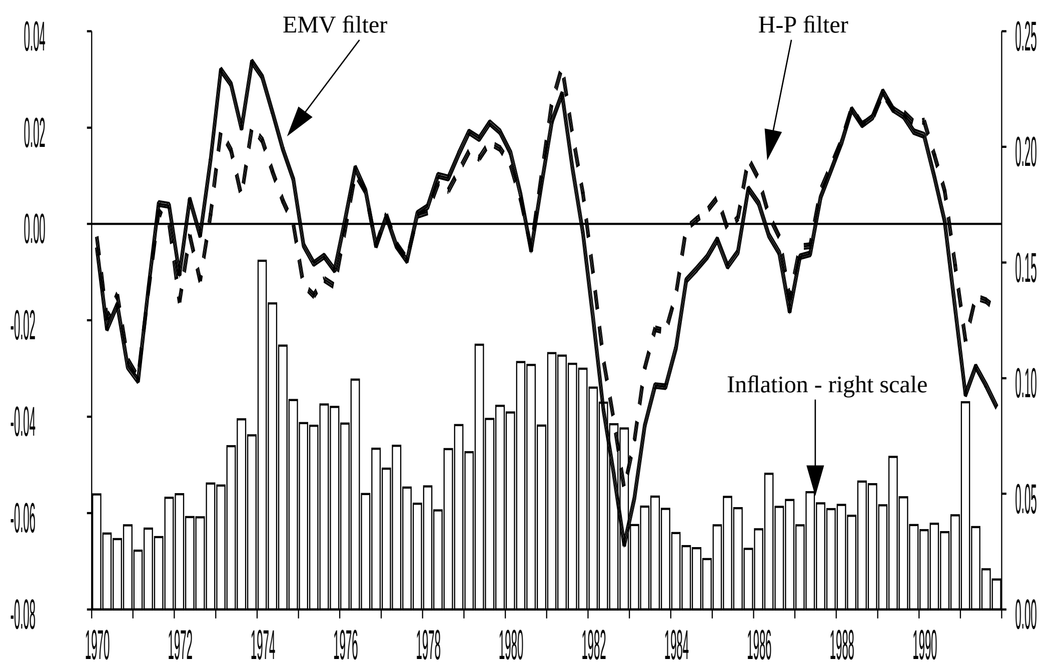
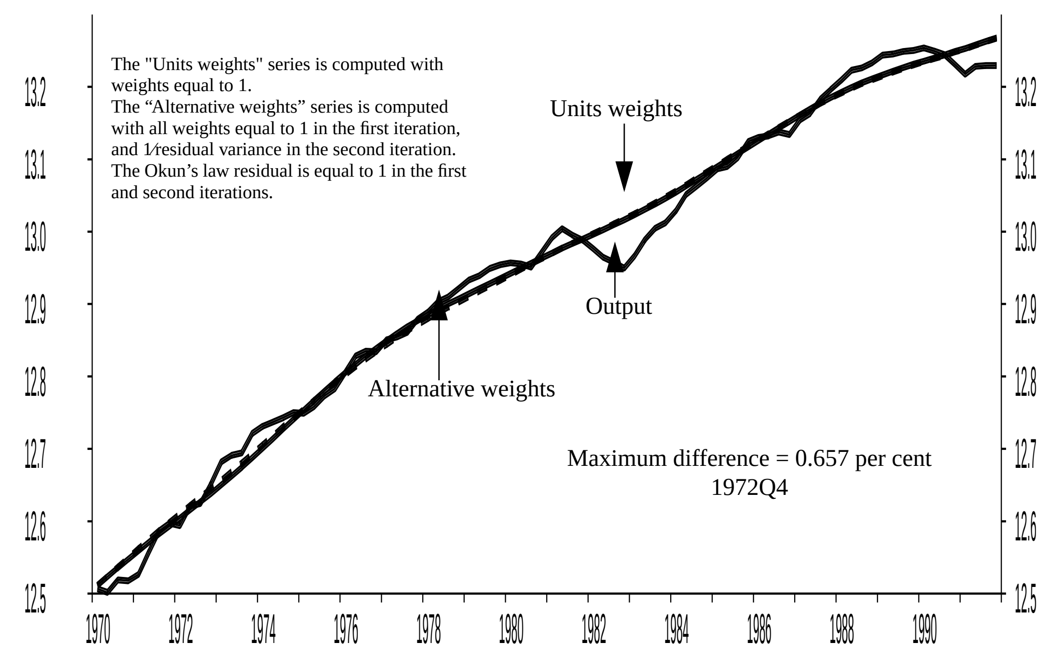


