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ABSTRACT

In this paper the simulation properties of a small, dynamic, open-economy IS-LM-Aggregate Supply model are examined under a variety of alternative policy rule assumptions. These assumptions include rigid money stock, exchange rate and nominal income targets, as well as less rigid policy rules that recognize information limitations. The model that is used consists of four dynamic structural equations describing aggregate demand, aggregate supply, money demand and the exchange rate. The parameters are chosen on the basis of existing empirical macro-models. Price expectations are adaptive in the short run but fully consistent in the long run. The implications of transitory and permanent shocks, both domestic and foreign, for the choice of policy regime are analyzed in the context of the model. The paper also highlights the important role played by real exchange rate adjustment in achieving full equilibrium in the presence of permanent shocks. The results bear strong similarities to the cross-policy rankings found in theoretical rational expectations models.

RÉSUMÉ

Dans cette étude, les auteurs examinent les propriétés que présente un petit modèle dynamique d'économie ouverte, de type IS-LM et comportant une fonction d'offre globale, avec lequel ils effectuent des simulations en utilisant différentes hypothèses de politique : l'hypothèse d'une contrainte sur le stock de monnaie, celles d'une cible de taux de change et d'une cible de revenu nominal ainsi que quelques autres hypothèses moins contraignantes qui reflètent le caractère limité de l'information. Le modèle comporte quatre équations structurelles dynamiques qui formalisent respectivement la demande globale, l'offre globale, la demande de monnaie et le taux de change. Le choix des paramètres est inspiré des modèles macro-économiques empiriques existants. Les anticipations de prix sont adaptatives à court terme mais entièrement cohérentes à long terme. Les auteurs analysent dans le contexte du modèle les conséquences des chocs passagers ou permanents, qu'ils soient d'origine intérieure ou extérieure. L'étude fait en outre valoir le rôle important que joue, dans un contexte de chocs permanents, le mécanisme d'ajustement qu'est le taux de change réel dans la réalisation d'un équilibre complet. Les résultats obtenus conduisent les auteurs à établir une hiérarchie des politiques très similaire à celle obtenue avec les modèles théoriques à anticipations rationnelles.

1 INTRODUCTION

The purpose of this paper is to examine the implications for the key variables of the macroeconomy of several alternative policy rules. The approach taken is to construct a small dynamic model of an open economy, impose parameter values based on previous empirical work, and then solve the model numerically under different assumptions about policy. With this framework, one can compare the impact and dynamic effects of exogenous shocks across the different policy rules. The analysis has its roots in that of Poole (1970), who compared nominal interest rate and money supply rules in a standard IS-LM model. A major difference between our analysis and Poole's, however, is that the latter was purely theoretical whereas the present study might be described as "semi-empirical". The use of numerical methods, while allowing the analysis of a much richer model, runs the risk of generating results that are very model-specific. For this reason, we have attempted to strike a balance between simplicity and realism, keeping the model as compact as possible while allowing for credible dynamics and taking account of the key linkages. In most cases we are able to intuit the impact and steady-state solutions using a textbook IS-LM-Aggregate Supply (AS) model, and the simulation model simply describes a plausible adjustment path.

At the outset we wish to acknowledge the Lucas (1976) critique of econometric policy evaluation. According to Lucas, because the structure of a model emerges from an optimal decision-making process that takes account of policy rules in place, any change in these rules will be accompanied by a change in model structure. Although we recognize this important qualification, we do not address the problem in this study. A related point is that the model developed here is characterized by rational or model-consistent expectations in the long run only. In the short run we assume that various institutional constraints and information and adjustment costs give rise to adaptive expectations processes.

The analysis focuses on three simple policy rules, and in each case a particular variable is held fixed. The variables in question are the money stock, the nominal exchange rate, and nominal spending. We also examine the properties of the continuum of conditional rules that lie between these three extremes. Questions of the feasibility of the implementation of the rules are not considered, although the investigation follows a sequence in which we gradually relax our initial information assumptions. Also, because the model contains only a single monetary aggregate, issues surrounding the choice between alternative financial variables, such as narrow or broad definitions of the money stock or credit, are not considered. In the model, money is viewed as being narrowly defined so that its demand is interest-elastic.

The central conclusion that emerges from this study is that, in the model under consideration, policy directed towards the stabilization of nominal income dominates the other two rules in terms of minimizing the variance of real output and the price level. This result is in accord with those generated by static rational expectations models. With the dynamic model used here, we are able to make statements about relative speeds of convergence in response to particular shocks. We also are able to generate estimates of the trade-offs between variance in real output and prices and variance in the financial variables implied by our simple model. Finally, the implications of specific types of information limitations are examined.

The paper is organized as follows. In Section 2 a brief overview of the related literature is provided. Section 3 presents the model, discusses its basic properties and justifies the parameter assumptions. In the subsequent three sections we maintain the assumption of full information for the monetary authorities. In Section 4 we analyze several individual shocks, focusing in particular on real foreign interest rate shocks. Section 5 analyzes the responses to changes in target levels and growth rates, both sudden and gradual. In Section 6, the model undergoes simultaneous arrays of random shocks and the resulting variances of the key macro variables under the alternative policy rules are compared. The seventh section is concerned with limiting the information available to the monetary authorities. In the first part it is assumed that the authorities cannot respond to a shock until the quarter following its realization, and the implications of lengthening the lag with which the targeted variable is returned to control are examined. In the second part we focus on the operation of a nominal spending rule, using an interest rate reaction function that contains only observable variables. Finally, in Section 8 we provide a summary and some concluding remarks.

2 RELATED LITERATURE

As noted in the introduction, this study has its roots in the analysis of Poole (1970). The essence of this type of approach is to solve a structural model under various assumptions as to policy rule and then to calculate and compare across regimes the impacts of particular shocks on the variables of concern -- typically real output. One can think of the structural equations as a type of filter between the exogenous shocks and the endogenous variables. Changing one of these equations, namely that which describes the policy rule, changes the filter and therefore changes the way in which shocks are allocated across the various endogenous variables. A common feature of such exercises is that the rankings of policies are either shock-specific, model-specific, or both. Thus, conclusions about policy can only be generalized by accepting assumptions regarding both the relative importance of the various types of shocks and the parameters of the model. Usually some form of combination rule turns out to be optimal.

Poole's (1970) analysis dealt with the choice between fixing interest rates or the money stock within the context of a simple fixed-price IS-LM model. Extensions of this analysis to an open economy have been made by Sparks (1979) and Henderson (1979), while the extension to a fully rational model with endogenous price determination has been made by Parkin (1978). However, in none of these analyses is an activist role for monetary policy necessarily presumed. Rather, as argued most clearly in Parkin, even when there is no explicit activist role for policy, there remains the issue of policy choice, namely, which variable to target in the short run. This choice affects the automatic shock-allocating properties of the economy, and therefore can have important implications for the variances of the variables of concern.

In the present study we restrict attention to three extreme policy rules, which simply exogenize the money stock, the nominal exchange rate or nominal spending, and to the various combination rules that lie in between. All three of these targets satisfy the initial requirement that policy provide the economy with a nominal anchor, although the exchange rate rule does so only provided that the foreign economy has an anchored price level as well.[1] The extensive literature on fixed versus flexible exchange rates may be brought to bear on the issue of choosing between the money stock and the exchange rate as policy target. Specific comparative analyses of exchange rate and money stock rules include Parkin (1977), Sparks (1979), Minford (1981) and Artis and Currie (1981). The latter paper, for example, analyzes a small, open-economy IS-LM-AS model with rational expectations and cost-mark-up pricing on the supply side. The conclusion reached is that stabilization of the exchange rate provides a more stable price level than does money stock targeting, unless foreign prices are the dominant source of exogenous shocks. This leads the authors to suggest that money supply targets be made conditional on the exchange rate so as to avoid wide swings in the latter. Placing a weight on the exchange rate in this way is also convenient for the policy-maker, given that this variable is observed virtually continuously. This type of policy rule is explored in some detail below.

The suggestion of targeting nominal income was first made by Meade (1978) and has since been taken up by Tobin (1980, 1983), Hall (1983), McCallum (1984) and Gordon (1983, 1985), and related discussion). The argument has been made that, as a policy target, nominal income has several advantages over the money stock and the exchange rate. It provides the economy with a nominal anchor, and represents a reasonable compromise between real output targeting, which provides no anchor for the price level, and price level targeting, which can result in rather large movements in real output. The equal weighting of prices and real output implied by a nominal income target may not suit the preferences of everyone but, as Tobin (1983) has noted, this approach has the distinct advantage of simplicity. Although nominal income must be viewed as an intermediate target, it clearly comes much closer to describing the ultimate goals of policy, namely, stable paths for prices and real output, than do either the money supply or the exchange rate. Nominal income targeting therefore is not nearly as subject to arguments against

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1. This is not strictly true, since a possible exchange rate target would be to set the rate of appreciation equal to the foreign rate of inflation. However, in the analysis to follow we consider mainly fixed-level rules, to which the statement in the text obviously applies.

intermediate targeting as are money stock or exchange rate targeting (see B. Friedman, 1975, 1977). Most of the drawbacks to the proposal seem to stem from problems of implementation (Poole, 1980), which relate mainly to information requirements and the difficulty of achieving a nominal income target in a real world setting. These practical considerations have led some (e.g., McCallum, 1984) to recommend that monetary rules be conditioned on the performance of nominal income relative to target. This class of policy rules is also considered below.

A static analysis of the implications of nominal income targeting for the key variables of the macroeconomy has been conducted by Bean (1983). Bean analyzed a simple stochastic rational expectations aggregate demand/aggregate supply (AD-AS) model, where aggregate supply presumes overlapping contracts, under money supply and nominal income targeting. He demonstrated that the nominal income rule will minimize the variance of real output if labour supply is inelastic. If labour supply is elastic, this dominance persists for AD shocks, but is reversed for AS shocks.

In a previous Bank of Canada Technical Report, Masson (1983) provided an analytic comparison of money stock and nominal income targeting in a simple dynamic IS-LM-AS model. The shock used by Masson was a step reduction in the growth rate of the targeted variable. The central conclusion was that policy aimed at nominal income will produce less severe cycles in real variables and inflation than will a similar policy aimed at the money stock, the demand for which is assumed to be interest-elastic. Masson demonstrated that this result carries over to generalizations of his model which account for asset stocks and open economy considerations. Masson noted, however, that the generality of this conclusion is limited by the information assumptions on which the analysis is based, so that practical considerations such as uncertainty about the structure of the economy and lags in data availability might reverse the ranking of the two strategies.

In a pair of studies whose objectives closely parallel our own, Currie and Levine (1984a, b) specified a small, continuous time, open-economy rational expectations model, imposed a set of parameter values and simulated a variety of individual shocks under several policy rules. In addition, optimal rules were generated by minimizing loss functions that contain weighted averages of the variances of output, the price level, the nominal rate of interest and the exchange rate. Their results indicate that, given the assumed loss functions, all rules considered tended to be dominated by a simple price level rule, at least from a single-economy perspective. This result seems to arise because expectations in the model were completely rational even in the short run, so that only unexpected changes in the price level could have real effects. The model to be analyzed below attempts to proxy observed structural rigidities that prevent agents from developing or acting upon their expectations in a completely rational fashion on a continuous basis, at least with regard to their decision to supply labour. It is clear that in such an economy, attempts to hold the price level fixed in the face of exogenous shocks would result in rather larger movements in real output than would be seen under an alternative rule that allowed some price flexibility. Indeed, the model used here implies highly oscillatory responses of real output and other variables to shocks when the price level is held fixed. These considerations led us to drop the price rule from the analysis at an early stage.

A recent paper by Taylor (1985) has explored several alternative forms of the nominal income rule. First, Taylor estimated a simple two-equation dynamic system for real output and inflation and solved for its infinite moving-average representation. He then replaced the estimated aggregate demand curve with a particular characterization of the nominal income rule, and solved for the new moving-average representation. Taylor concluded that in examining alternative policy rules, the dynamic implications may be at least as important as the impact effects. He also provides evidence that more stability would be achieved by relating the deviation of the <u>level</u> of real output from trend to the negative of the rate of inflation, rather than relating the real <u>growth rate</u> to the negative of the rate of inflation. Both these rules differ from the policy rule considered below, which relates the deviation of the <u>level</u> of real output from trend to the negative of the deviation of the price <u>level</u> from its steady-state target, and which therefore precludes the base drift that can occur when the real growth rate is related to the negative of the rate of inflation and the objective is not achieved in every period.

3 THE MODEL

The model we use is a small system of quarterly equations that is representative of a small open economy. It consists of 11 equations, of which four provide the basic structure, two describe the formation of expectations, four are identities, and one describes the monetary rule. The functional form of the model is presented in Table 1, where all variables are in logarithmic form except the interest rates. For analytical convenience variables are normalized such that in the control solution all take the value zero.

3.1 Basic Structure

The four equations that give the basic structure of the model are the aggregate demand for real output (Y), aggregate supply determining the domestic price or GNE deflator (P), the determination of the spot exchange rate (S, the price of foreign exchange) and the demand for money (M). As is typical in an open-economy macro-model, aggregate demand depends negatively on the real interest rate and positively on the real price of foreign exchange (S+P*-P, the spot exchange rate adjusted for the foreign price level relative to the domestic price level), which is denoted SR below. Lagged adjustment is incorporated into the equation to capture the empirical finding that effects of changes in real interest rates or international competitiveness build up over time. Supply behaviour is modelled by a price equation that resembles an inflation-augmented Phillips curve. (This can be seen in equation (2) by subtracting P_{-1} from both sides of the equation.) In this equation the deviation of real output from its equilibrium level represents the demand pressure term.

In the supply curve, price expectations for the current period are formed one period ago. This captures the effect of implicit or explicit contracts. In the demand curve, however, the price expectations are those formed this period for next period's price level. This is because the relevant real interest rate is the expected real interest rate on assets purchased in the current period and held to the next period. To capture the effect that financial markets clear quickly relative to the labour

Table 1

THE MODEL IN FUNCTIONAL FORM

(1) $Y = -\alpha_1(RN-4(PE-P)) + \alpha_2(P^*+S-P) + \alpha_3 Y_{-1} + U_Y$

(2) $P = PE_{-1} + \alpha_4 Y + U_P$

(3) $S = \alpha_6(P-P^*) + (1-\alpha_6)[S_{-1} + 0.25(GRTAR_{-1}+GRTAR_{-2}+GRTAR_{-3}+GRTAR_{-4})] - \alpha_7(RN-4(PE-P) - R^* + 4(P^*E-P^*)) + \alpha_6\alpha_5 RRADJ + U_S$

(4) $M = P + \alpha_8 Y - \alpha_9 RN + \alpha_{10}(M_{-1}-P_{-1}) + U_M$

(5) $PE = P + \alpha_{11}(P-P_{-1}) + \alpha_{12}(P_{-1}-P_{-2}) + \alpha_{13}(P_{-2}-P_{-3}) + \alpha_{14}(P_{-3}-P_{-4}) + (1-\alpha_{11}-\alpha_{12} - \alpha_{13} - \alpha_{14})^*(S+P^* -S_{-1}-P^*_{-1})$

(6) $P^*E = P^* + \alpha_{15}(P^*-P^*_{-1}) + (1-\alpha_{15})(P^*_{-1}-P^*_{-2})$

(7) $RR = RN - 4(PE-P)$

(8) $EPDOT = 4(PE-P)$

(9) $YN = P+Y$

(10) $RRADJ = \sum_{i=1}^{8} w_i \left[((\alpha_3-1)/\alpha_2\alpha_4)U_{P(-i)} - U_{Y(-i)}/\alpha_2 + (\alpha_1/\alpha_2)(R^*_{-i} - 4(P^*E_{-i} - P^*_{-i})) - U_{S(-i)}/\alpha_6\right]$

(11) Policy rule

a) $M = 0$
b) $S = 0$
c) $YN = 0$
d) $RN = RN1 \equiv 5.18PE_{-1} + 3.75Y_{-1} + 1.14(S+P^*) - 0.64(S+P^*)_{-1} - 0.24(P_{-1} + P_{-2} + P_{-3} + 2P_{-4})$

e) $RN = RN1 + 4.55(1.048\sigma^2_{UP} + 0.11\sigma^2_{UY})(1.028\sigma^2_{UP} + 0.02\sigma^2_{UY} + \sigma^2_{UM})^{-1} * [M-1.028PE_{-1}-0.152Y_{-1}-0.037(S+P^*)+0.026(S+P^*)_{-1} + 0.33RN-0.9(M-P)_{-1} + 0.0097(P_{-1} + P_{-2} + P_{-3} + 2P_{-4})]$

Table 1 (continued)

All variables are in logarithmic form except the interest rates (RN, RR, R*), which are expressed as fractions at annual rates. The variables are defined as follows:

EPDOT = expected domestic inflation over next quarter (annual rates)
GRTAR = growth rate of targeted variable (explained in Section 3.4)
RRADJ = factor that ensures real interest parity in long run after a permanent shock (explained in Section 3.4)
M = money supply
P = domestic price level (analogous to GNE deflator)
PE = expected domestic price level for next quarter
P* = foreign price level
P*E = expected foreign price level for next quarter
RN = domestic nominal interest rate (3 months)
RR = real domestic interest rate
R* = foreign nominal interest rate (3 months)
S = spot exchange rate (price of foreign exchange)
U_M = error in demand for money equation
U_P = error in price equation
U_S = error in spot exchange rate equation
U_Y = error in aggregate demand (Y) equation
Y = real output
YN = nominal GNP

All variables are endogenous except GRTAR, P*, R*, U_M, U_P, U_S, and U_Y.

market, we assume that financial market participants know the current price level when forming their expectations of the future price level.

The exchange rate equation is built on the assumption that purchasing-power parity holds in the long run when the international real interest rate differential is zero and there are no permanent real shocks. We do not impose rational expectations in the exchange market since the empirical evidence rejects the joint hypothesis of rational expectations and no risk premium. (See, for example, Longworth et al. (1983).) The variables GRTAR and RRADJ are included for technical reasons and are discussed fully below.