Figure 5: Alternative weights

Log levels of real GDP in 1986$



Log output gaps

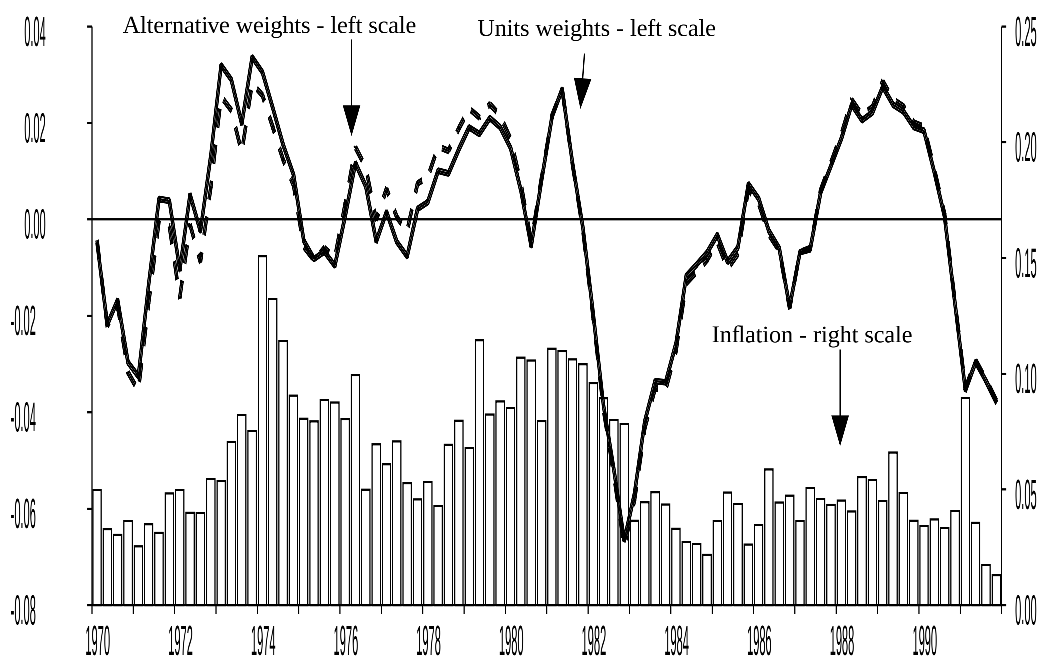
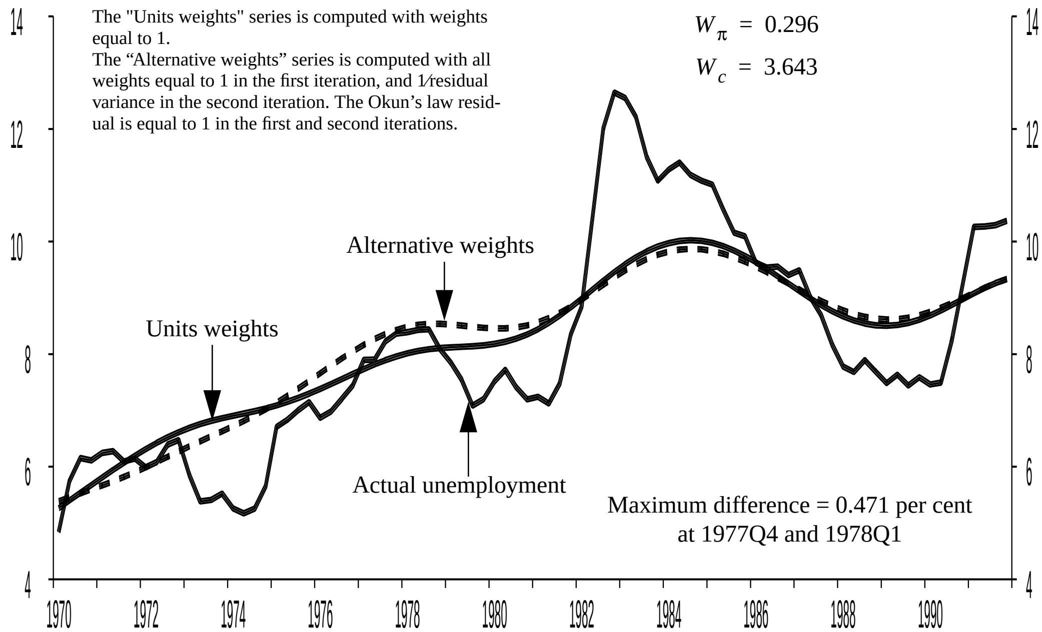


Figure 6: Alternative weights

Unemployment rates



Gaps

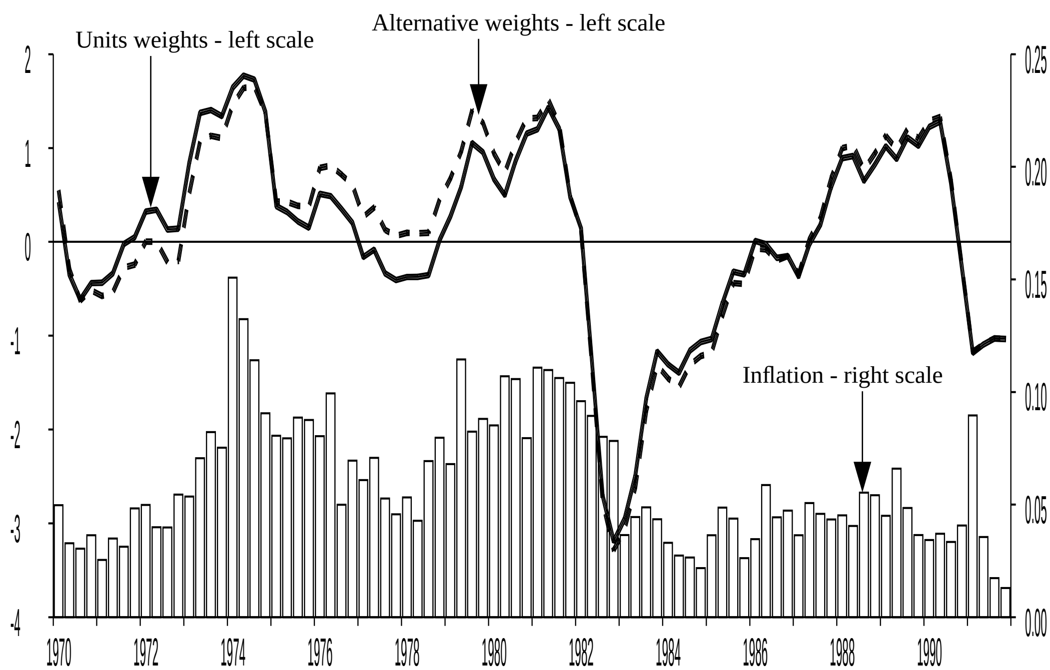
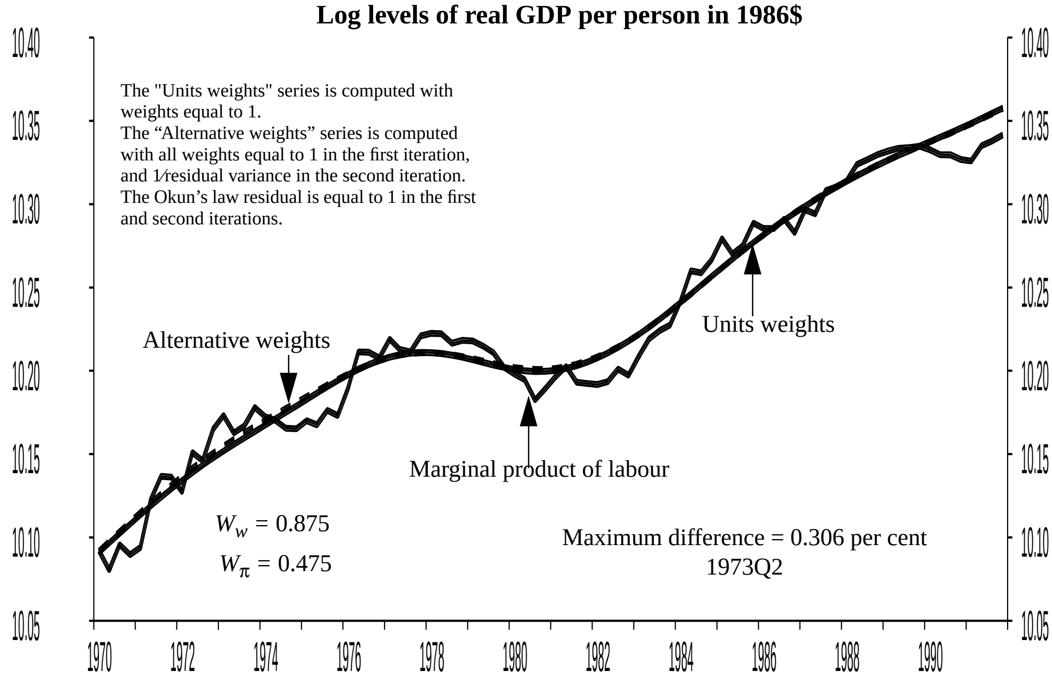


Figure 7: Alternative weights

Log levels of real GDP per person in 1986$



Log gaps

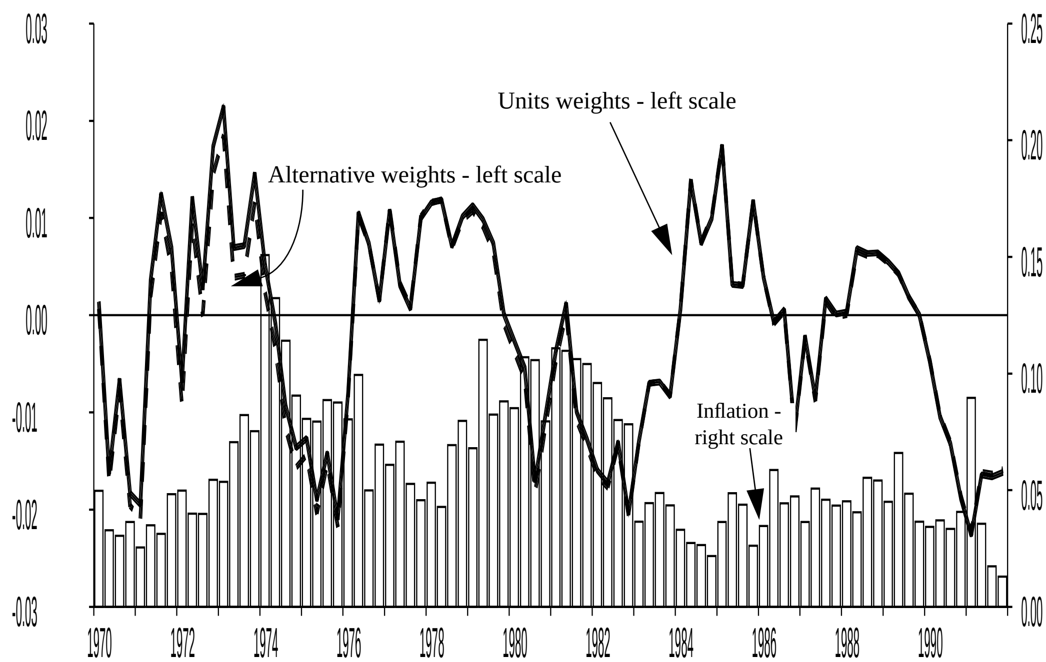
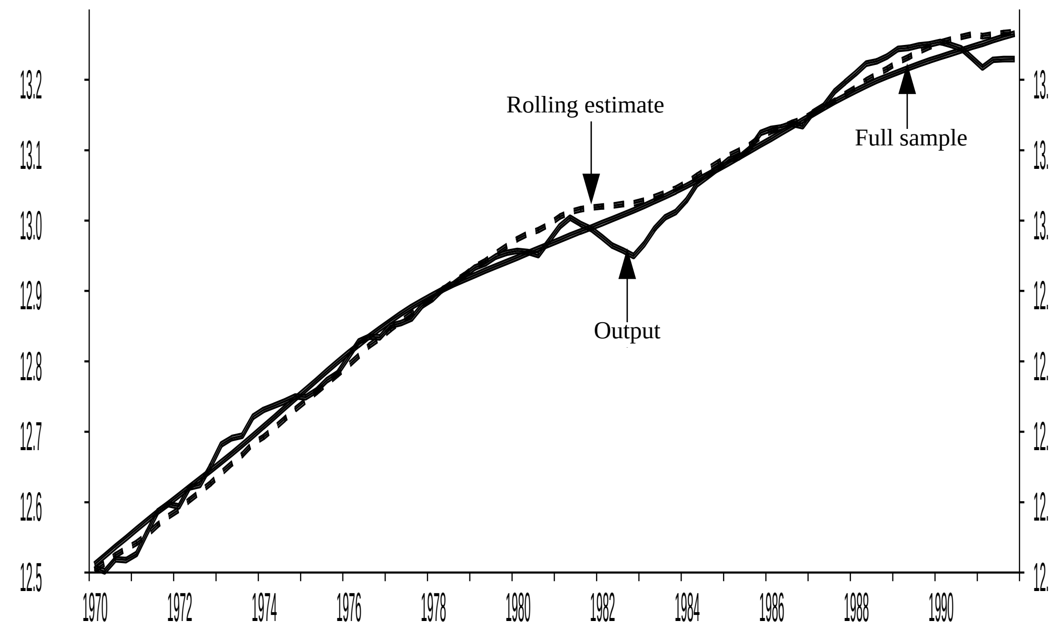