The demand-for-money equation is a typical real partial adjustment equation where the demand for real balances depends on real output and the nominal interest rate.

3.2 Expectations

The model contains endogenously determined expectations for domestic and foreign prices. These expectations are not determined rationally in the sense of being consistent in the short run with the model in which they are imbedded. However, in the long run they are consistent with the model, in the sense that in full equilibrium the expected and actual price levels will be the same, and for small shocks may approximate short-run rational behaviour when the costs of gathering and processing information are taken into account. Expected domestic inflation from time t to t+1 (PE-P) is a weighted sum of past domestic inflation and past exchange-rate-adjusted foreign inflation, with the weights summing to one. Expected foreign inflation (P*E-P*) is also a weighted sum of past inflation, with the assumption that domestic inflation does not affect foreign inflation.[2]

3.3 Identities

The real domestic interest rate (RR), the expected rate of domestic inflation at annual rates (EPDOT) and nominal income (YN) are determined

2. In most of our experiments, the foreign price level is kept constant so the precise form of this expectations process matters little.

by appropriate identities. The variable RRADJ is an identity that determines the adjustment in the exchange rate equation necessary to give perfect international asset substitutability (equality of domestic and foreign real interest rates) in the long run.

3.4 A Digression on Exchange Rates, RRADJ and GRTAR

Recently, there has been considerable analysis of models in which the exchange rates are determined in asset markets. One implication of this theory is that the exchange rate is free to jump at any time -- there is no reason for it to adjust sluggishly as goods prices do. However, by themselves, asset market considerations can determine only the expected rate of change of the nominal exchange rate and therefore its level relative to that expected for the next period. Assumptions about the goods market must be invoked if we are to explain the determination of the expected future level of the exchange rate. A traditional assumption has been the concept of long-run purchasing-power parity, one interpretation of which is an infinite elasticity of substitution between domestic and foreign goods in the long run. Once this assumption is abandoned, however, the determination of the level of the exchange rate becomes much more complex, because it depends on all the permanent factors affecting the supply and demand for output, as well as on asset market considerations.

For example, consider the case where there is perfect international substitutability among assets, which is a reasonable long-run assumption for Canada (see Boothe et al. (1985)). Since in the long run, the Canadian real interest rate will have to equal the world real interest rate, the long-run demand for domestic output will depend on the world real interest rate, the real exchange rate, and possibly other factors.[3] Since the world real interest rate is given to the small domestic economy, the only variable that is free to adjust aggregate demand to aggregate supply is the real exchange rate. As well as monetary (or price)

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3. In a model more fully articulated than our own, the long-run supply of domestic output would also depend on the world real interest rate and the real exchange rate through the size of the capital stock and the interaction of labour demand and labour supply.

factors, the exchange rate equation would therefore have to include the factors influencing the demand for and supply of output, constrained in such a way that the goods market will clear in the long run.[4] The approach we have taken here is to have the exchange rate determined by traditional factors in the short run, but by goods-market equilibrium in the long run.

Long-run output in the model is given by the supply condition (equation (2)) which can be written as:

$$Y = \frac{P-PE_{-1}}{\alpha_4} - \frac{U_P}{\alpha_4} = \frac{-U_P}{\alpha_4} \qquad (12)$$

since in the long run, price expectations must be fulfilled ($P = PE_{-1}$). Since perfect international asset substitutability implies the equality of real interest rates (RN-4(PE-P)) = (R*-4(P*E-P*)), long-run aggregate demand (from equation (1)) will be given by

$$Y = \frac{-\alpha_1}{1-\alpha_3}(R^*-4(P^*E-P^*)) + \frac{\alpha_2}{1-\alpha_3}(S+P^*-P) + \frac{U_Y}{1-\alpha_3} \qquad (13)$$

When equating supply and demand for real output, one finds that the relationship between the real exchange rate (S+P*-P) and the real interest rate (R*-4(P*E-P*)) in equilibrium is given by:

$$S+P^*-P = \frac{\alpha_1}{\alpha_2}(R^*-4(P^*E-P^*)) - \frac{1}{\alpha_2}U_Y - \frac{(1-\alpha_3)}{\alpha_2\alpha_4}U_P \qquad (14)$$

The inclusion of a four-quarter moving average of the variable GRTAR, the growth rate of the targeted variable, ensures that in the steady state the exchange rate equation (3) with real interest rates equal

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4. This is in stark contrast with empirical exchange rate equations, which almost never contain explicit goods-market factors. In many rational expectations macro-models, however, the uncovered interest parity condition gives the expected change in the exchange rate while the solution for the level of the long-run equilibrium exchange rate is generated through the imposition of goods-market equilibrium.

internationally can be written as[5]:

$$S = P - P^* + \frac{U_S}{\alpha_6} + \alpha_5 \text{ RRADJ} \tag{15}$$

Equating equations (14) and (15) gives the steady-state value of RRADJ as

$$\text{RRADJ} = \frac{1}{\alpha_5}\left[-\left(\frac{1-\alpha_3}{\alpha_2\alpha_4}\right)U_P - \frac{1}{\alpha_2}U_Y + \frac{\alpha_1}{\alpha_2}(R^*-4(P^*E-P^*)) - \frac{U_S}{\alpha_6}\right] \tag{16}$$

The coefficient α_5 is arbitrary, so we have normalized it to unity. We permit RRADJ to adjust to its equilibrium value in the eight quarters following a permanent shock by applying the weights w_i, assumed to be along a normal probability density function, which sum to unity (Table 2). Thus, in the case of a transitory shock, one of the terms in RRADJ will be temporarily non-zero, and the weight on that term will rise gradually to about 0.25 and then fall back to zero. This is meant to be a rough approximation to the sort of learning lag one might expect to observe. On the other hand, for a permanent shock, one of the terms in RRADJ becomes non-zero permanently, and the weight with which the term influences the exchange rate equation will rise along the cumulative distribution, reaching unity after eight quarters.

3.5 Policy Rules — Closing the Model

The model has eleven endogenous variables and seven exogenous variables (see Table 1). The equation that closes the model is a policy rule that sets one of the endogenous variables (money stock, exchange rate, or nominal income) equal to a constant; since the model is written in deviations from equilibrium values, this constant in each case takes on the value zero. This allows for the solution of the domestic interest

5. Because of the linear homogeneity of the model, GRTAR and S will grow at the same rate in steady-state equilibria where P* is constant. If the term in GRTAR were not added to S_{-1}, although the growth rate of S (with P* constant) eventually would converge to the growth rate of the targeted variable, the real exchange rate level would differ across steady states with differing targeted growth rates.

rate and all other endogenous variables. In a later section, the model is closed by exogenizing linear combinations of the three target variables and by interest rate reaction functions. A rule setting a weighted average of two or three nominal variables to zero provides the economy with a nominal anchor because it holds the sum of two or three nominal variables,[6] which is itself a nominal variable, constant.

3.6 Coefficients for the Basic Structure

The coefficients for the equations that describe the basic structure of the model (see Tables 2 and 3) were taken from a number of empirical studies for Canada, including RDXF, studies of M1 demand and reduced-form equations estimated at the Bank of Canada.

On the basis of previous empirical studies, including those of Pierre Duguay (1979), it is estimated that the long-run impact of a 1 percentage point rise in the real interest rate on real output is in the neighbourhood of 0.8 per cent. The mean lag is about 3.5 quarters, which makes it convenient to assume a coefficient on the lagged dependent variable (α_3) of 0.75 and an impact coefficient (α_1) of 0.2. A long-run effect of a 1 per cent change in the real exchange rate on real output of between 0.3 and 0.4 per cent, which is consistent with RDXF simulations of an exchange rate shock,[7] implies an impact coefficient (α_2) of 0.1.

The coefficient on real output in the aggregate supply equation implies that a 1 per cent reduction in real output will reduce the price level by 0.1 per cent in the first quarter, and therefore will reduce the annual rate of inflation by 0.4 per cent in the first quarter.

The exchange rate equation has a coefficient on the lagged dependent variable ($\alpha_6 = 0.75$) and a coefficient on the interest rate differential ($\alpha_7 = -0.5$) which resemble those in RDXF. However, the influence of the relative price term comes with an immediate impact of 0.25, rather than

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6. S is in truth a relative price of two currencies. However, S + P* is a nominal variable which, like M and YN, can provide a nominal anchor. Since for most of our experiments P* is held fixed, we treat S itself as a target variable.

7. Bank of Canada Technical Report 26, page 72.

Table 2

COEFFICIENTS

$\alpha_1 = 0.2$	$\alpha_9 = 0.3$
$\alpha_2 = 0.1$	$\alpha_{10} = 0.9$
$\alpha_3 = 0.75$	$\alpha_{11} = 0.30$
$\alpha_4 = 0.1$	$\alpha_{12} = 0.24$
$\alpha_5 = 1.00$	$\alpha_{13} = 0.18$
$\alpha_6 = 0.25$	$\alpha_{14} = 0.12$
$\alpha_7 = 0.5$	$\alpha_{15} = 0.72$
$\alpha_8 = 0.1$	

$w_1 = w_8 = 0.0228,\ w_2 = w_7 = 0.069,\ w_3 = w_6 = 0.1596,\ w_4 = w_5 = 0.2486$

Table 3

EQUATIONS WITH IMPOSED COEFFICIENTS

(1') $Y = -0.2(RN-4(PE-P)) + 0.1(P*+S-P) + 0.75Y_{-1} + U_Y$

(2') $P = PE_{-1} + 0.1Y + U_P$

(3') $S = 0.25(P-P*) + 0.75[S_{-1} + 0.25(GRTAR_{-1} + GRTAR_{-2} + GRTAR_{-3} + GRTAR_{-4})] - 0.5(RN-4(PE-P)-R*+4(P*E-P*)) + 0.25RRADJ + U_S$

(4') $M = P + 0.1Y - 0.3RN + 0.9(M_{-1}-P_{-1}) + U_M$

(5') $PE = P + 0.30(P-P_{-1}) + 0.24(P_{-1}-P_{-2}) + 0.18(P_{-2}-P_{-3}) + 0.12(P_{-3}-P_{-4}) + 0.16(S + P* - S_{-1} - P*_{-1})$

(6') $P*E = P* + 0.72(P*-P*_{-1}) + 0.28(P*_{-1}-P*_{-2})$

having the coefficient of 0.25 applied to a four-quarter moving average of relative prices, as in RDXF.[8]

The money-demand equation has a long-run real income elasticity of 1.0 ($\alpha_8/(1-\alpha_{10})$) and a long-run semi-elasticity with respect to the interest rate of -3.0 ($-\alpha_9/(1-\alpha_{10})$). Since in the model interest rates are entered as fractions, this semi-elasticity is consistent with an elasticity of -0.3 at an interest rate of 10 per cent.

3.7 Coefficients in Price Expectations Equations

The price expectations equations (PE and P*E) have been constructed to give the model accelerationist properties. If the equations are written in inflation form (PE-P, for example) it can be seen that the sum of the coefficients on past inflation is one. We began by estimating regressions of inflation on its past values (and, in the case of PE-P, on past inflation in exchange-rate-adjusted foreign prices) over the 1970Q4-1982Q4 period.[9] We then lengthened the average lag slightly in order to increase the output change required to produce a lower rate of inflation to that consistent with results from other empirical work.

3.8 Comments on Properties of the Model

The output equation is homogeneous of degree zero in prices and the price and exchange rate equations are homogeneous of degree one in prices. As mentioned above, the model has accelerationist features -- the reduction in the steady-state rate of inflation is proportional to the integral of the deviation of output from its normal value of zero.

The long-run solution of the model can be found by solving the first four equations (1' through 4') above for a long-run static solution where PE = P and P*E = P*. Then:

$$Y = -0.8RR + 0.4(P^*+S-P) + 4U_Y \quad \text{(Demand)} \qquad (17)$$

8. An equation much like the RDXF equation is found in Boothe et al. (1985).

9. We started with long lags and dropped insignificant coefficients at the longer lags.

$$Y = -10U_P \quad \text{(Supply)} \tag{18}$$

$$RR = RR^* \quad \text{(Perfect Asset Substitutability)} \tag{19}$$

$$M - P = Y - 3RN + 10U_M \quad \text{(Money Demand)} \tag{20}$$

where the U_i shocks are taken to be permanent.

By equating long-run aggregate supply to long-run aggregate demand and using (19) we obtain the following expression for the real exchange rate:

$$S + P^* - P = -25\ U_P - 10U_Y + 2RR^* \tag{21}$$

The solutions for the nominal variables P, S and M depend on the policy rule being followed. Table 4 presents some examples.

Table 4

STEADY-STATE SOLUTIONS FOR NOMINAL VARIABLES UNDER VARIOUS POLICY RULES

	Policy Rule		
Variable	P+Y=0	S=0	M=0
P	$10U_P$	P^*-2RR^*	$3RR^*-10U_M$
		$+10U_Y+25U_P$	$+10U_P$
S	$-P^*+2RR^*$	0	$-P^*+5RR^*-10U_M$
	$-10U_Y-15U_P$		$-10U_Y-15U_P$
M	$-3RR^*+10U_M$	$P^*-5RR^*+10U_M$	0
		$+10U_Y+15U_P$	

Notice that, since long-run aggregate supply does not depend on the real interest rate, the model is unsuitable for answering certain long-run questions that relate to the response of the capital stock to interest rate changes.

4 INDIVIDUAL SHOCKS UNDER FULL INFORMATION

In this and in the two following sections, we maintain the hypothesis that the monetary authority operates with full information. This means that shocks are observed (or may be deduced by observing all variables) and may be responded to in the same quarter, so that the targeted variable never deviates from control. This assumption is more restrictive for some shocks and some rules than it is for others. For example, it does not seem restrictive in the case of an exchange rate rule, since this variable may be observed and targeted more or less continuously. Also, the assumption is not restrictive in the case of foreign interest rate shocks, which are also observable on a continuous basis. However, in the case of IS shocks in general, or under a nominal income rule, this assumption is obviously rather strong, and so in Section 7 this premise is relaxed.

In this section we examine the properties of the model under the three rules when it is faced with ten shocks, each treated as arising individually. The shocks considered are transitory and permanent IS, AS, exchange market (EM) and P* shocks, and temporary and permanent foreign interest rate shocks. Transitory shocks are one quarter in duration, while the temporary foreign interest rate shock is assumed to last four quarters. Treating transitory shocks as being of one-period duration is standard in the literature, while in the case of foreign real interest rate shocks it was felt that a somewhat longer duration was more typical and therefore would be more interesting. All shocks take on the value unity, which may be interpreted as a one per cent increase, or one percentage point in the case of interest rate shocks. As noted in the preceding section, the control solution of the model is zero, so all shock solutions are in deviation-from-control form.

The results are presented in two ways. First, in Table 5 we present the Root-Mean-Squared deviations from control calculated over cumulative horizons of 1, 3 and 10 years for each of the shocks, for the three rules, and for eight key macro variables. These data provide an impression of the relative variances that might be observed under each rule, assuming that each shock arose in isolation. In addition to these data, we present Figures 1-48, which focus on the transitory and permanent IS and AS shocks, as well as on the two foreign interest rate shocks. In each graph the solutions from the three different rules are compared. The results are extensive and are presented for completeness only. The following discussion will focus on the general statements that may be made and on the more interesting results.

4.1 General

Referring to the data in Table 5, we note in particular that even when one chooses the variance of real output as the sole criterion for selecting a policy option, the ranking depends not only on the shock but also on the horizon over which the variance is calculated. For example, in the case of the permanent IS shock (a permanent increase in government expenditure, perhaps, or in export demand because of a rise in U.S. output), at the 1-year horizon one would choose the nominal income rule in order to minimize the variance of real output, but at the 10-year horizon the money supply rule dominates. These differences arise because the model is dynamic and the different rules give rise to different speeds of adjustment. This point will become clearer below.

Focusing on the 1-year horizon and using the variance of real output as the criterion, we see that the nominal income rule dominates for all 10 shocks with the exception of the two aggregate supply shocks. The exchange rate rule shows the worst performance for all except the two aggregate supply shocks, in which the exchange rate rule is preferred. The money supply rule lies in an intermediate position between the other two rules. Interestingly, at longer horizons the nominal income rule begins to dominate the other two for both aggregate supply shocks as well. This occurs because the nominal income rule provides a faster-converging system in which most of the adjustment to these shocks is completed in the first year or two.