Figure 8: Full sample and rolling estimate of potential output

Log levels of real GDP in 1986$



Log output gaps

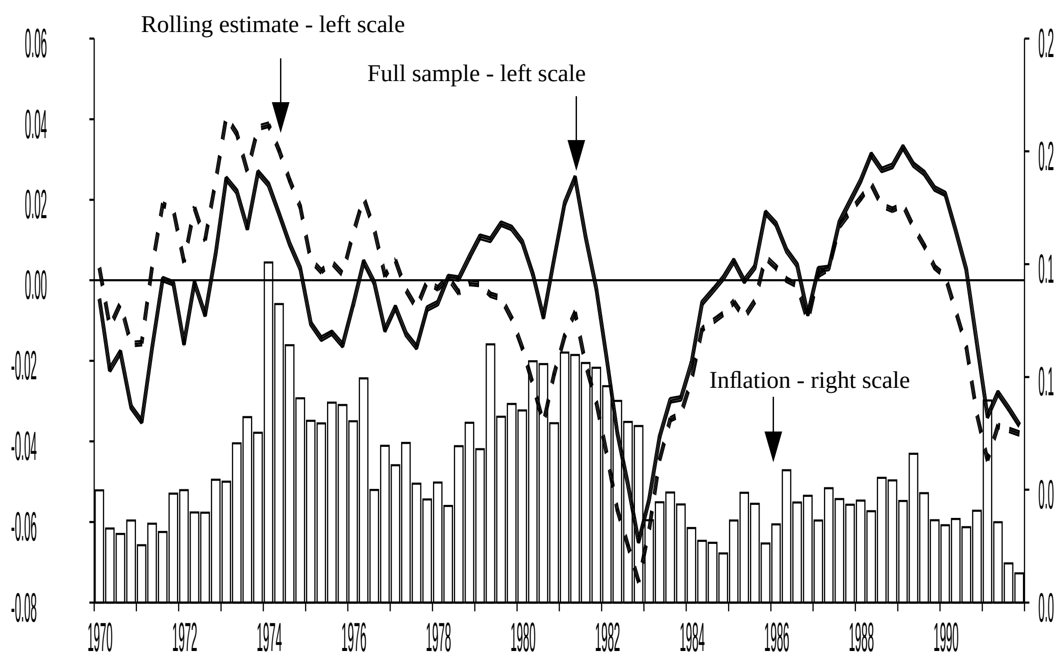
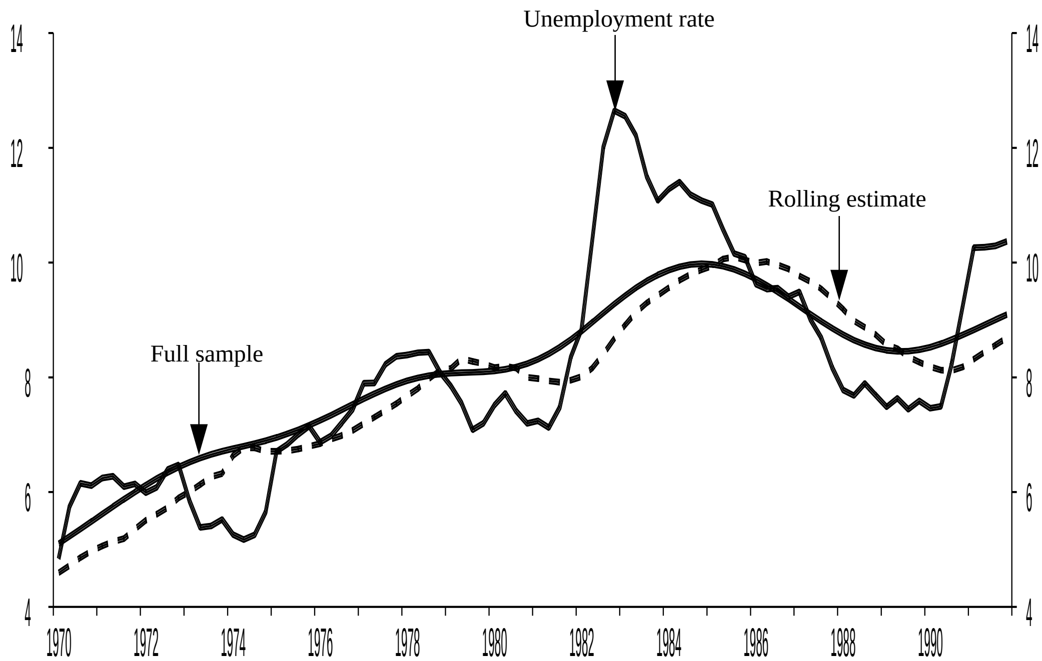


Figure 9: Full sample and rolling estimate of NAIRU

Levels in per cent



Per cent gaps

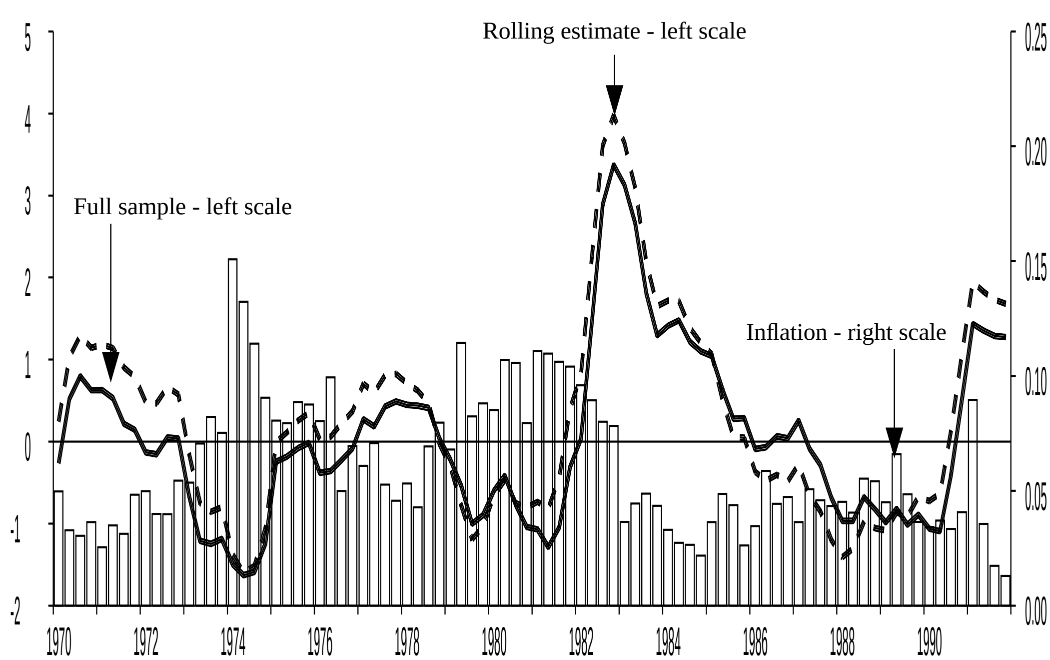
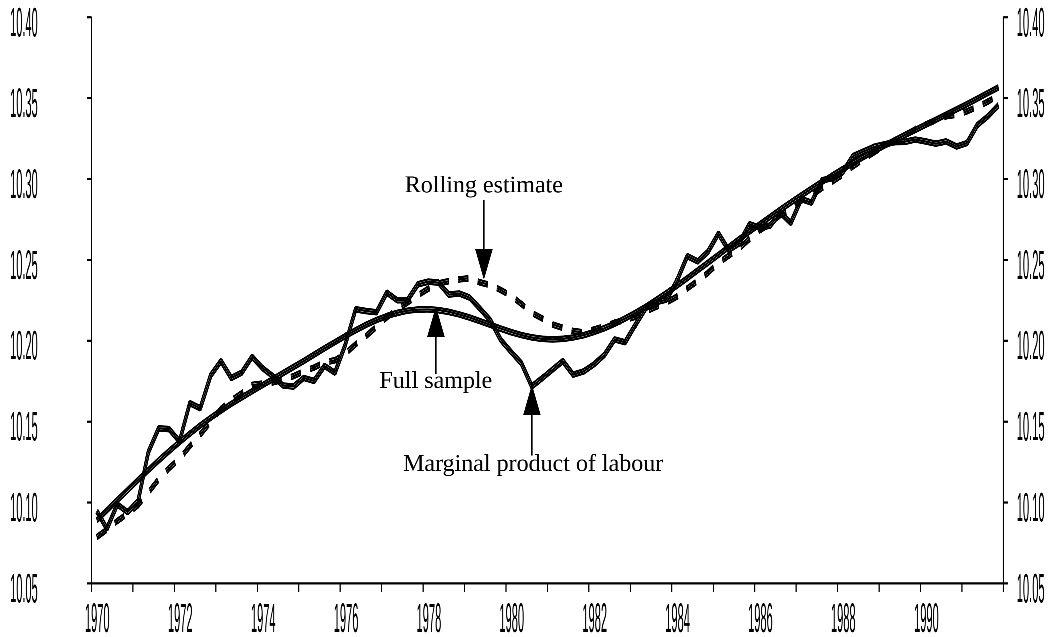