Despite the qualifications that must be made, the dominance of the nominal income rule over the other two rules is, as predicted by the literature, fairly general. However, with a given set of exogenous shocks

Table 5

RESPONSES TO TRANSITORY AND PERMANENT SHOCKS UNDER FULL INFORMATION (1 per cent shock, per cent deviation from control)

(a) Transitory IS Shock

		Y	P	YN	RN	RR	M	S	SR
M-Rule									
RMS*	1 year	0.513	0.137	0.611	0.352	0.380	0.000	0.561	0.696
	3 years	0.339	0.139	0.428	0.288	0.268	0.000	0.926	0.936
	10 years	0.187	0.124	0.249	0.164	0.149	0.000	0.512	0.514
S-Rule									
RMS	1 year	0.709	0.287	0.945	0.176	0.308	0.387	0.000	0.287
	3 years	0.528	0.852	0.985	0.586	0.544	0.842	0.000	0.852
	10 years	0.500	0.716	0.596	0.523	0.403	0.581	0.000	0.716
YN-Rule									
RMS	1 year	0.190	0.190	0.000	1.537	2.191	0.554	1.201	1.110
	3 years	0.160	0.160	0.000	0.966	1.311	0.436	0.893	0.784
	10 years	0.123	0.123	0.000	0.539	0.720	0.252	0.494	0.453

(b) Permanent IS Shock

		Y	P	YN	RN	RR	M	S	SR
M-Rule									
RMS	1 year	1.426	0.335	1.747	0.993	1.030	0.000	1.255	1.590
	3 years	1.104	0.447	1.515	0.751	0.993	0.000	6.312	6.594
	10 years	0.651	0.860	1.022	0.559	0.566	0.000	9.465	8.865
S-Rule									
RMS	1 year	2.020	0.620	2.619	0.338	0.379	0.823	0.000	0.620
	3 years	3.189	4.565	7.418	1.489	1.742	5.402	0.000	4.565
	10 years	2.563	10.038	9.631	2.924	1.740	7.037	0.000	10.038
YN-Rule									
RMS	1 year	0.418	0.418	0.000	1.699	2.704	1.581	3.275	2.882
	3 years	1.138	1.138	0.000	1.431	1.664	1.264	6.699	5.570
	10 years	0.786	0.786	0.000	0.870	0.931	0.855	9.256	8.958

* Root mean square deviation from control

Table 5 (continued)

(c) Transitory AS Shock

		Y	P	YN	RN	RR	M	S	SR
M-Rule									
RMS	1 year	0.571	1.245	0.705	1.663	1.325	0.000	0.678	1.840
	3 years	0.847	1.206	0.465	0.995	0.812	0.000	1.446	2.529
	10 years	0.499	0.834	0.437	0.583	0.453	0.000	0.847	1.403
S-Rule									
RMS	1 year	0.302	1.421	1.134	1.297	0.622	0.413	0.000	1.421
	3 years	0.807	2.364	1.817	1.341	1.218	1.256	0.000	2.364
	10 years	0.995	1.738	1.149	1.211	0.884	1.089	0.000	1.738
YN-Rule									
RMS	1 year	0.976	0.976	0.000	1.982	1.726	0.641	1.410	2.381
	3 years	0.821	0.821	0.000	1.281	1.117	0.407	1.554	2.342
	10 years	0.468	0.468	0.000	0.713	0.614	0.256	0.855	1.292

(d) Permanent AS Shock

		Y	P	YN	RN	RR	M	S	SR
M-Rule									
RMS	1 year	1.499	3.142	1.655	4.254	1.796	0.000	1.453	4.575
	3 years	5.912	9.120	3.282	4.624	1.650	0.000	8.981	18.051
	10 years	10.022	13.180	3.898	3.268	1.265	0.000	11.530	24.253
S-Rule									
RMS	1 year	0.796	3.558	2.765	3.690	0.628	1.009	0.000	3.558
	3 years	2.812	15.126	12.480	5.813	2.675	7.721	0.000	15.126
	10 years	10.993	25.143	14.891	7.068	3.798	9.645	0.000	25.143
YN-Rule									
RMS	1 year	2.597	2.597	0.000	5.266	3.776	1.504	3.638	6.235
	3 years	6.409	6.409	0.000	4.119	2.629	2.508	11.568	17.965
	10 years	9.396	9.396	0.000	2.431	1.497	2.312	14.220	23.589

Table 5 (continued)

(e) Transitory EM Shock

		Y	P	YN	RN	RR	M	S	SR
M-Rule									
RMS	1 year	0.148	0.212	0.296	0.420	0.635	0.000	0.785	0.704
	3 years	0.118	0.165	0.184	0.257	0.371	0.000	0.489	0.496
	10 years	0.071	0.134	0.126	0.149	0.204	0.000	0.275	0.275
S-Rule									
RMS	1 year	0.250	0.101	0.318	1.014	1.015	0.549	0.000	0.101
	3 years	0.213	0.112	0.200	0.658	0.645	0.370	0.000	0.112
	10 years	0.132	0.114	0.124	0.371	0.357	0.217	0.000	0.114
YN-Rule									
RMS	1 year	0.071	0.071	0.000	0.516	0.388	0.266	0.433	0.414
	3 years	0.055	0.056	0.000	0.354	0.260	0.177	0.279	0.263
	10 years	0.033	0.033	0.000	0.195	0.143	0.103	0.153	0.145

(f) Permanent EM Shock

		Y	P	YN	RN	RR	M	S	SR
M-Rule									
RMS	1 year	0.363	0.477	0.820	0.900	0.427	0.000	2.137	1.703
	3 years	0.290	1.214	1.247	0.859	0.443	0.000	1.814	1.143
	10 years	0.296	1.096	0.936	0.600	0.298	0.000	1.129	0.719
S-Rule									
RMS	1 year	0.698	0.229	0.915	1.573	1.741	1.569	0.000	0.229
	3 years	0.553	0.814	0.936	1.102	1.090	1.627	0.000	0.814
	10 years	0.492	0.677	0.566	0.720	0.649	0.950	0.000	0.677
YN-Rule									
RMS	1 year	0.157	0.157	0.000	1.210	0.905	0.743	1.114	0.972
	3 years	0.160	0.160	0.000	0.777	0.608	0.939	0.805	0.669
	10 years	0.120	0.120	0.000	0.437	0.337	0.530	0.445	0.390

Table 5 (continued)

(g) Transitory P* Shock

		Y	P	YN	RN	RR	M	S	SR
M-Rule									
RMS	1 year	0.105	0.109	0.152	0.296	0.505	0.000	0.997	0.513
	3 years	0.076	0.109	0.114	0.199	0.300	0.000	0.593	0.337
	10 years	0.047	0.092	0.083	0.114	0.165	0.000	0.327	0.187
S-Rule									
RMS	1 year	0.394	0.202	0.485	1.621	1.980	0.570	0.000	0.503
	3 years	0.276	0.211	0.299	0.961	1.165	0.372	0.000	0.340
	10 years	0.171	0.165	0.180	0.539	0.641	0.228	0.000	0.220
YN-Rule									
RMS	1 year	0.044	0.044	0.000	0.418	0.301	0.136	0.788	0.312
	3 years	0.034	0.034	0.000	0.274	0.201	0.102	0.463	0.192
	10 years	0.021	0.021	0.000	0.151	0.110	0.063	0.254	0.106

(h) Permanent P* Shock

		Y	P	YN	RN	RR	M	S	SR
M-Rule									
RMS	1 year	0.181	0.259	0.420	0.452	0.336	0.000	0.200	0.840
	3 years	0.161	0.773	0.819	0.572	0.307	0.000	2.189	0.676
	10 years	0.195	0.756	0.647	0.412	0.212	0.000	1.568	0.446
S-Rule									
RMS	1 year	0.834	0.496	1.288	1.418	1.877	1.542	0.000	0.622
	3 years	0.638	1.553	1.788	1.124	1.199	1.932	0.000	0.847
	10 years	0.632	1.387	1.276	0.816	0.751	1.398	0.000	0.864
YN-Rule									
RMS	1 year	0.084	0.084	0.000	0.599	0.449	0.380	1.542	0.481
	3 years	0.010	0.010	0.000	0.221	0.145	0.384	2.264	0.188
	10 years	0.099	0.099	0.000	0.470	0.380	0.620	1.505	0.434

Table 5 (continued)

(i) Temporary R* Shock

		Y	P	YN	RN	RR	M	S	SR
M-Rule									
RMS	1 year	0.214	0.255	0.462	0.500	0.195	0.000	1.248	1.007
	3 years	0.179	0.886	0.956	0.669	0.311	0.000	1.419	0.767
	10 years	0.230	0.911	0.781	0.493	0.235	0.000	0.919	0.515
S-Rule									
RMS	1 year	0.404	0.124	0.524	0.937	1.043	0.892	0.000	0.124
	3 years	0.338	0.577	0.735	0.638	0.683	1.271	0.000	0.577
	10 years	0.378	0.563	0.459	0.487	0.441	0.758	0.000	0.563
YN-Rule									
RMS	1 year	0.084	0.084	0.000	0.712	0.517	0.419	0.655	0.576
	3 years	0.116	0.116	0.000	0.519	0.430	0.730	0.614	0.505
	10 years	0.092	0.092	0.000	0.299	0.239	0.437	0.344	0.303

(j) Permanent R* Shock

		Y	P	YN	RN	RR	M	S	SR
M-Rule									
RMS	1 year	0.214	0.255	0.462	0.500	0.195	0.000	1.248	1.007
	3 years	0.340	1.471	1.769	1.353	0.419	0.000	3.000	1.614
	10 years	0.490	4.070	3.710	1.550	1.126	0.000	5.307	1.458
S-Rule									
RMS	1 year	0.404	0.124	0.524	0.937	1.043	0.892	0.000	0.124
	3 years	0.638	0.913	1.484	0.887	1.293	2.560	0.000	0.913
	10 years	0.513	2.008	1.926	1.063	1.061	3.815	0.000	2.008
YN-Rule									
RMS	1 year	0.084	0.084	0.000	0.712	0.517	0.419	0.655	0.576
	3 years	0.052	0.052	0.000	1.195	0.849	1.986	1.795	1.241
	10 years	0.157	0.157	0.000	1.000	0.971	2.439	1.851	1.792

Notes:

1. Figures shown are root-mean-squared deviations from control over the three post-shock horizons.
2. Shocks are equal to unity; since the model is expressed in logarithms this may be thought of as 1 per cent or, in the case of interest rates, 100 basis points.
3. Transitory refers to a shock one quarter in duration; permanent refers to a shock that becomes unity and remains there for the duration of the simulation; temporary refers to a shock that is four quarters long.

there is a given amount of total variation that must be absorbed by the economy, and changing the policy rule simply reallocates this variation between the principal variables. Thus, a reduction in the variance of real output usually will be reflected in an increase in variance elsewhere in the economy. Once again focusing on the 1-year horizon, Table 4 reveals that the financial variables -- the nominal and real interest, exchange rates and the money stock -- generally have higher variances under the nominal income rule. For example, with the IS and AS shocks, the lowest variance of nominal interest rates is generally obtained under the exchange rate rule, with the most variance under the nominal income rule. In contrast, the EM, P* and R* shocks result in a high nominal interest rate variance under the exchange rate rule, with the lowest variance being observed under the money stock rule. Thus, the nominal income rule is not always associated with the highest interest rate variance, but there does appear to be a trade-off between variance in real output and the price level on the one hand and in the financial variables on the other. The nature of the trade-off implied by our model will be examined in more detail below.

4.2 Solution Paths for Specific Shocks

We now turn to a comparison of the three rules for specific shocks, referring to Figures 1-48. We focus on transitory and permanent IS(UY) and AS(UP) shocks, and then on temporary and permanent foreign interest rate shocks.

4.2.1 Transitory IS shock (Figures 1-8)

Real output (Y) initially responds virtually one-for-one to the IS shock under the exchange rate (S) rule, and only slightly less so under the money stock (M) rule. The nominal income (YN) rule allows Y to respond only about one-third as much as the shock. Furthermore, this response occurs in the quarter after the shock, since the YN-rule can insulate Y and P from IS shocks completely in the first period.[10] Thereafter, all three rules allow cycles in real output, with output under the YN-rule damping to zero after about 10 years and output in the other two cases still deviating significantly from control after 25 years. Under the M- and S-rules the price level (P) behaves similarly, rising about 0.2 and 1.2 per cent, respectively, and then cycling persistently. The cycles are considerably more damped under the M-rule than under the S-rule. Of course, under the YN-rule, P must fall (in the quarter after the shock) to mirror the movements in Y, a movement which is accomplished through a 2 per cent appreciation of the exchange rate. Thereafter, the return to control is very rapid under the YN-rule.

Under the YN-rule, less movement in Y and P is obtained by allowing greater movements in the financial variables. For example, the nominal rate of interest (RN) rises nearly 3 percentage points above control, while the real rate (RR) rises 4 percentage points; both fall to a level about 100 basis points below control in the next period, a movement that is necessary to provide a "soft landing" (in this case, no change) for YN. Under the M- and S-rules, on the other hand, the initial movements in RN and RR are much more modest, on the order of 50 basis points for the M-rule, and nearly zero for the S-rule. Notice that the response under the S-rule is somewhat more gradual than under the M-rule. Both rules then lead to cycles lasting for more than 25 years. The exchange rate appreciates sharply under both the YN- and M-rules, but damps fairly quickly.

4.2.2 Permanent IS shock (Figures 9-16)

The permanent IS shock leads to much larger output responses, with Y under the M- and S-rules still cycling between ± 1 per cent after about 15 years. By that time the YN-rule has brought Y and P back to control. Under the S-rule, the price level cycles around a level that is 10 per cent above control. This occurs because the real exchange rate must

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10. This is so because the authorities react to the IS shock with an offsetting shift in the LM curve in the opposite direction, thus keeping real output constant, and the resulting effect on the exchange rate does not affect the price level until the following quarter.

appreciate 10 per cent in steady state to bring output back to control and the S-rule prevents the nominal exchange rate from adjusting. This fact also reduces the rate of convergence of the S-rule relative to the other rules.

4.2.3 Transitory AS shock (Figures 17-24)

The responses of the model to AS (UP) shocks are somewhat more parameter-specific than for AD shocks, since even the direction of the appropriate policy response may be reversed by a different choice of parameters. Initially, Y declines under all three rules, with the largest drop for the YN-rule and smallest for the S-rule. Return to control is fastest for the YN-rule. The price level rises in all three cases, with the largest increase under the S-rule and the smallest under the YN-rule. The movements in RN and RR are broadly similar across regimes. Given our choice of parameters, under all three rules both real and nominal interest rates must rise in response to AS shocks. The response is sharpest under the YN-rule, where real and nominal rates rise more than 300 basis points, but the M-rule response is nearly as strong. Under the S-rule, on the other hand, the increase in rates is only about 150 basis points. The subsequent overshooting of control, which is required for a soft landing, is smallest under the M-rule. The exchange rate appreciates at impact under both the YN- and M-rules and then cycles around control. The speed of convergence is very slow under the M-rule.

4.2.4 Permanent AS shock (Figures 25-32)

An example of a permanent AS shock would be a permanent increase in the price of oil. All three rules produce the expected decline in output, each converging at different speeds at a level 10 per cent below control. The overshooting of the new equilibrium level is greatest under the S-rule, while the initial reduction in real output is lowest under the YN-rule. The same comments may be made with respect to the price level, which of course rises in all three cases. The nominal rate of interest rises 5 to 6 percentage points above control during the first three years, and subsequently falls off for the M- and YN-rules. Under the S-rule there is a second round that takes RN to 12 percentage points above control, and the subsequent rate of convergence is very slow. The nominal exchange rate appreciates 15 per cent under the YN- and M- rules, whereas the steady-state real appreciation is 25 per cent for all three rules.

4.2.5 Foreign interest rate shocks (Figures 33-48)

We have chosen to examine foreign interest rate shocks in some detail for two reasons. First of all, the appropriate policy response to an increase in foreign interest rates is of special concern in Canada. Second, these experiments do not require the strong information assumptions that have been implicit in the experiments discussed so far. Unlike IS and AS shocks, in the case of foreign interest rate shocks there is no recognition lag. This means that the authorities can react immediately in a previously determined way to keep their targeted variable constant. For the other shocks, more realistic information assumptions are imposed in Section 7.

It should be noted at the outset that because ours is a one-country model there is no entirely satisfactory way of incorporating a reason for a foreign interest rate shock. The shocks examined are nominal and real interest rate shocks, since the foreign price level is held constant. Furthermore, the model cannot distinguish between an arbitrary shock and a rise in rates prompted by a surge in foreign real output. Obviously, a two-country model would be required in order to address properly such questions.[11]

The temporary foreign interest rate shock considered here is an increase of 100 basis points in the foreign nominal (and real) interest rate that lasts four quarters. The results are depicted in Figures 33-40. Under all three rules, the authorities respond by raising domestic nominal interest rates, but to varying degrees depending on which nominal variable is held constant. Under the S-rule the initial response is, not

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11. Some effort has been made to answer these concerns by running a combination shock with the model. Under the assumption that the foreign interest rate increase was due to a surge in real foreign output growth, we added to the foreign interest rate shock simulation a small AD shock and a small negative shock to the exchange rate. The results were not very different from those presented below.

surprisingly, virtually one-for-one; after the shock disappears, RN moves below control and then cycles around control for some time. Under the M-rule, RN rises just over 50 basis points after two quarters. However, in contrast with the path taken under the S-rule, RN subsequently rises further and remains above control for two more years before falling below and then cycling around control. The path followed by RN under the YN-rule falls between that of the other two, rising about 70-80 basis points, falling below control immediately after the shock disappears, and then quickly converging to control. A similar description is true of real interest rates under the S- and YN-rules. However, under the M-rule real interest rates actually fall in the first instance. This movement is due to the large depreciation of the exchange rate, which enters the determination of expectations of future prices. Notice also that under the M-rule, the real rate remains above control for about five years after the shock, unlike the other two rules. Thus, relatively lower interest rates at the time of the shock must be paid for with a much longer period of above-normal rates later on.

Because the nominal rate of interest rises only about 70 basis points under the YN-rule, the authorities must accept some depreciation of the exchange rate in the short run. Figure 39 indicates that this depreciation is on the order of 0.75 per cent, and virtually disappears after the shock dissipates. In contrast, the depreciation under the M-rule is about twice as large and remains above 0.3 per cent for about five years after the shock disappears.

On the real side, we notice that the moderate rise in interest rates under the M-rule allows the effect of the depreciation to dominate and cause an expansion of real output. Both the YN- and S-rules lead to contractions in real output, with by far the sharpest contraction seen under the S-rule. The same may be said for the price level, which rises about 1.4 per cent under the M-rule but falls 0.5 per cent under the S-rule. Under the YN-rule, P rises slightly more than 0.1 per cent, mirroring an opposite movement in real output.

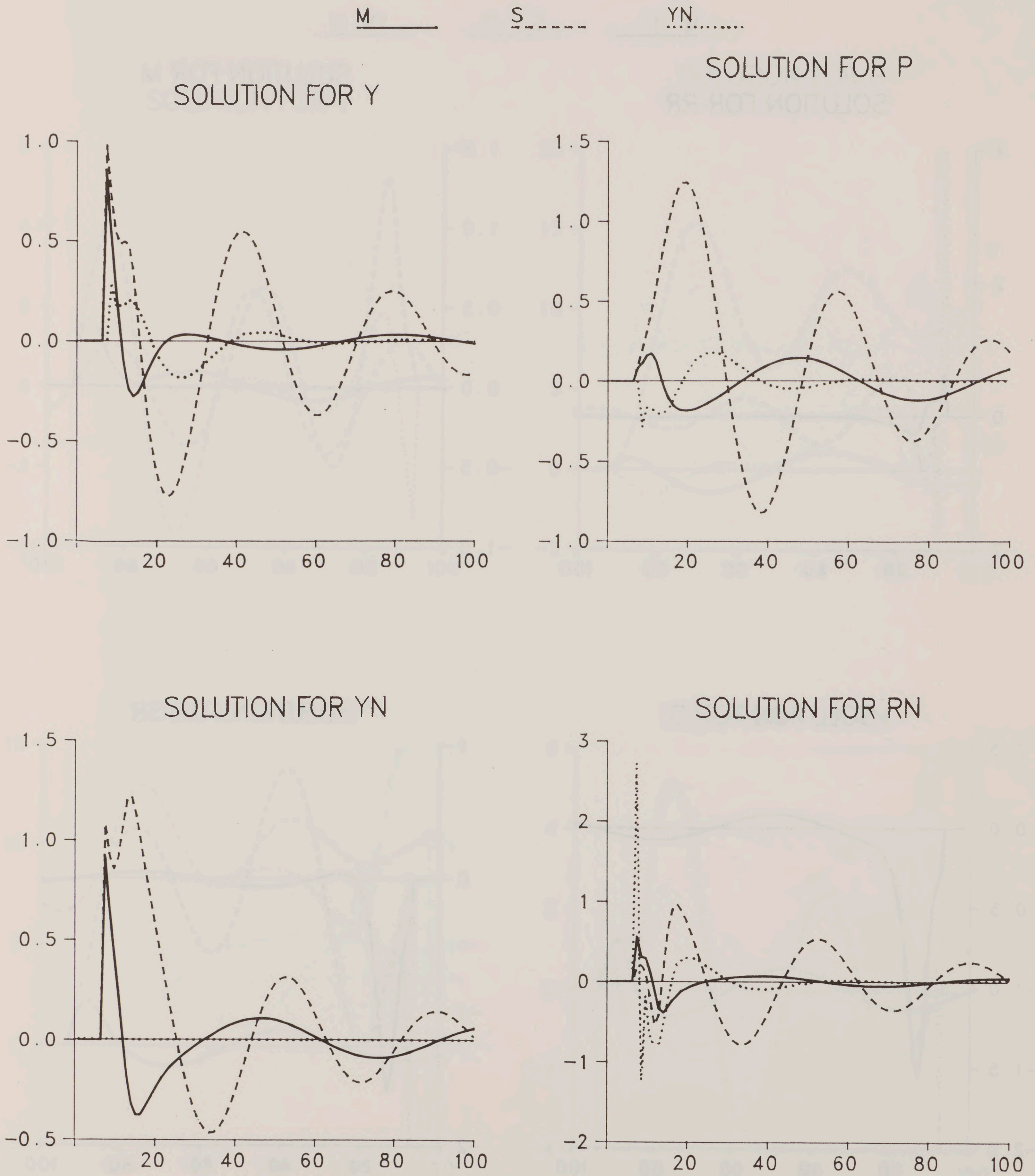
The case of a permanent increase in the foreign interest rate is examined next. In this case, the foreign nominal (and real) interest rate rises 100 basis points for the duration of the simulation. The results are presented in Figures 41-48.

Under the S-rule and YN-rule, the nominal and real interest rate solutions are very similar to those for the temporary shock, except that now they converge on the long-run value of 1 per cent. Under the M-rule, however, nominal rates continue to rise for about three years post-shock to a level 2 percentage points above control. At the same time, real rates reach a peak of 1.5 percentage points above control after about five years. Both RN and RR then cycle around the long-run value of 1 per cent. Under the YN-rule, S depreciates immediately, without cycles, by 2 per cent, while under the M-rule the depreciation peaks at over 6 per cent and then cycles around 5 per cent. A similar path is followed by the price level under the M-rule. Under the S-rule, the required real depreciation causes P to fall 2 per cent while under the YN-rule, both P and Y remain virtually unchanged. Real output expands initially under the M-rule and contracts under the S-rule.

These results indicate a fairly strong preference in the context of our model for the YN-rule over the other two when faced with either a temporary or a permanent foreign interest rate shock. In the case of the temporary shock, in particular, we have seen that both the M- and S-rules result in large fluctuations in real output and prices which may be reduced at what seems to be a small cost in terms of variability in financial markets. Specifically, the short-run nominal interest rate solutions under the M- and YN-rules are very similar during the first four quarters, with interest rates under the YN-rule rising only slightly more quickly. Evidently, the large cycles that arise under the M-rule (but do not occur under the YN-rule) result from keeping interest rates down initially, thereby inducing a large exchange rate depreciation and the attendant increase in output and prices. The S-rule, in contrast, generates an RN response that is initially very strong, and therefore causes a sharp decline in real output and prices. The resulting real cycle is on the order of three times the amplitude of that generated under the YN-rule, and is far more persistent as well. The YN-rule takes the middle course with nominal interest rates adjusting more quickly and by slightly more than under the M-rule, but less than under the S-rule. In the short run the nominal income rule therefore results in a temporary depreciation of the exchange rate, but the size of the latter movement is only about half that experienced under the M-rule. As a result, real output and prices deviate from control by much less than under the M- or S-rules. The real advantage of the YN-rule over the other two comes from allowing one additional financial variable to adjust in response to shocks; this implies that real output and prices need to adjust less in order to equilibrate the system. In other words, allowing a foreign interest rate shock to impact partly on domestic interest rates and partly on the exchange rate is preferable to a policy that forces all the necessary adjustment onto only one variable.