Figure 10: Full sample and rolling estimate of equilibrium marginal product of labour

Log levels of real GDP per person employed in 1986$



Log gaps

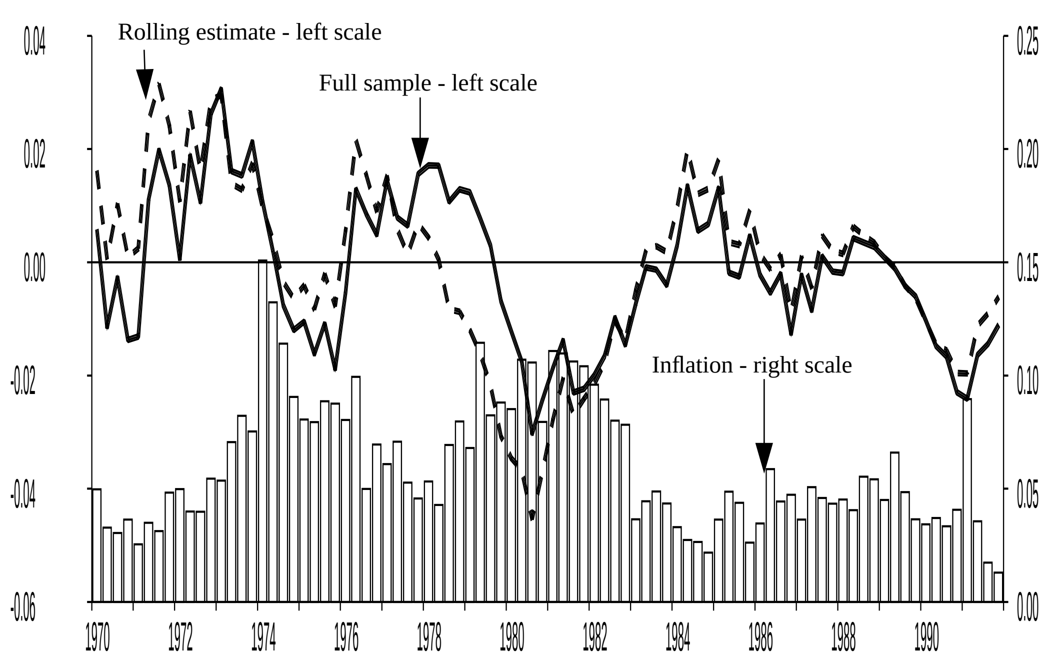
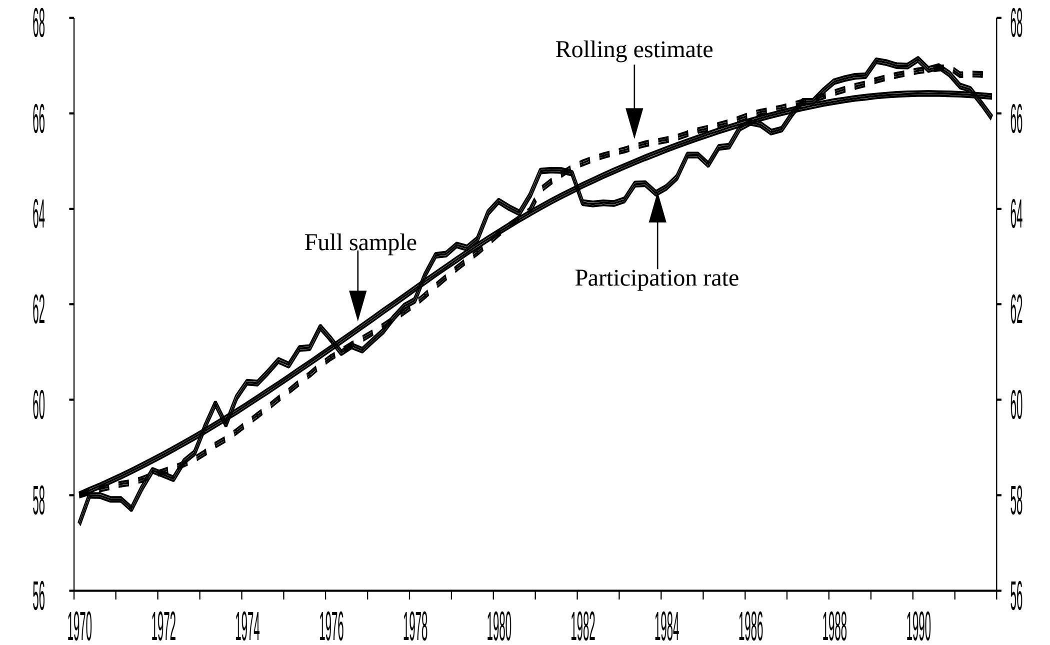