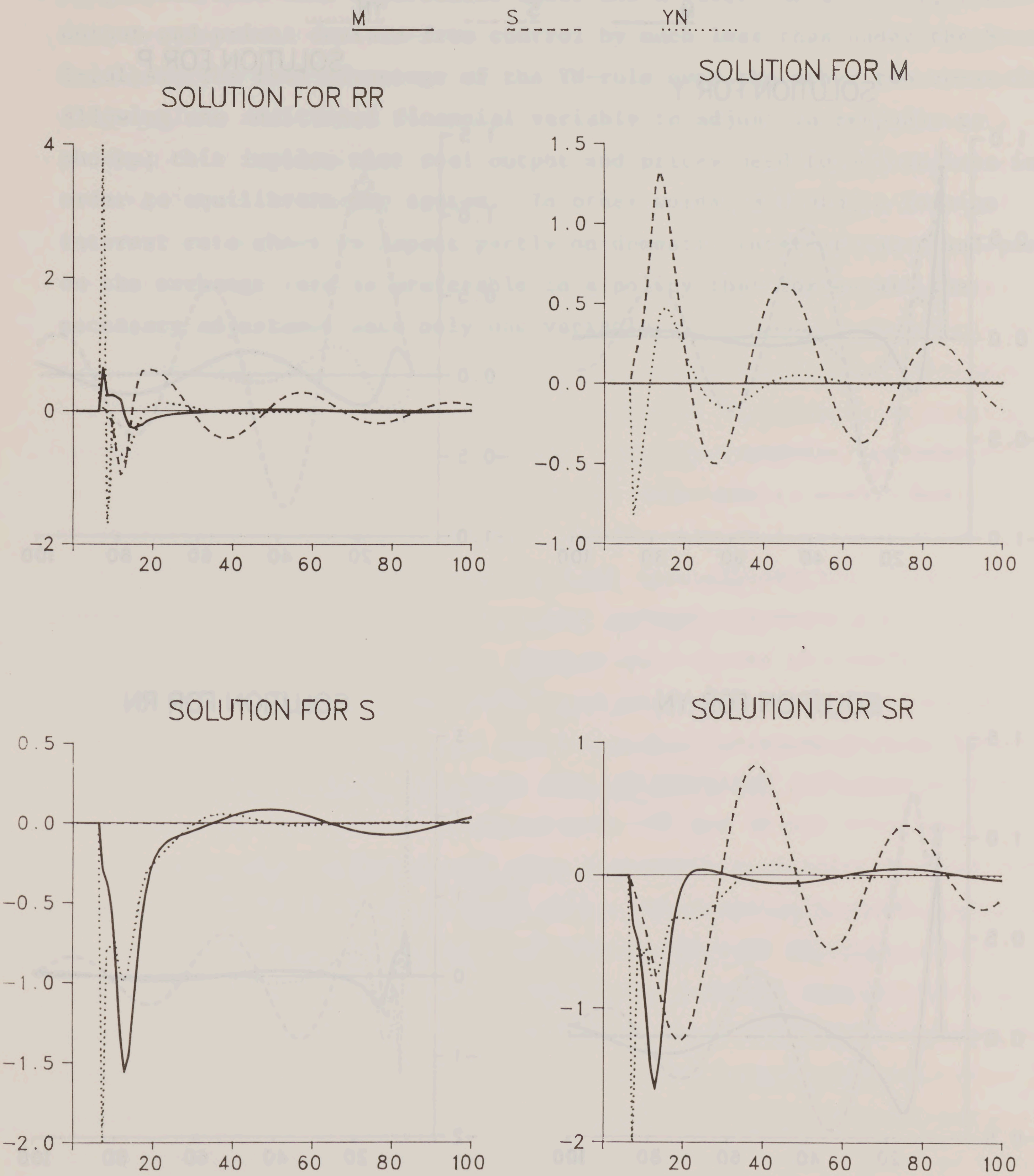
Figures 1-4

TRANSITORY UY SHOCK



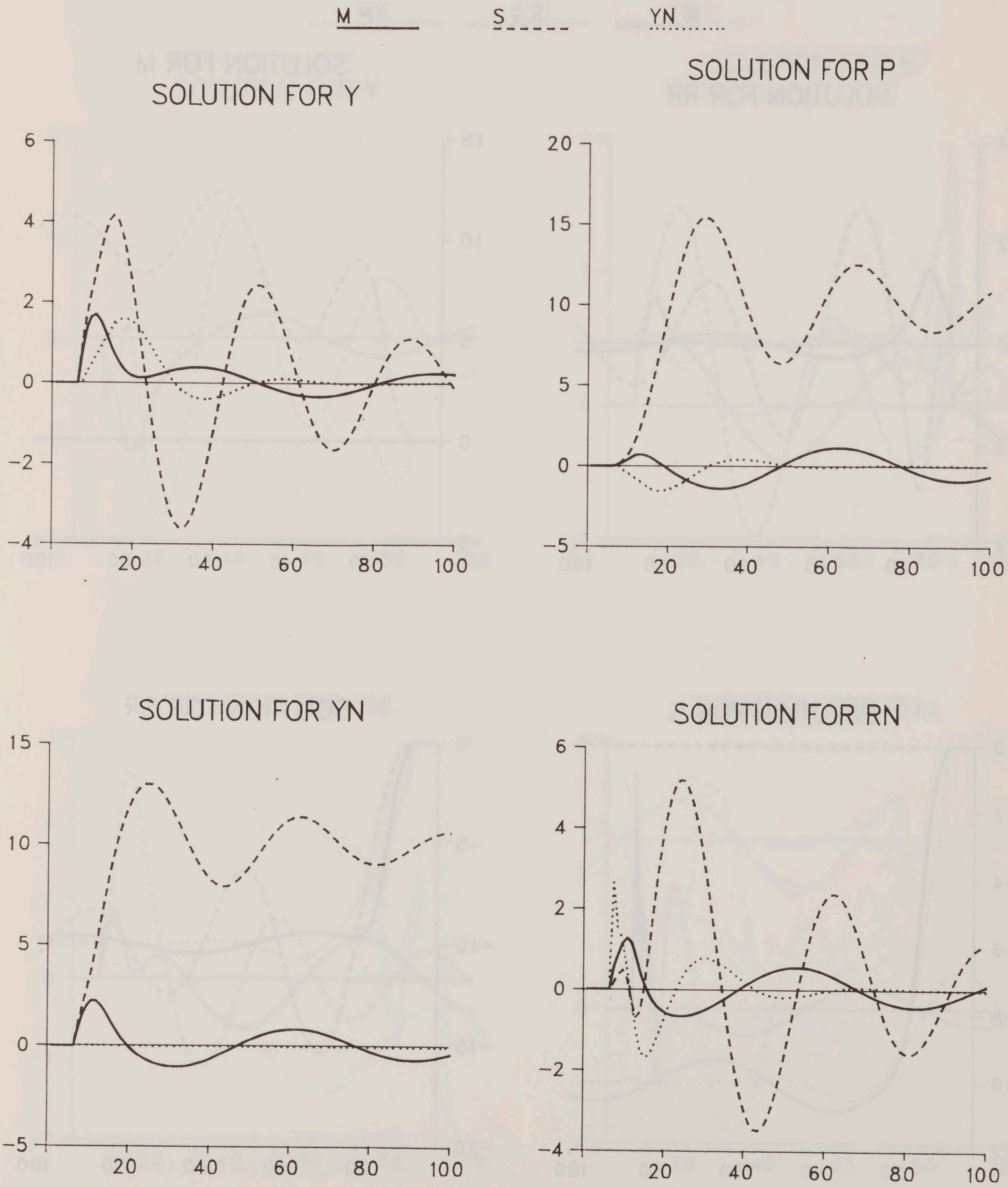
Figures 5-8

TRANSITORY UY SHOCK



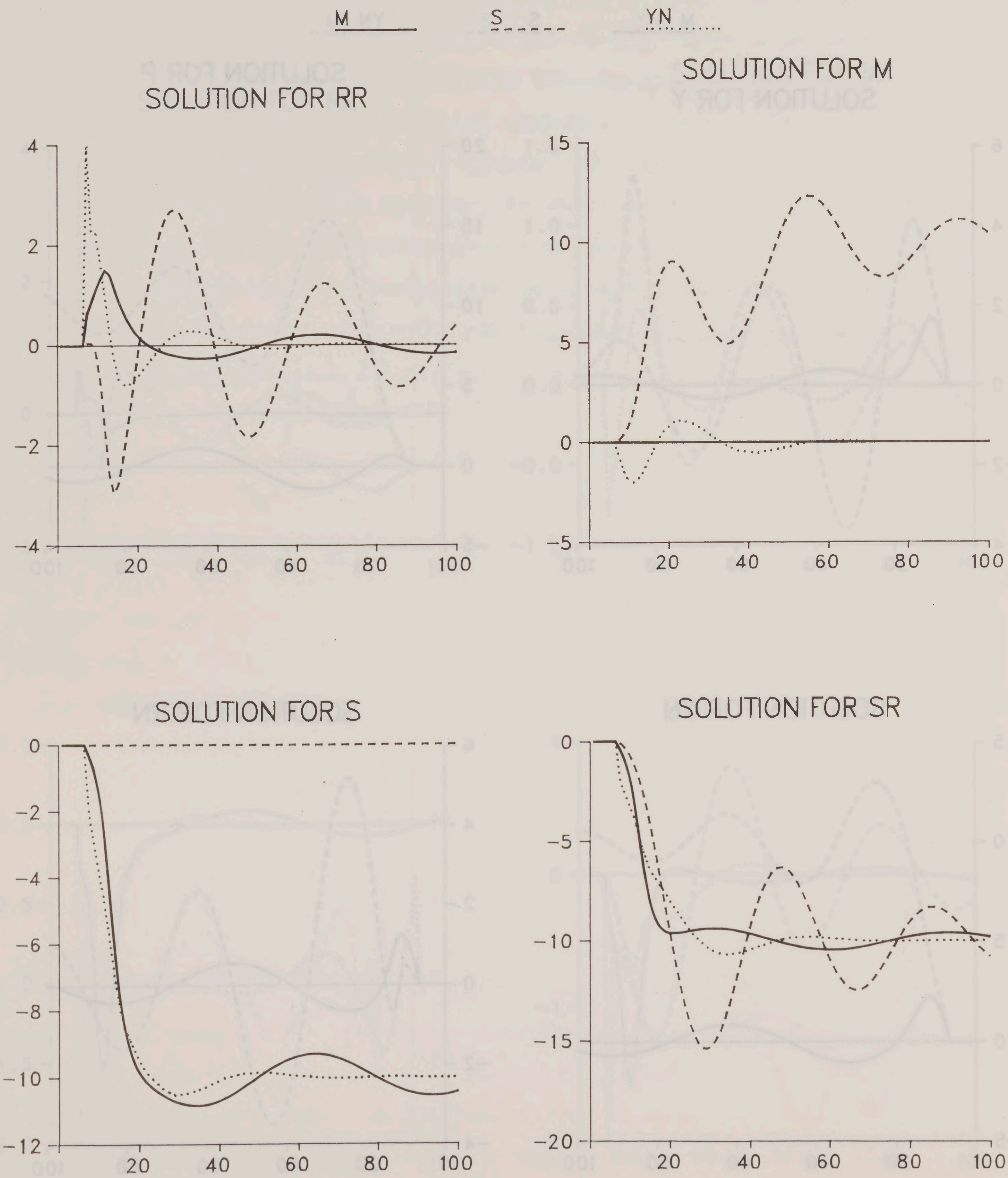
Figures 9-12

PERMANENT UY SHOCK



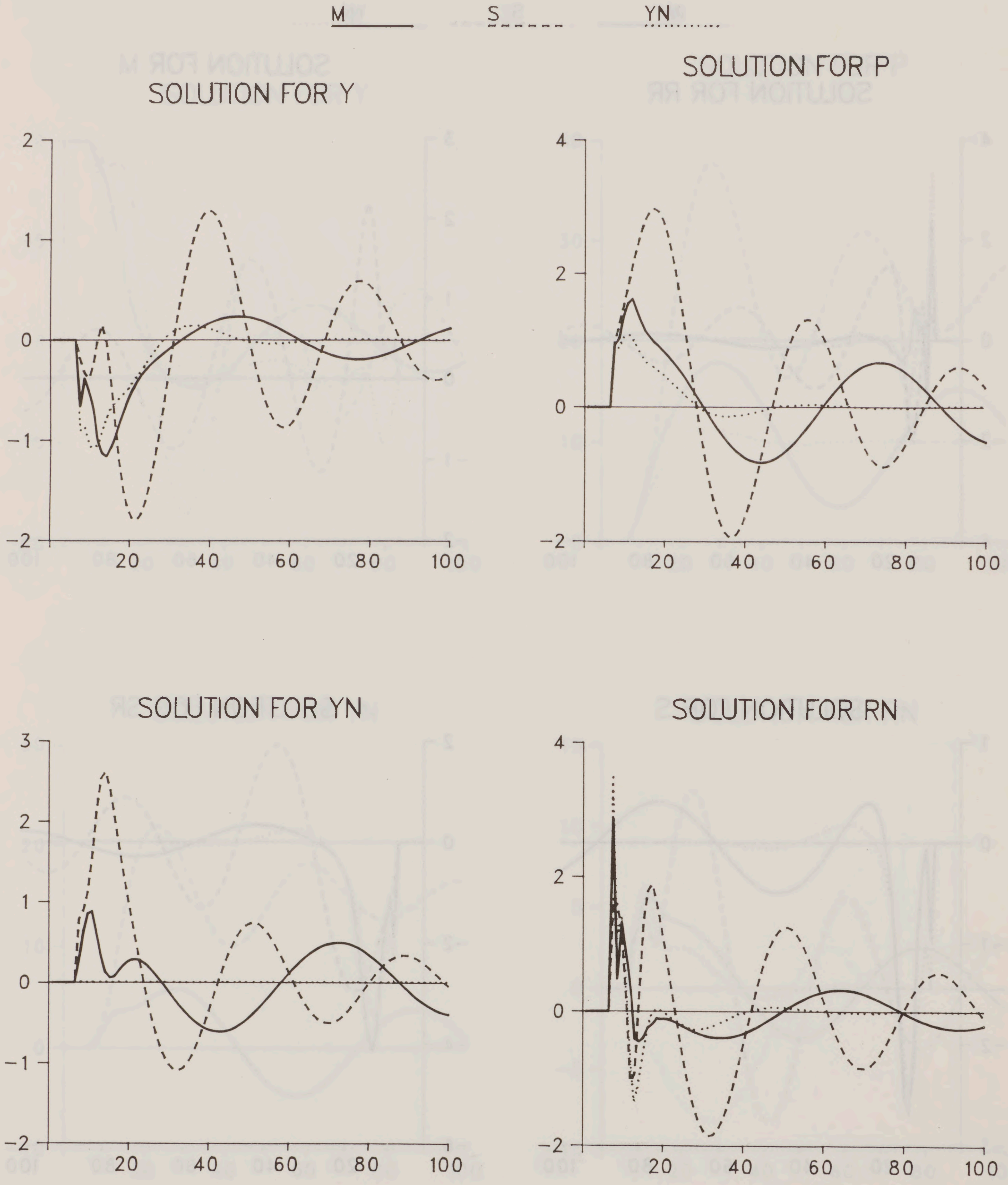
Figures 13-16

PERMANENT UY SHOCK



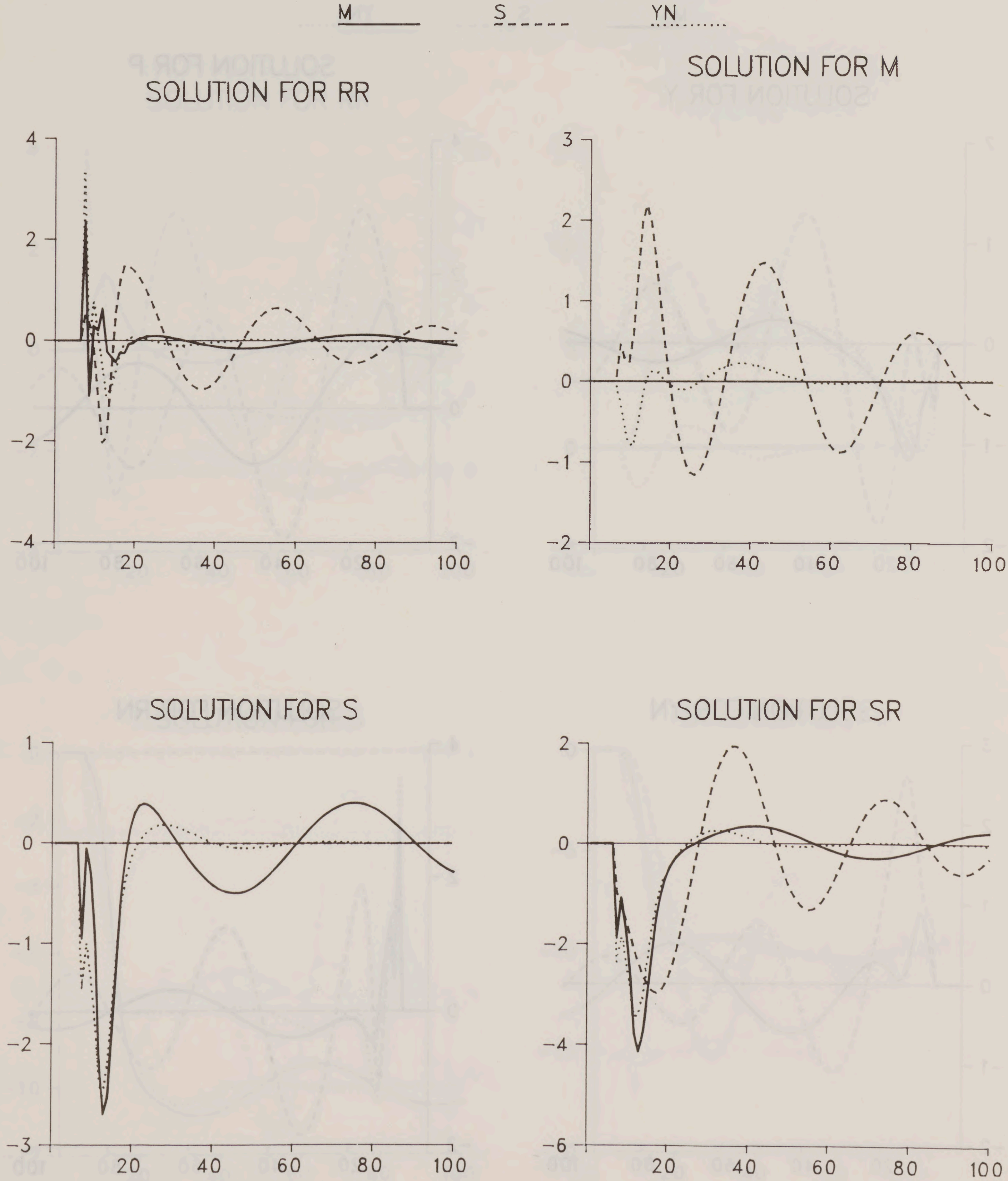
Figures 17-20

TRANSITORY UP SHOCK



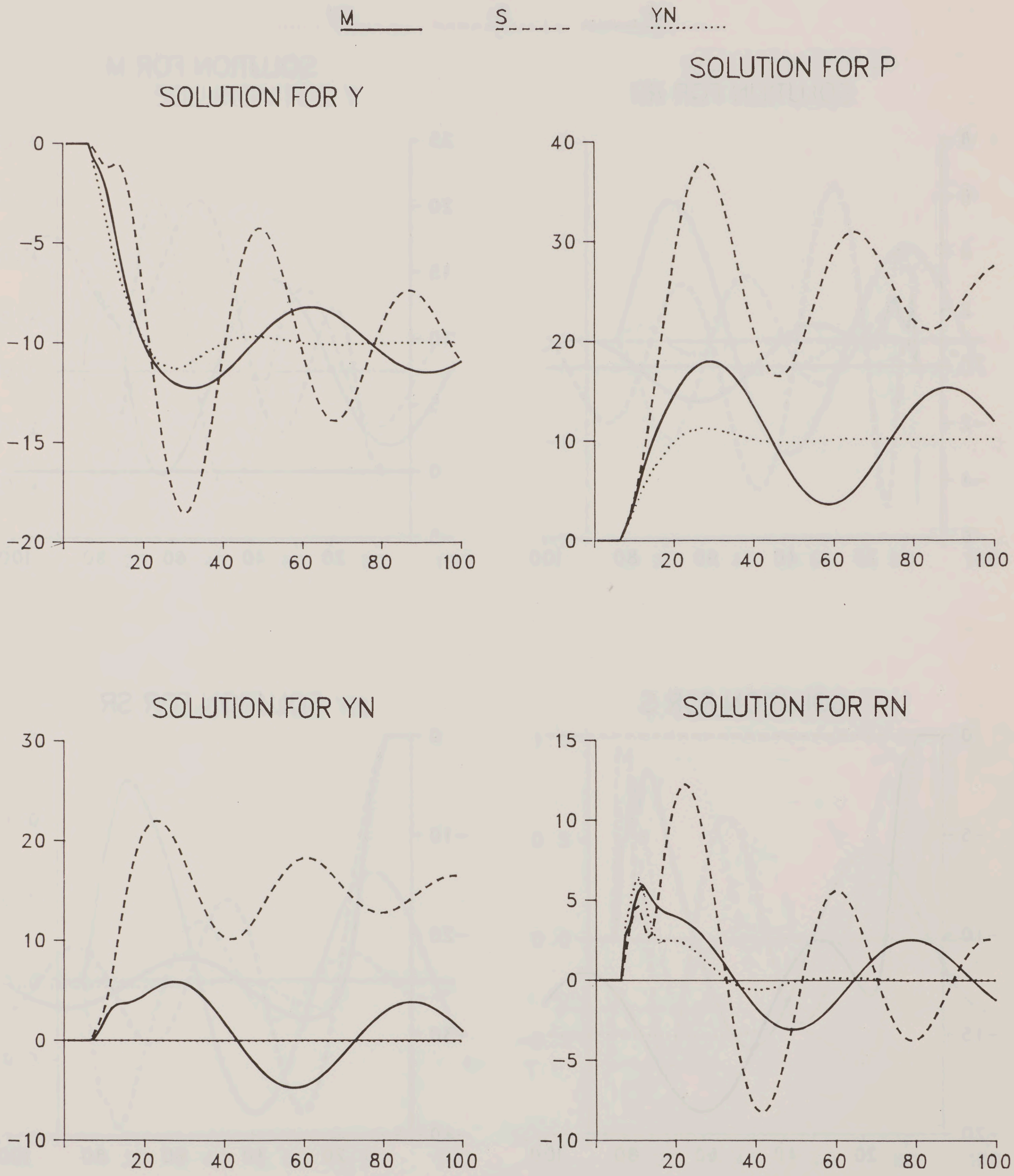
Figures 21-24

TRANSITORY UP SHOCK



Figures 25-28

PERMANENT UP SHOCK

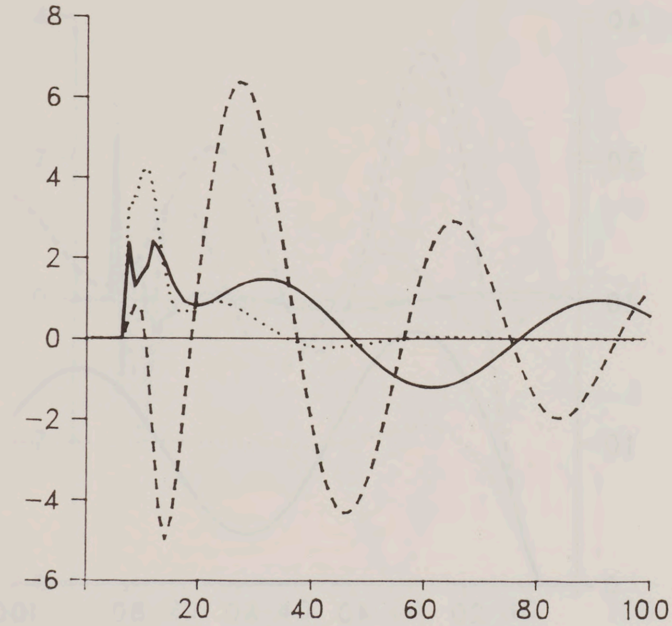


Figures 29-32

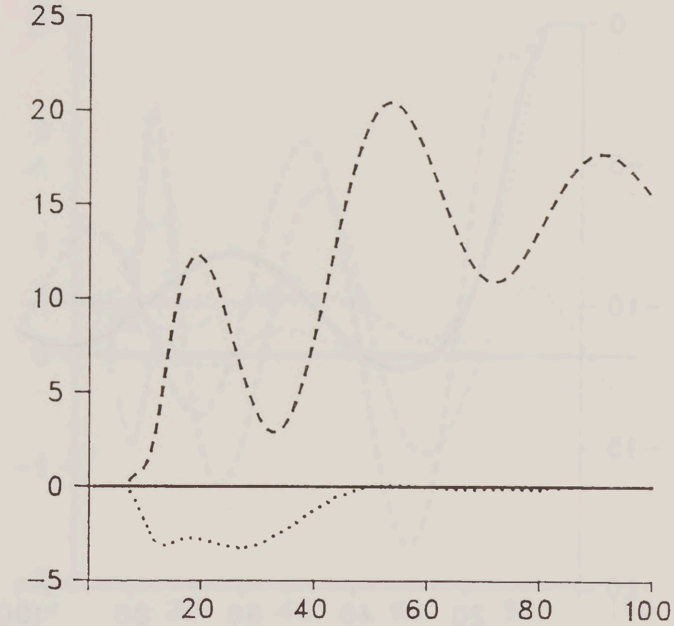
PERMANENT UP SHOCK

M ——— S - - - - - YN ....