Figure 11: Full sample and rolling estimate of equilibrium participation rate

Levels in per cent



Per cent gaps

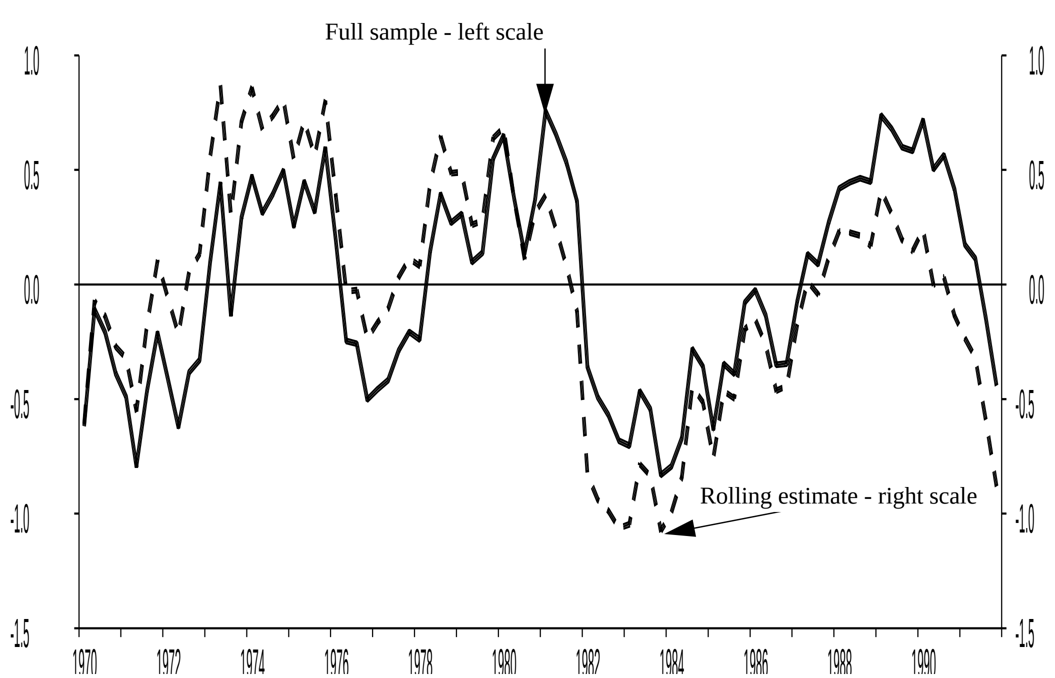
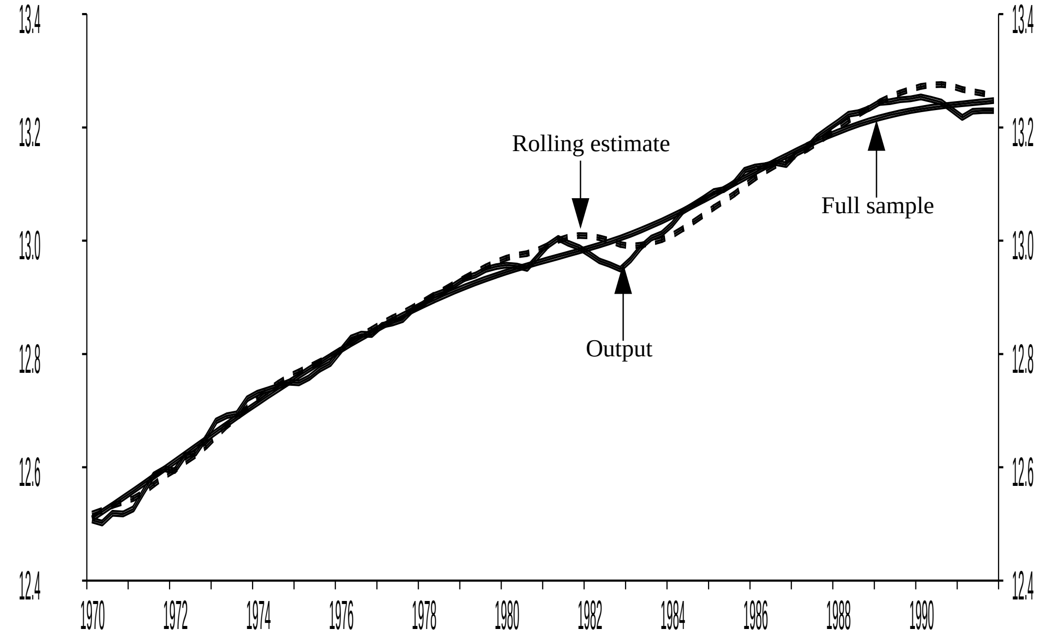


Figure 12: Full sample and rolling estimate of potential output: H-P filter

Log levels of real GDP in 1986$



Log output gaps

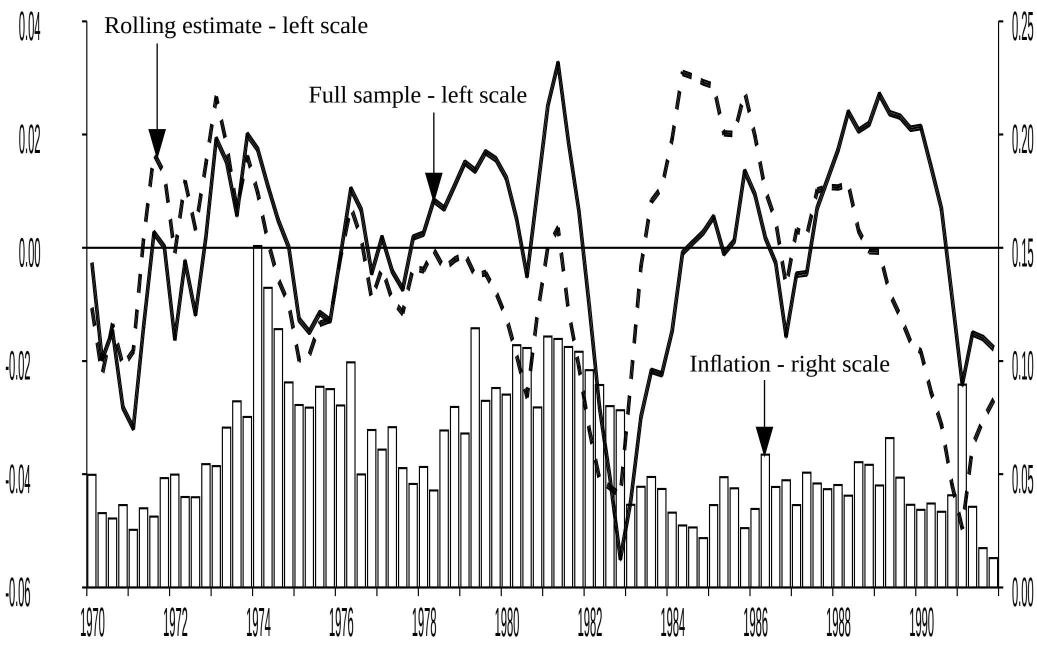
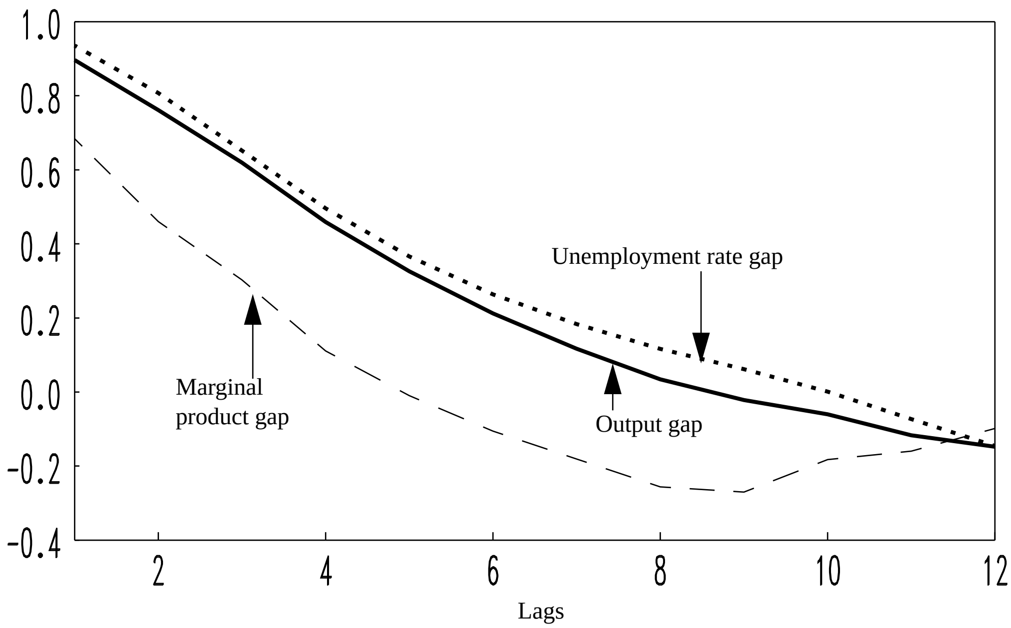


Figure 13: Autocorrelations of output gap, unemployment rate gap, and marginal product of labour gap



T-statistics of autocorrelations of output gap, unemployment rate gap, and marginal product of labour gap