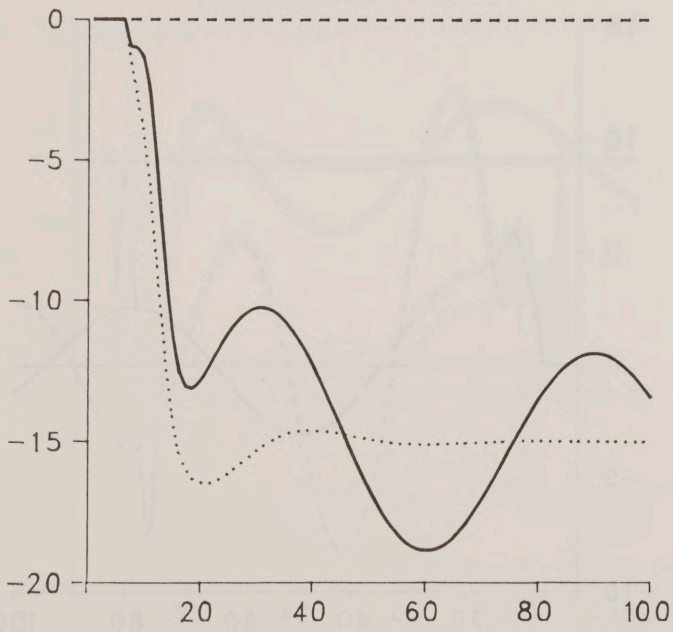
SOLUTION FOR RR



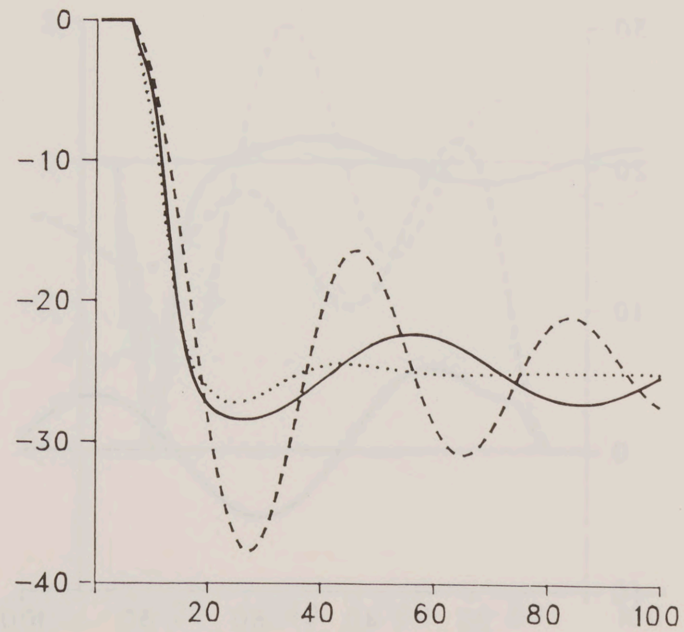
SOLUTION FOR M



SOLUTION FOR S

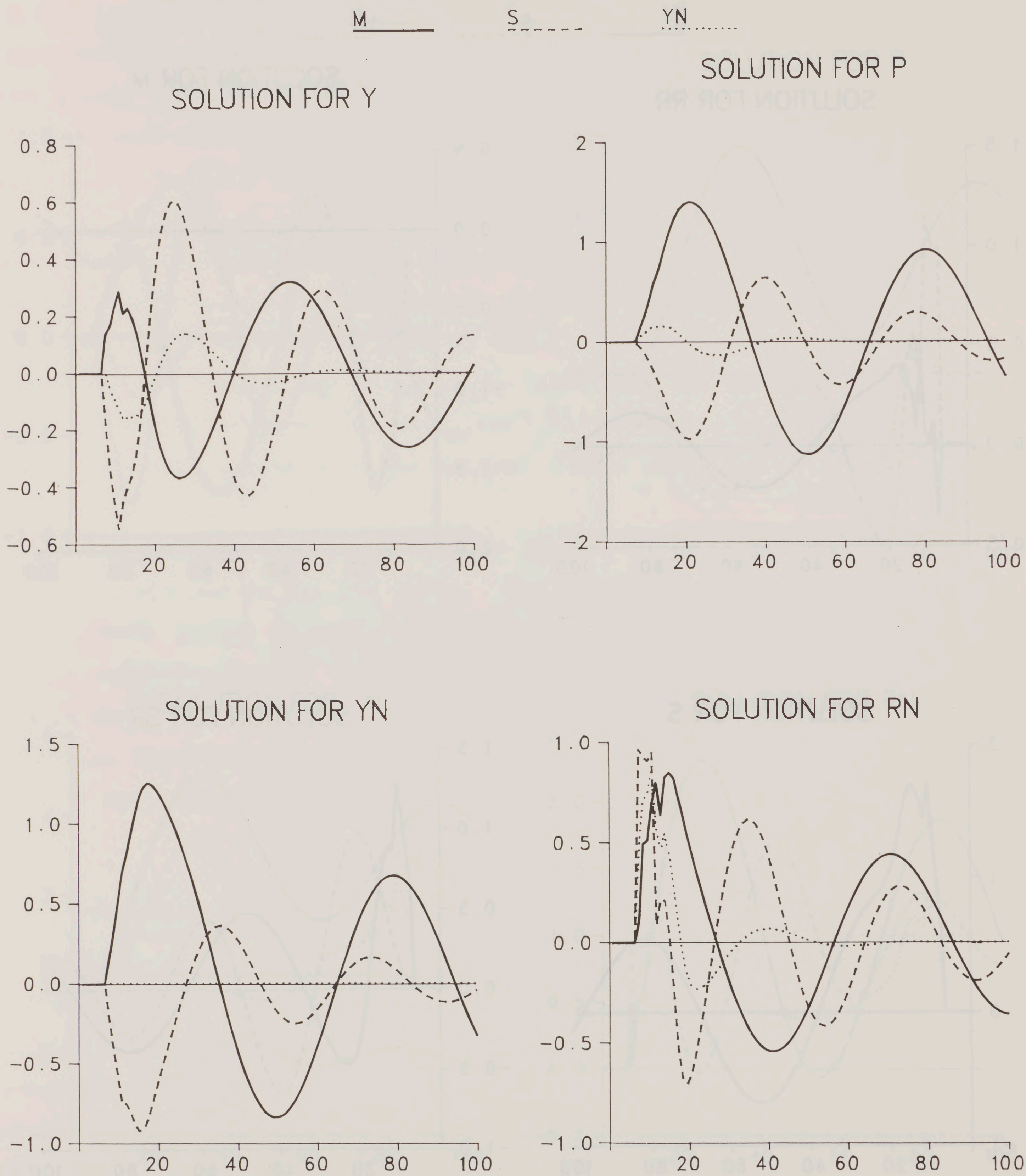


SOLUTION FOR SR



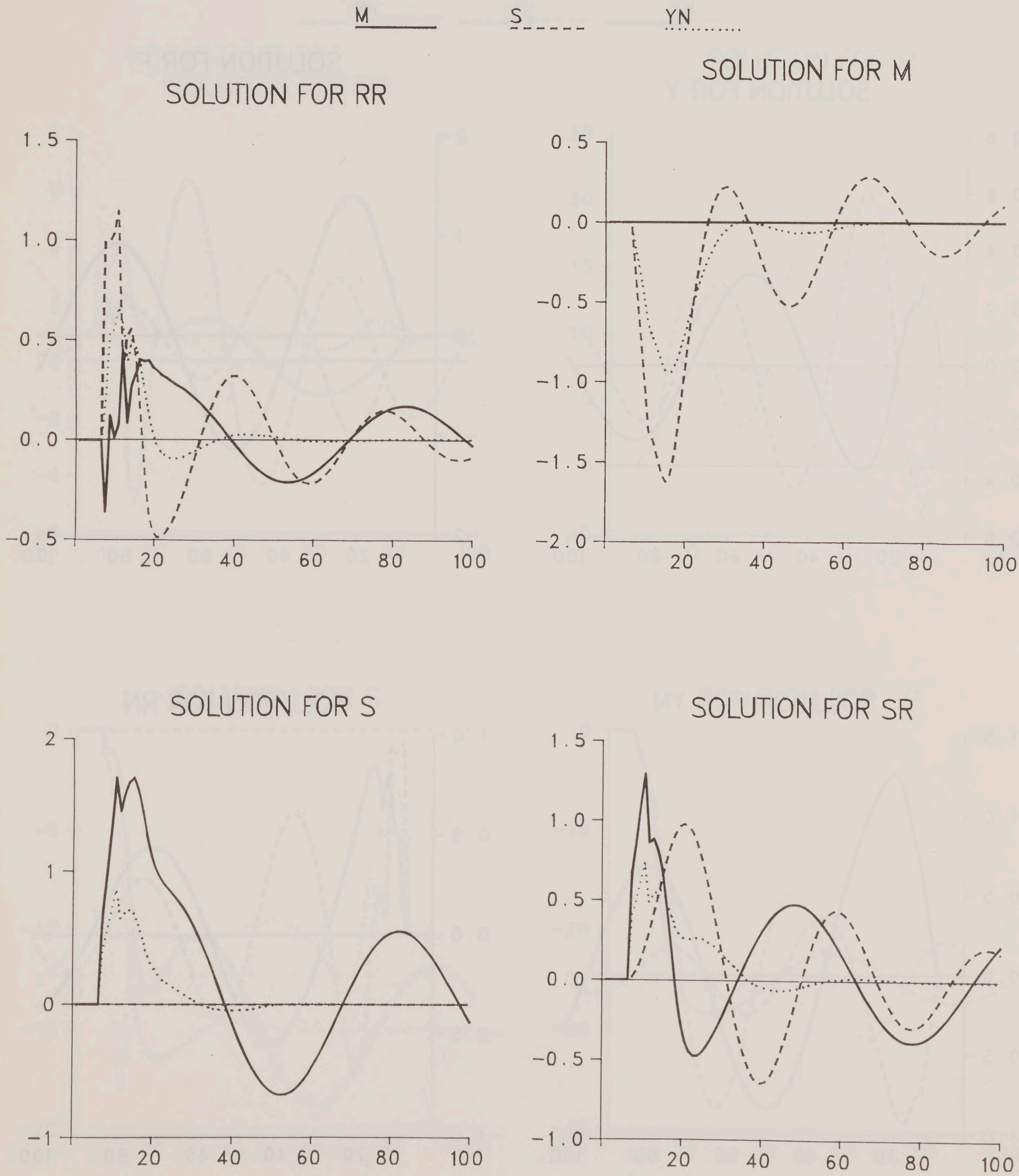
Figures 33-36

TEMPORARY RUS SHOCK



Figures 37-40

TEMPORARY RUS SHOCK

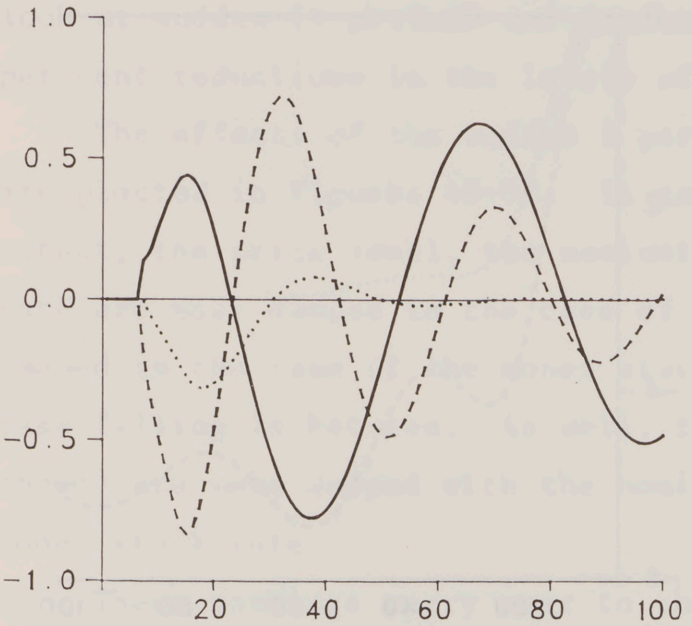


Figures 41-44

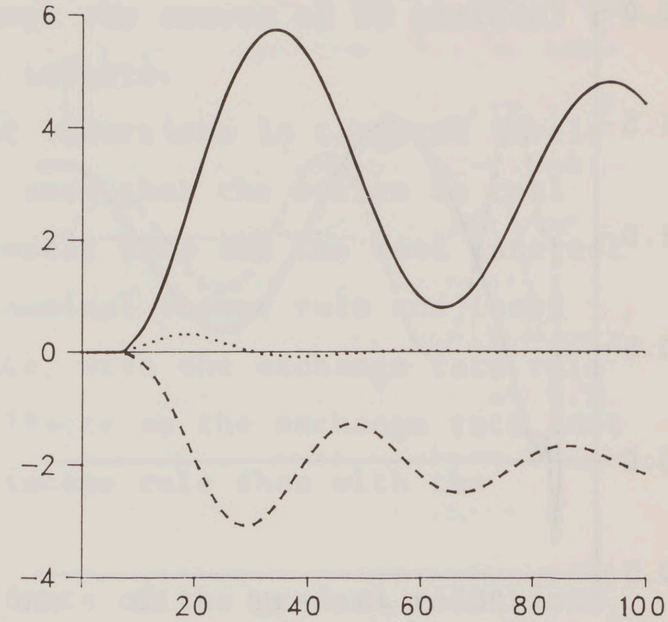
PERMANENT RUS SHOCK

M ——— S - - - - - - YN ....

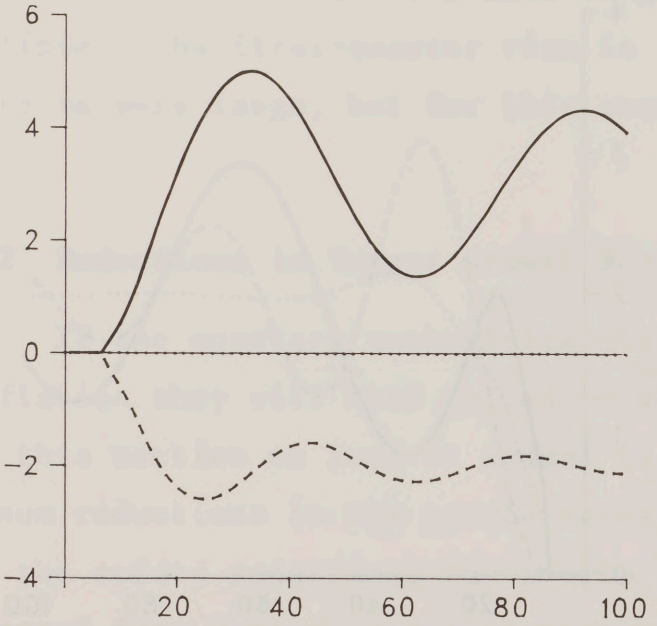
SOLUTION FOR Y



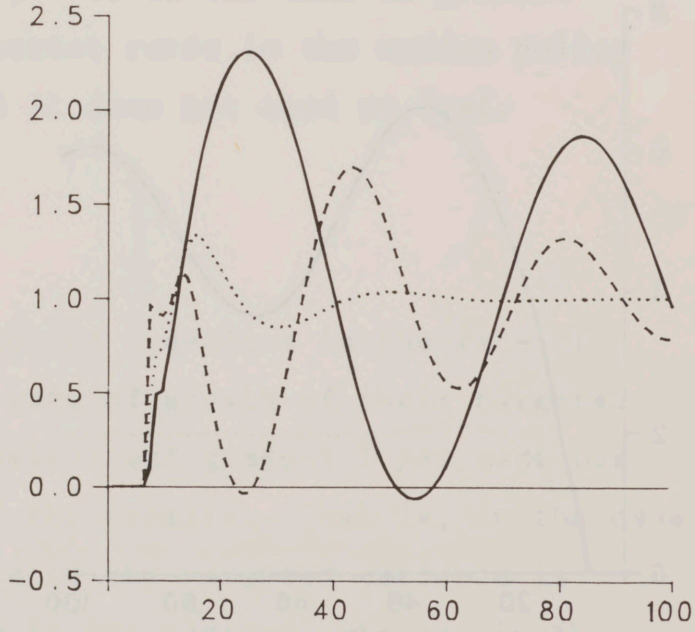
SOLUTION FOR P



SOLUTION FOR YN

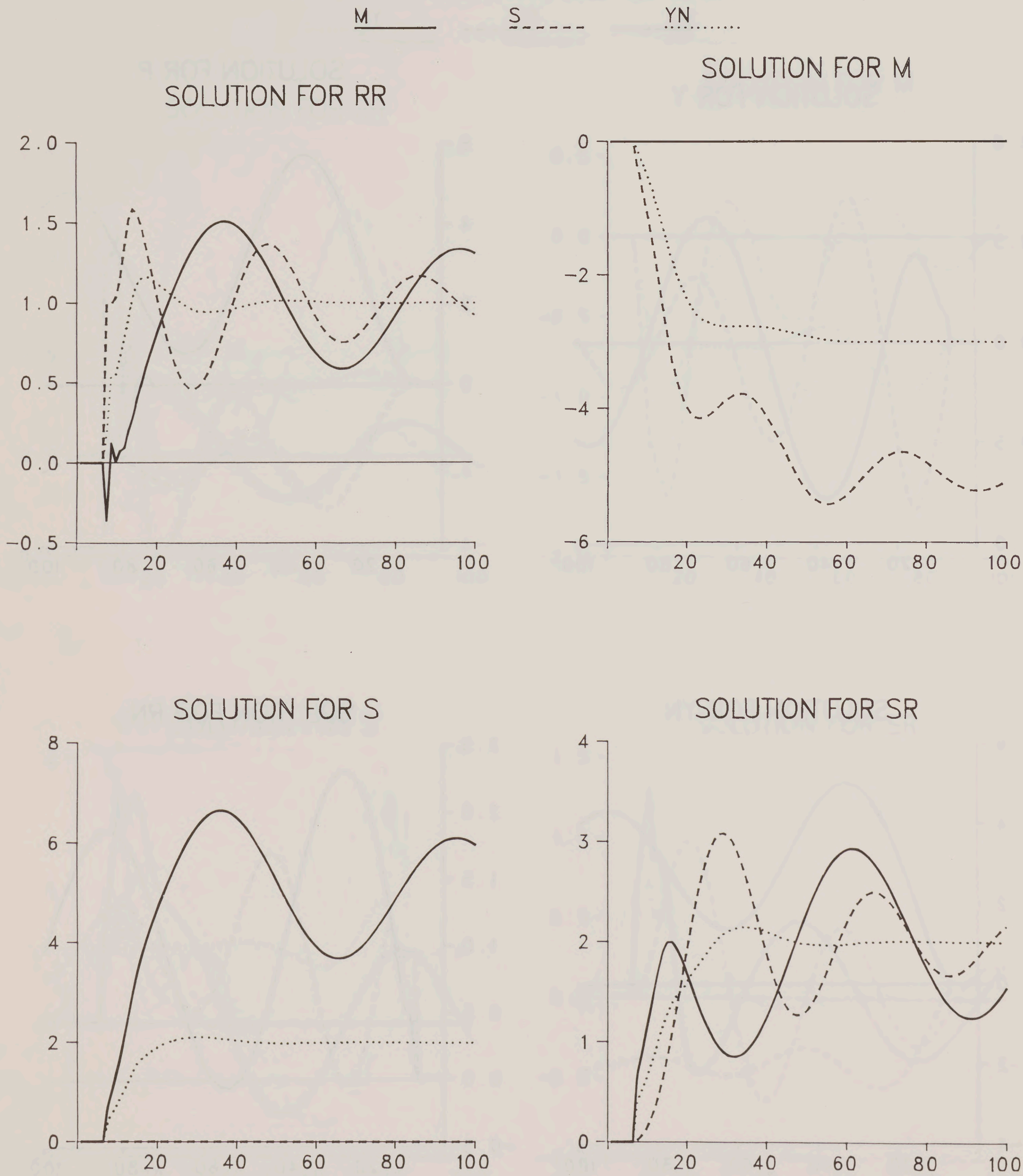


SOLUTION FOR RN



Figures 45-48

PERMANENT RUS SHOCK



5 REDUCTIONS IN TARGET LEVELS OR TARGET GROWTH RATES

5.1 Reductions in Target Levels

An examination of the dynamic adjustment of the economy to reductions in the levels of the three targeted variables is of some interest as an intermediate step towards an examination of the effects of reductions in the growth rates of targeted variables. In this section, therefore, we look at sudden (1 period) and gradual (over the course of 20 periods) 1 per cent reductions in the levels of the targets.

The effects of the sudden 1 per cent reductions in targeted levels are plotted in Figures 49-52. It can be seen that the cycles in real output, the price level, the nominal interest rate and the real interest rate are most damped in the case of the nominal income rule and least damped in the case of the money stock rule, with the exchange rate rule case falling in between. As well, the effects on the exchange rate (not shown) are more damped with the nominal income rule than with the money stock rule.

These rankings carry over to the effects of the gradual reductions, which are plotted in Figures 53-56. The rise in the nominal interest rate in the first period in the gradual reduction cases is 1/20 of that in the sudden reduction cases. Moreover, the nominal and real interest rates remain above control for a much longer period in the case of gradual policies; the first-quarter rise in interest rates in the sudden policy case is very large, but for this reason it does not need to last.

5.2 Reductions in Target Growth Rates

If the monetary authorities wish to see a decline in the rate of inflation they will need to reduce the rate of growth of their targets. In this section we look at sudden (1 period) and gradual 1 per cent per annum reductions in the growth rates of the targets. That is, in the case of the sudden reduction, the growth rate of the targeted variable is changed from 0 to -1 per cent per annum in one quarter, whereas in the second shock the growth rate adjusts from 0 to -1 per cent per annum gradually, over a period of 20 quarters. Homogeneity of degree one in the growth rates of nominal variables implies that inflation converges to -1 per cent per annum in all the simulations.[12]

The effects of sudden reductions in the growth rates of targeted variables are plotted in Figures 57-60. As in the case of the target-level shock, the paths of the economic variables of primary interest are least cyclical with the nominal income rule and most cyclical with the money stock rule. Initially, interest rates rise by less in the case of the exchange rate rule than in the case of the other two rules. This results in a smaller initial loss in real output. However, in subsequent quarters the output loss eventually becomes greater than under a nominal income rule.

As with the level shocks, changing the shocks from sudden to gradual (as defined in Section 5.1) has no effect on the ranking of the cyclical effects of the three policies (see Figures 61-64). Nominal income targets again produce less cyclical effects than the other two targets.

Conditional on our choice of model and its parameters, these two sets of results taken together, provide further evidence of the potential usefulness of nominal income as the guiding variable when attempting to reduce inflation. It is worth noting that this same result has been demonstrated both analytically and within the context of a small dynamic simulation model in Masson (1983).

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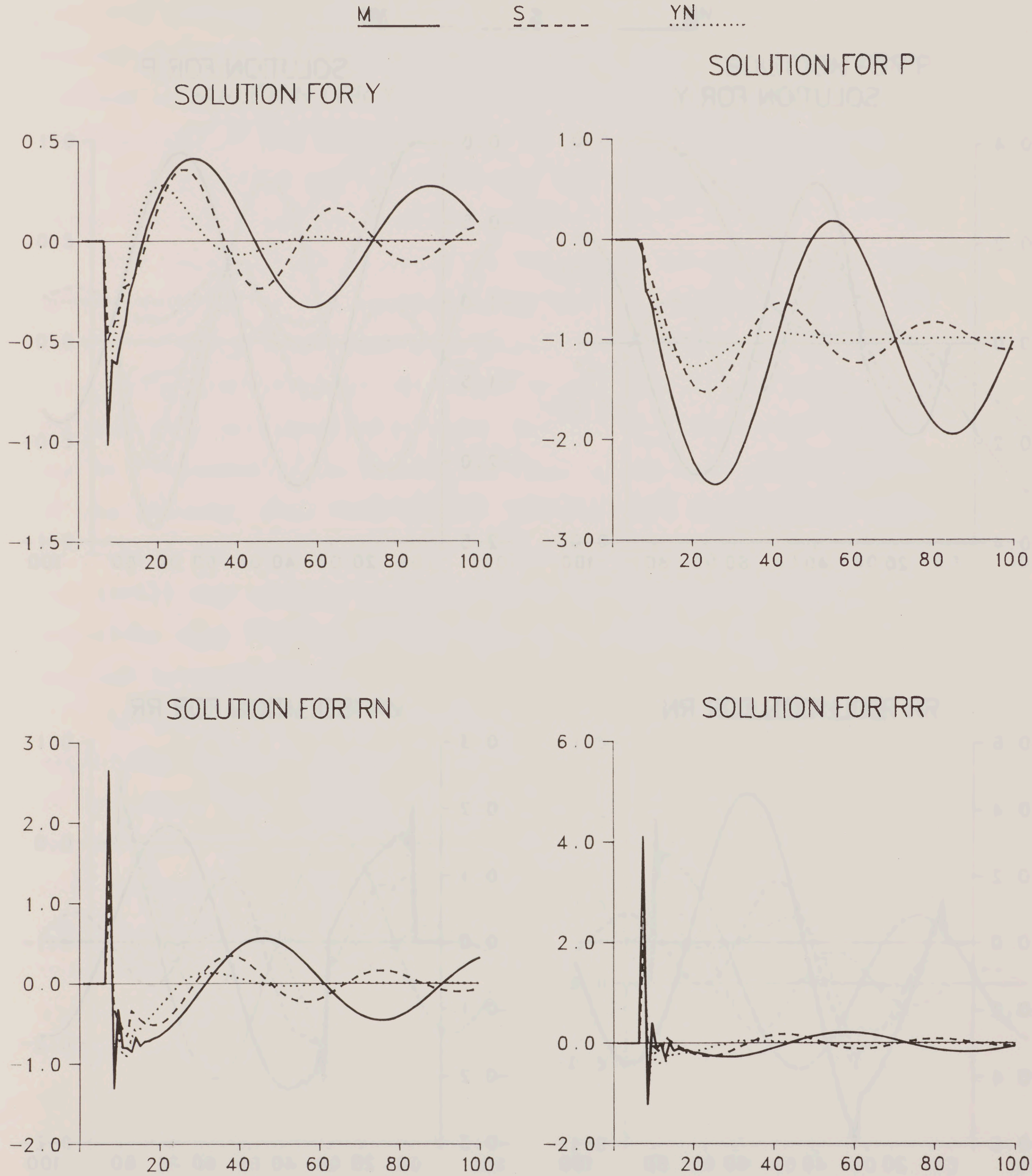
12. The level of prices, however, will fall by more in the case of the money stock rule because the required fall in nominal interest rates in equilibrium leads to an increase in the demand for real balances which, with the money stock held fixed, can only be satisfied by a decline in the price level.

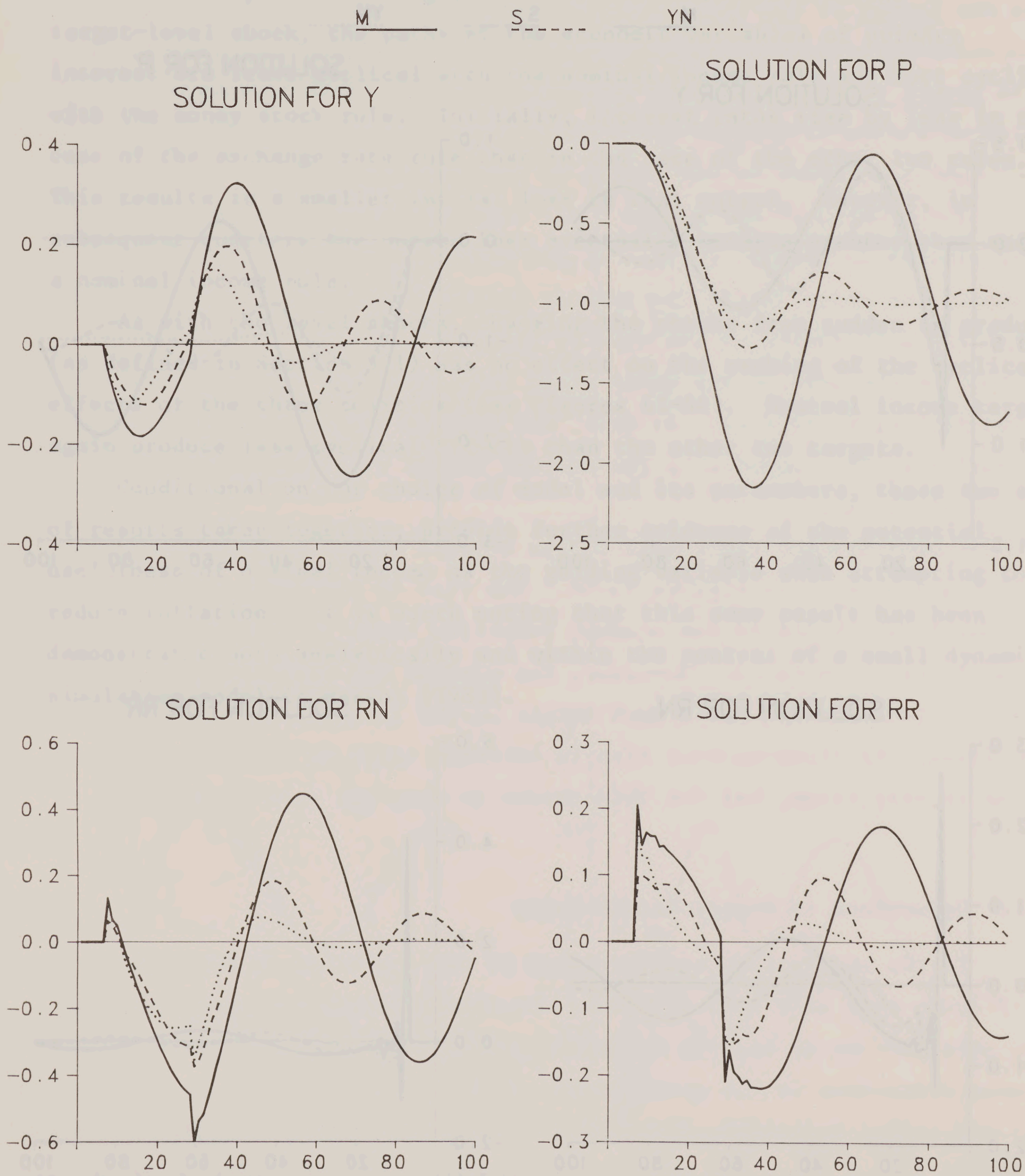
Figures 49-52

SUDDEN LEVEL REDUCTION



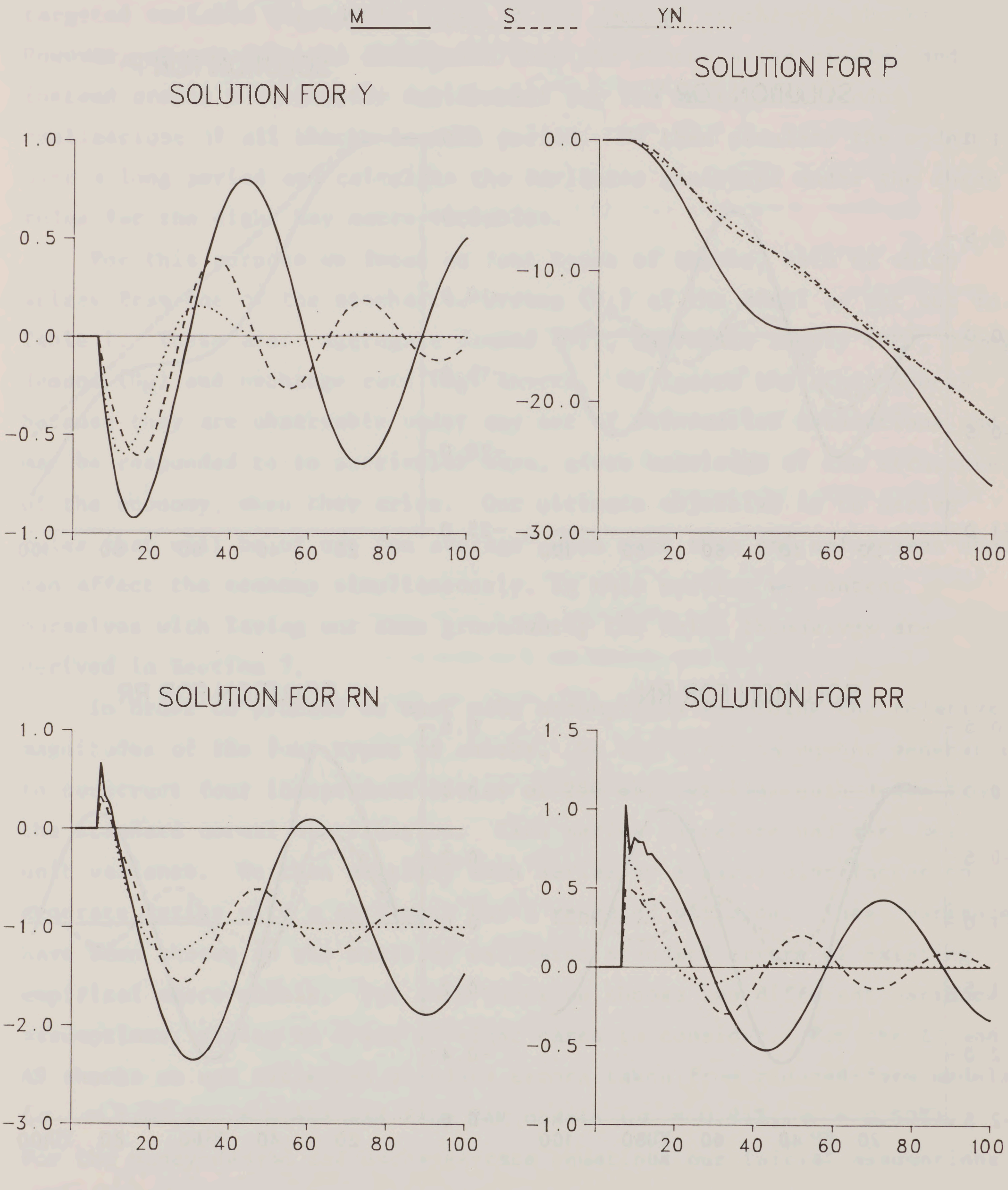
Figures 53-56

GRADUAL LEVEL REDUCTION



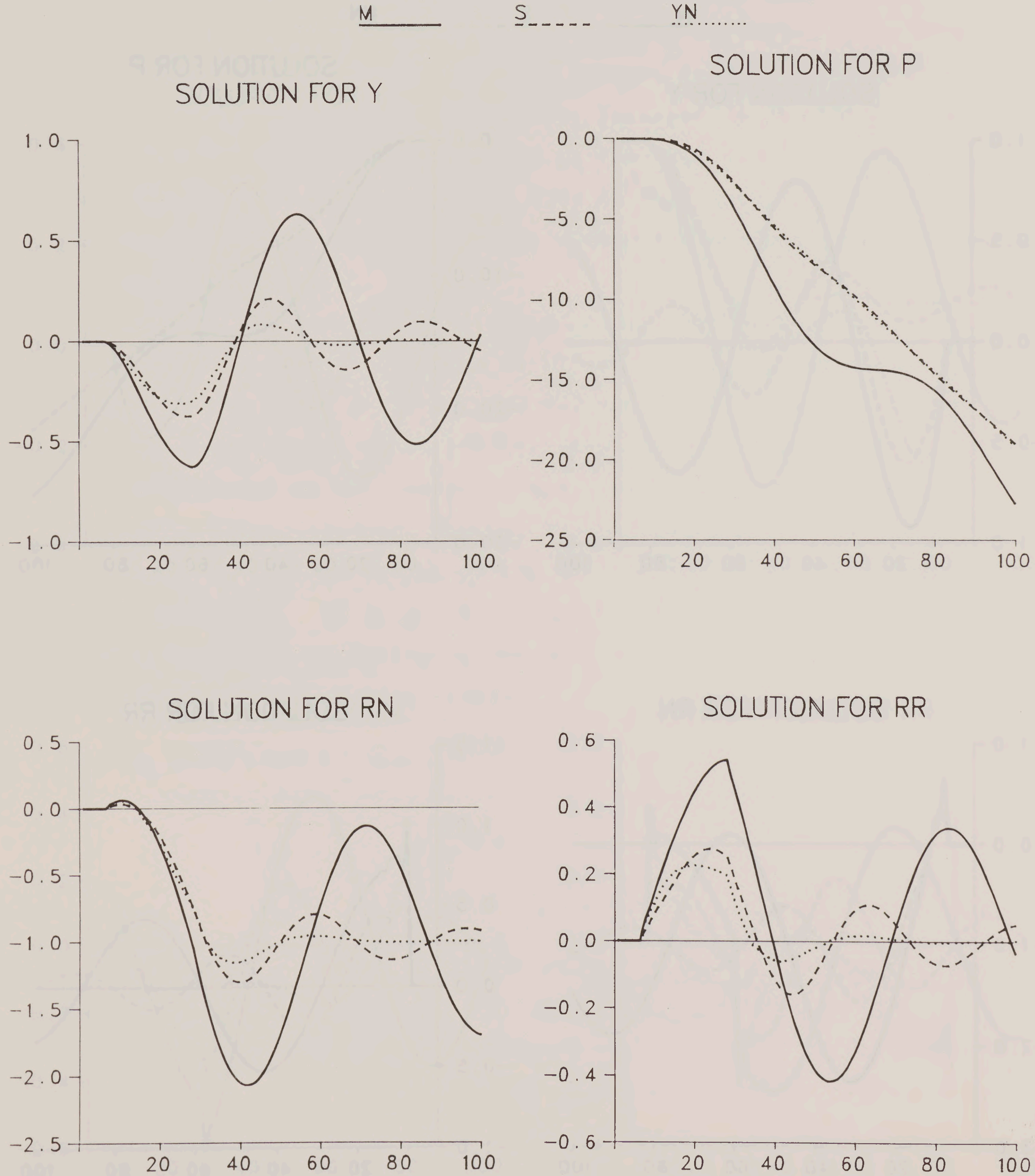
Figures 57-60

SUDDEN GROWTH RATE REDUCTION



Figures 61-64

GRADUAL GROWTH RATE REDUCTION



6 ARRAYS OF SHOCKS UNDER FULL INFORMATION

In this section we once again retain the assumption that the monetary authorities have full information and, as a consequence, can keep the targeted variable absolutely fixed in the face of stochastic shocks. However, we now drop the assumption that the shocks arise singly, and instead create a stochastic environment for the model with random realizations of all shocks in each period. We then simulate the model over a long period and calculate the variances generated under the three rules for the eight key macro variables.

For this purpose we focus on four types of shocks, each of which arises from one of the stochastic errors (U_i) of the model as set out in Table 1. These are: aggregate demand (UY), aggregate supply (UP), money demand (U_M) and exchange rate (U_S) shocks. We ignore the other shocks because they are observable under any set of information assumptions and may be responded to in particular ways, given knowledge of the structure of the economy, when they arise. Our ultimate objective is to derive rules that will be of use "on average" when more than one unforeseen shock can affect the economy simultaneously. In this section we content ourselves with laying out some groundwork; the rules themselves are derived in Section 7.