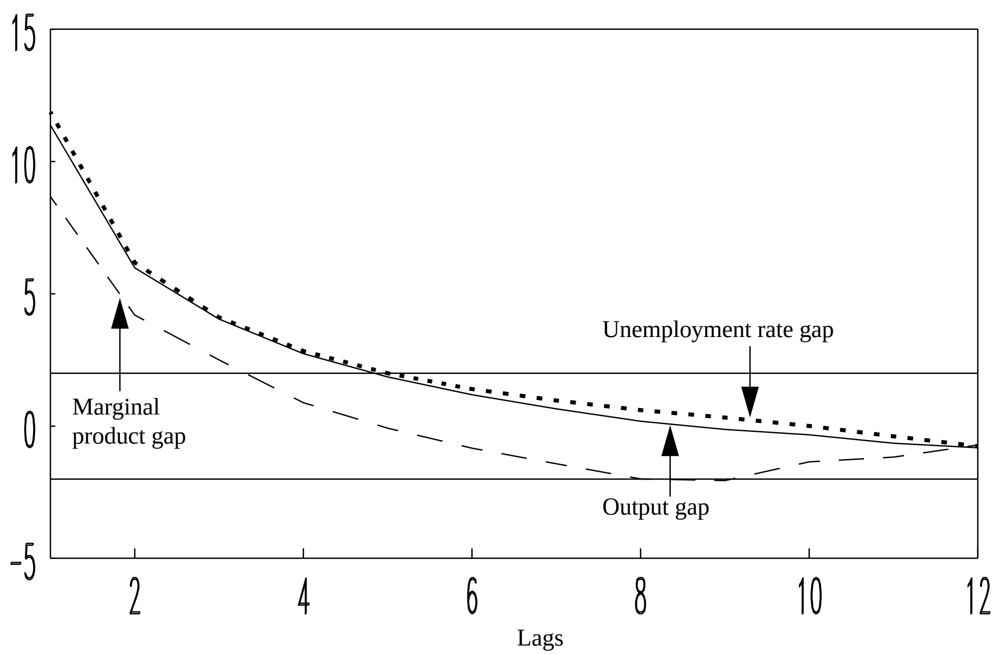


Figure 14: Auto- and cross-correlations of marginal product residuals

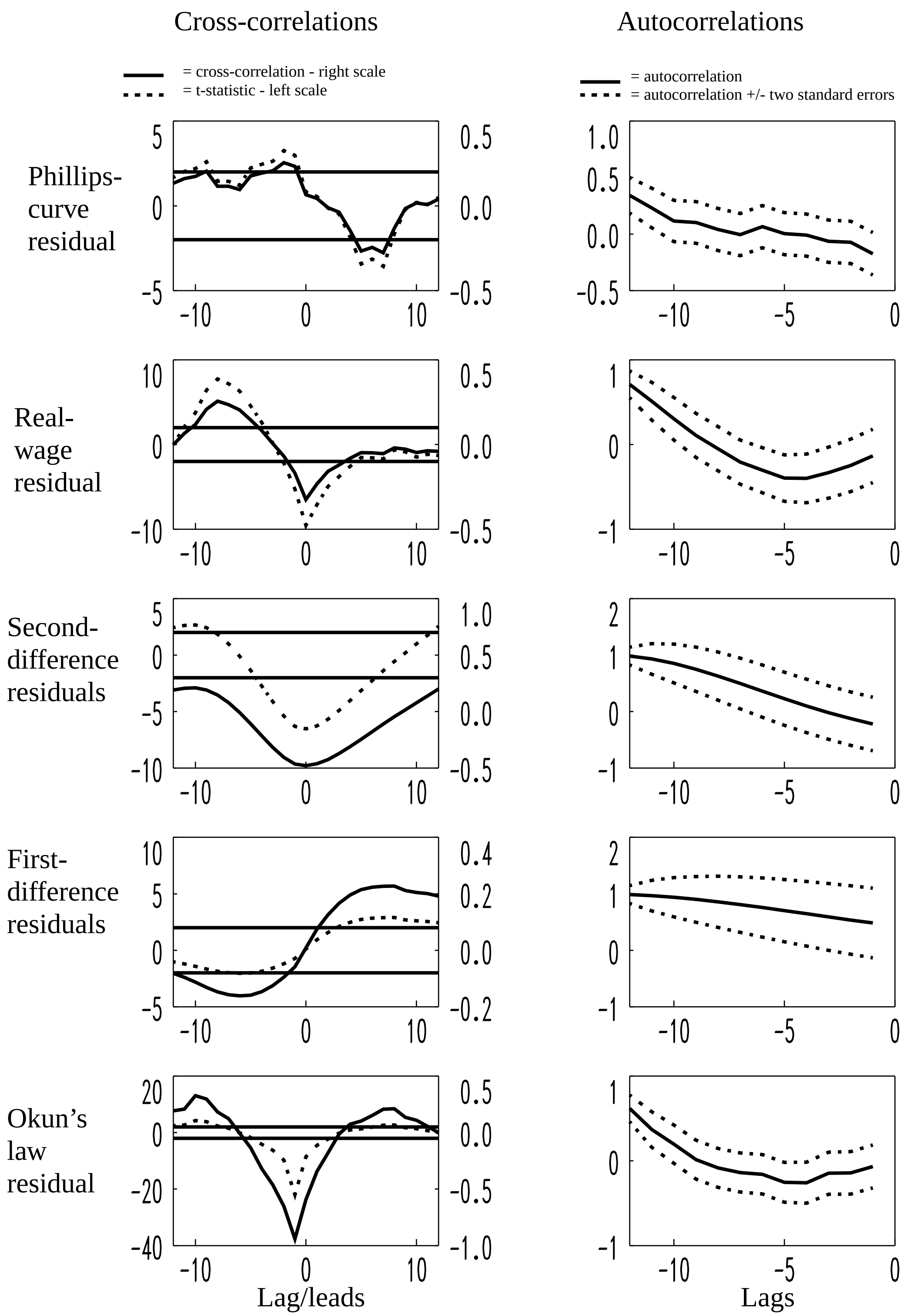
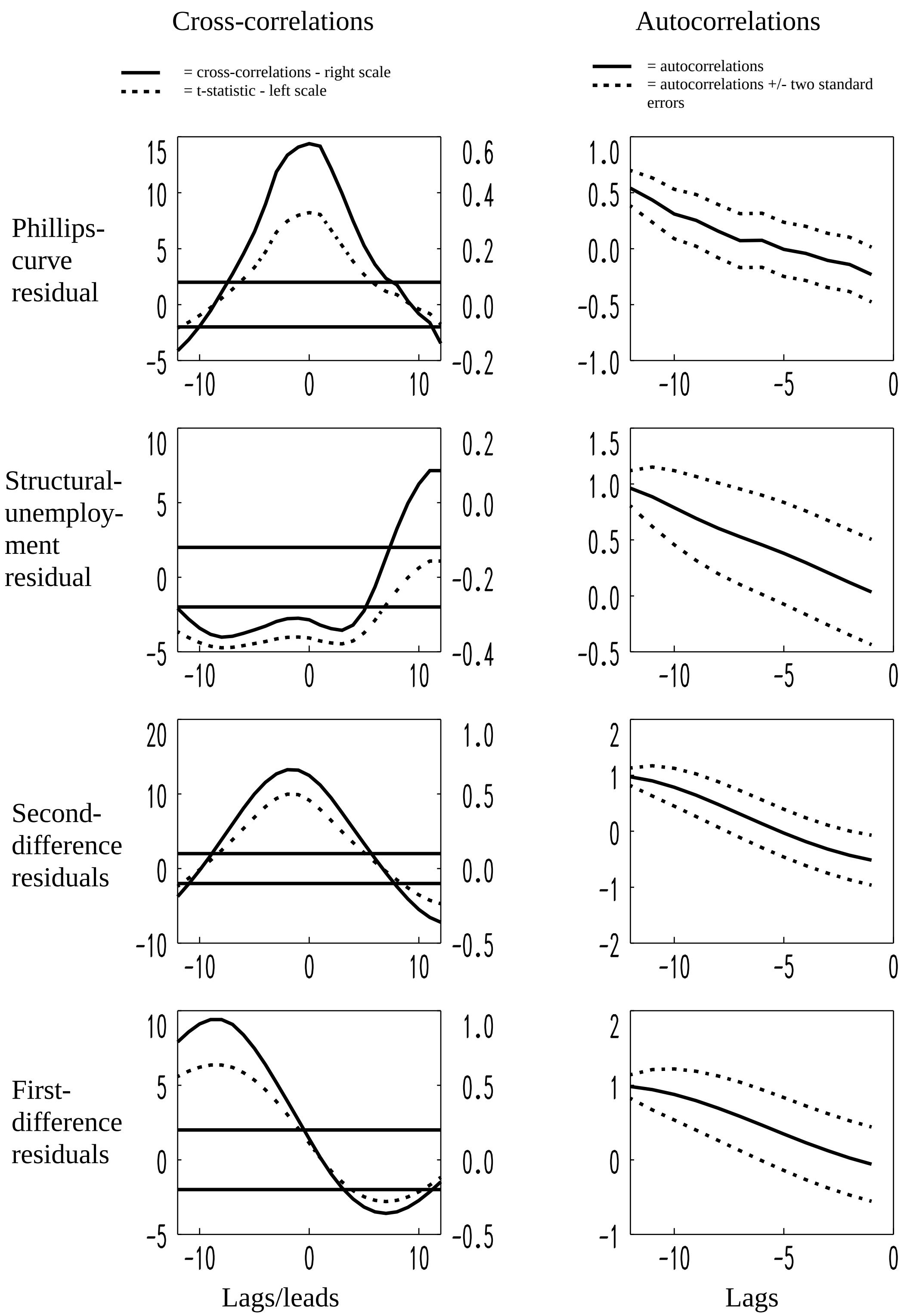


Figure 15: Auto- and cross correlations of unemployment rate residuals



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