In order to proceed we must make assumptions regarding the relative magnitudes of the four types of shocks. We use a random number generator to construct four independent series of 250 observations each drawn from the standard normal distribution. Each series therefore has zero mean and unit variance. We then multiply each series by a particular factor to generate series with a zero mean but a rescaled variance. These variances have been chosen on the basis of estimated standard errors of existing empirical macro-models. For each shock we choose two different variance assumptions, giving us a set of eight cases to consider. For the IS and AS shocks we use estimated standard errors taken from reduced-form models ($\sigma_{UY} = 1.0\%, \sigma_{UP} = 0.4\%$) and from VAR models ($\sigma_{UY} = 0.85\%$, $\sigma_{UP} = 0.60\%$). For the money-demand and exchange-rate equations our initial assumptions are taken from approximate standard errors of quarterly equations estimated over the 1970s ($\sigma_{UM} = \sigma_{US} = 0.80\%$), but we also examine the implications of an increase in variance due to instability or volatility in which we assume $\sigma_{UM} = 2.0\%$ and/or $\sigma_{US} = 1.5\%$. The combinations of assumptions we make are shown in Table 6.

Throughout the analysis, we use the same four generated N(0,1) variables for the four shocks. A large number of drawings were made in order that the four generated series be within ± 5 per cent tolerance ranges in terms of both mean and variance. The summary statistics for the four generated series are presented in Table 7.

We then simulated the model with all four shocks impacting on the system simultaneously over the sequence of 250 periods. After solving the model under the three rules for each of the eight combinations of shocks we calculated the simulated standard deviations over the 250 observations of the eight key macro variables. These data are given in Table 8.

The results are consistent with our earlier work in that the variances of output and prices are lower for the nominal income rule than for the other two rules, but we find that the M- and S-rules favour output relative to prices in the sense that the standard deviation of output is lower than that of the price level. The nominal income rule also yields a lower variance for the exchange rate than does the money stock rule.

The ranking of the rules on the basis of the variance of nominal or real interest rates depends very much on the case considered. When the variance of money demand (σ^2_{UM}) is high (Cases 2, 4, 6 and 8), the nominal income rule tends to have the lowest variance of the three rules for real and nominal interest rates -- the only exception being that the exchange rate rule has a lower variance for the real interest rate in Case 2. When the variance of money demand is low, the money stock rule has the lowest variance for interest rates, followed by the nominal income rule, and the exchange rate rule -- with the exception of Case 1 in which the exchange rate rule has a lower variance than the nominal income rule for the real interest rate.

We can gain a better impression of the trade-offs between the different types of variance by examining Figures 65-69. There we have repeated the above exercise for a number of policy rules that lie between the pure M-rule and the pure S-rule, between the pure M-rule and the pure

Table 6

EIGHT ASSUMED SHOCK ARRAYS (Standard error, %)

Case	U_Y	U_P	U_M	U_S
1	1.00	0.40	0.80	0.80
2	1.00	0.40	2.00	0.80
3	1.00	0.40	0.80	1.50
4	1.00	0.40	2.00	1.50
5	0.85	0.60	0.80	0.80
6	0.85	0.60	2.00	0.80
7	0.85	0.60	0.80	1.50
8	0.85	0.60	2.00	1.50

Table 7

SUMMARY STATISTICS ON FOUR RANDOM SERIES DRAWN FROM N(0,1) DISTRIBUTION (250 observations)

		X_1	X_2	X_3	X_4
Sample mean		-0.043947	-0.045316	-0.016362	0.024689
Sample variance		0.978954	1.049540	0.992426	0.979996
Correlation matrix	X_1	1.0			
	X_2	0.059 (0.93)	1.0		
	X_3	-0.046 (0.73)	-0.090 (1.42)	1.0	
	X_4	0.029 (0.46)	-0.027 (0.43)	0.030 (0.47)	1.0

Note:

1. Random series were generated from NORMAL function in TROLL. A large number of series were tried before we obtained four series for which $-0.05 \leq$ mean ≤ 0.05 and $0.95 \leq$ variance ≤ 1.05.
2. Figures in parentheses are t-statistics calculated from the formula $t = r(n-2)^{\frac{1}{2}}/(1-r^2)^{\frac{1}{2}}$ and must exceed 1.96 to be significant at the 0.95 level.

Table 8

SIMULATED STANDARD DEVIATIONS UNDER ALTERNATIVE SHOCK ASSUMPTIONS AND FULL INFORMATION (250 observations, %)

Exogenous shocks	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
U_Y	0.989	0.989	0.989	0.989	0.841	0.841	0.841	0.841
U_P	0.410	0.410	0.410	0.410	0.615	0.615	0.615	0.615
U_M	0.797	1.992	0.797	1.992	0.797	1.992	0.797	1.992
U_S	0.792	0.792	1.485	1.485	0.792	0.792	1.485	1.485
M-Rule								
Y	2.024	4.008	2.138	4.114	2.227	3.932	2.325	4.036
P	4.145	11.432	4.550	11.784	3.925	10.681	4.291	11.033
YN	3.815	9.655	4.143	9.930	3.356	9.048	3.678	9.323
RN	3.680	8.440	3.974	8.671	3.882	8.358	4.137	8.579
RR	4.199	9.669	4.546	9.854	4.355	9.714	4.690	9.898
M	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
S	4.964	8.816	5.513	9.199	5.337	8.921	5.820	9.280
SR	5.124	7.571	5.632	7.937	6.041	7.929	6.468	8.271
S-Rule								
Y	5.128	5.128	5.121	5.121	5.686	5.686	5.669	5.669
P	7.391	7.391	7.232	7.232	8.260	8.260	8.093	8.093
YN	5.372	5.372	5.378	5.378	5.877	5.877	5.877	5.877
RN	5.992	5.992	6.517	6.517	6.800	6.800	7.224	7.224
RR	4.505	4.505	5.250	5.250	5.227	5.227	5.859	5.859
M	5.737	6.404	5.933	6.601	6.510	7.098	6.689	7.281
S	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
SR	7.391	7.391	7.232	7.232	8.260	8.260	8.093	8.093
YN-Rule								
Y	1.396	1.396	1.408	1.408	1.814	1.814	1.820	1.820
P	1.396	1.396	1.408	1.408	1.814	1.814	1.820	1.820
YN	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
RN	3.945	3.945	4.218	4.218	4.130	4.130	4.354	4.354
RR	4.857	4.857	4.986	4.986	4.655	4.655	4.772	4.772
M	2.112	4.058	2.299	4.181	2.052	3.996	2.239	4.119
S	3.903	3.903	4.105	4.105	4.282	4.282	4.458	4.458
SR	4.303	4.303	4.468	4.468	5.347	5.347	5.476	5.476

YN-rule, and between the pure S-rule and the pure YN-rule. For example, in the case of the conditional money supply/exchange rate rule, the money supply is forced below (above) target whenever the exchange rate depreciates (appreciates). In the case of the conditional money supply/nominal income rule, money becomes countercyclical, rising whenever nominal income falls below target and falling whenever nominal income rises above target. Finally, the third set of rules engineers an exchange rate appreciation (depreciation) whenever nominal income rises above (falls below) target. Algebraically, the three classes of conditional rules are given as follows:

M conditional on S: $(M - \bar{M}) = -\beta(S - \bar{S})$ (i)

M conditional on YN: $(M - \bar{M}) = -\beta(YN - \overline{YN})$ (ii)

S conditional on YN: $(S - \bar{S}) = -\beta(YN - \overline{YN})$ (iii)

In each case, a bar over a variable denotes the target level, which for our purposes always takes the value zero. By changing the weights on the conditioning variable in each case, and resimulating the Case 1 array of 250 shocks, we generate three continua of rules and, therefore, of possible variance combinations. These have been plotted for several pairs of variables in Figures 65-69, in each case using only the Case 1 array of shocks.

Figure 65 illustrates the trade-off curves between variability in prices and output, as generated by the model. This trade-off is positively sloped, indicating that movement from the S-rule to the M-rule and then towards the YN-rule, or directly from the S-rule to the YN-rule, yields a fairly continuous decline in both variances. The costs of this change in policy in terms of nominal interest rate variability are illustrated in Figure 66. There it is clear that the pure S-rule is inefficient, at least in terms of these variables, since both variances could be reduced through use of a conditional S/M or S/YN rule, but the remaining combinations of output and interest rate variability are all in a fairly close neighbourhood. It does seem likely from this, however,

Figure 65

Trade-Off: Standard Deviation of Output vs. Prices

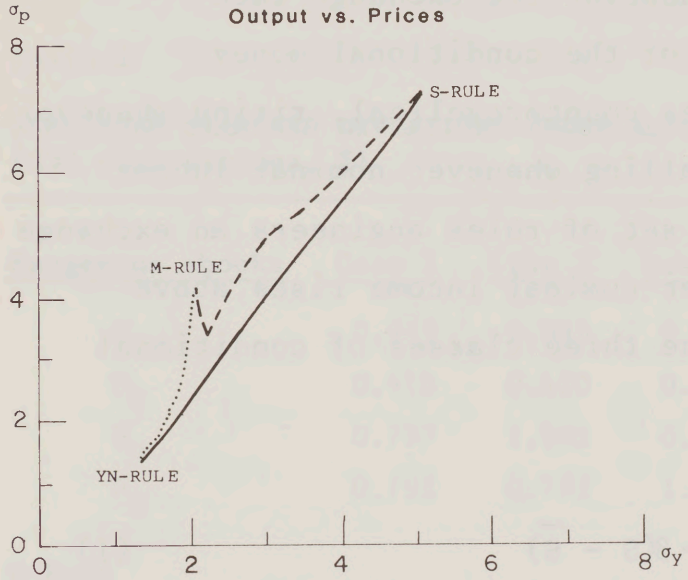


Figure 66

Trade-Off: Standard Deviation of Output vs. Nominal Interest Rate

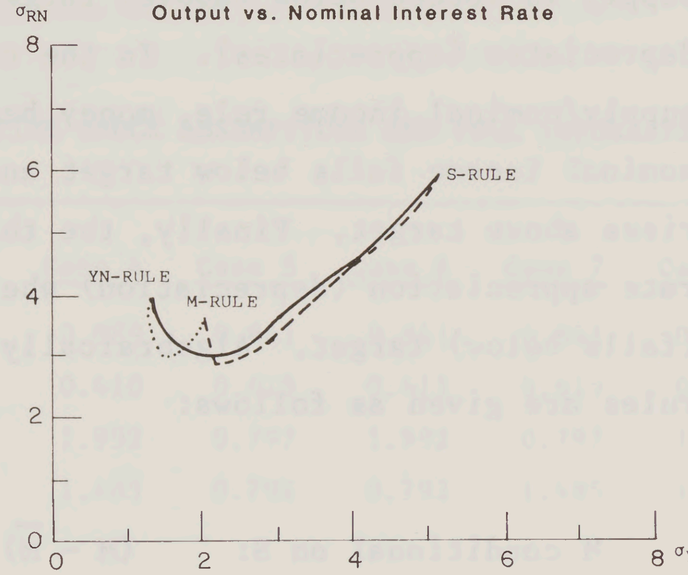


Figure 67

Trade-Off: Standard Deviation of Output vs. Exchange Rate

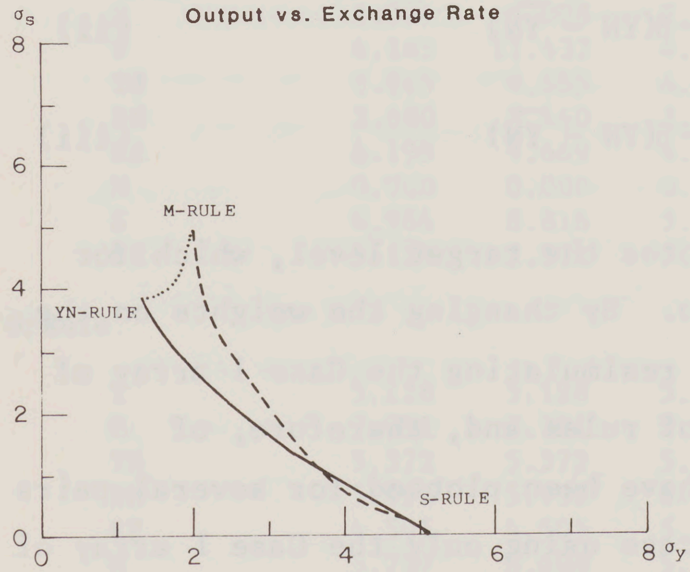


Figure 68

Trade-Off: Standard Deviation of Nominal Interest Rate vs. Exchange Rate

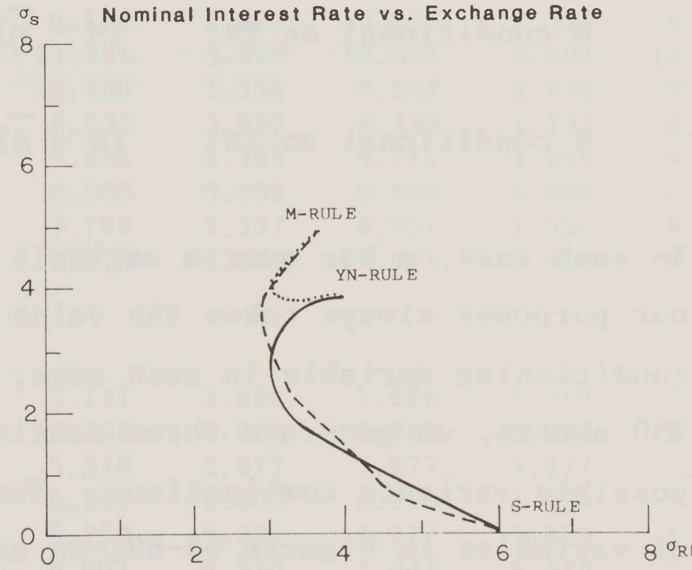
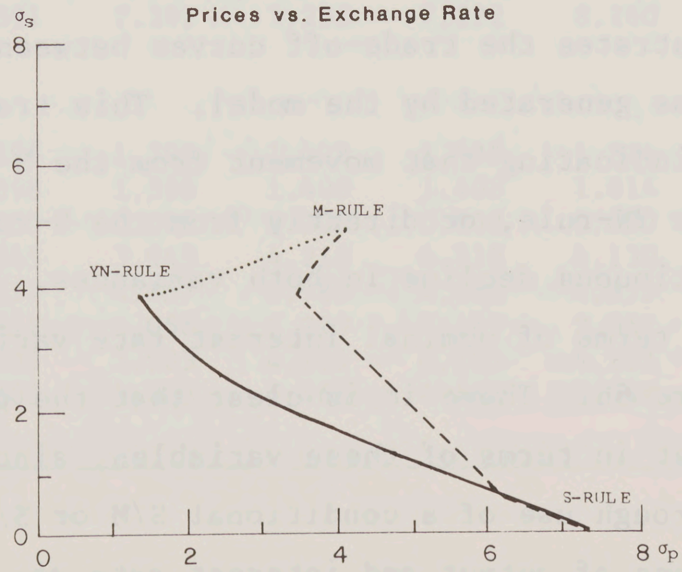


Figure 69

Trade-Off: Standard Deviation of Prices vs. Exchange Rate



that if variability of the interest rate were a matter of concern, no pure rule would be preferred.

The remaining three figures examine the trade-off curves between variability in exchange rates and in output, prices and nominal interest rates. All three are basically negatively sloped, indicating that no clear preference can be expressed without placing utility-related weights on the different types of variance and choosing a preferred point on each locus. However, there is a tendency for the loci between the pure M- and S-rules on the one hand, and between the pure M- and YN-rules on the other, to be dominated by the locus between the pure S- and YN-rules for these other trade-offs. That is, the locus of combination S/YN rules tends to lie closer to the origin, and therefore to suggest lower variances for both variables, in each of the other three comparisons.

7 PARTIAL INFORMATION CASE

7.1 Shock-Recognition and Policy-Response Lags

In this subsection we extend our analysis and comparison of nominal income, money supply and exchange rate targeting strategies by taking account of two types of policy lags. The first is a shock-recognition lag. The assumption that the monetary authority can and does respond contemporaneously to shocks is not an unrealistic assumption in the context of a quarterly model when the shocks in question are foreign interest rate shocks or when the policy is one of targeting the exchange rate, because both foreign interest rates and the exchange rate may be observed and domestic interest rate reactions formulated on a continuous basis. However, for money supply and nominal income rules in the presence of IS and aggregate supply (AS) shocks this assumption is clearly inappropriate, although to different degrees, since money stock data usually are more timely than are national accounts data. Of course, it is possible that the authorities will be able to guess within a quarter that a particular type of shock has occurred because of the movements of variables that are observed on a high-frequency basis. This notion of the gradual receipt of information can, in principle, be modelled, and an attempt to do so is made in Section 7.3. For the present, however, we will assume that no response is possible during the quarter in which the shock occurs. During the next quarter it is assumed that the authorities are able to deduce that a shock has occurred, calculate the appropriate policy response, and to respond completely or partially, as preferred.

The second type of lag considered is a policy-response lag. Since the authorities cannot respond to shocks comtemporaneously, the targeted variable will necessarily deviate from target in the shock period. As in previous work there are no target bands so the objective of the authorities will be to bring the variable in question back to control. However, the speed of this return to control will be allowed to vary between one, two and four quarters. Such a delay might be optimal, for example, when lags between movements in policy instruments and targets can result in problems of instrument instability. Concern that this problem could arise in practice under the M1 targeting policy of 1975-1982 was prompted by the empirical finding of a humped interest rate response pattern in the M1 demand equation.[13] Although this source of instrument instability has been ruled out in our model by the choice of a partial-adjustment formulation of the money-demand equation, there seems to be no obvious reason to rule out entirely the possibility of instrument instability. Thus, our second objective is to examine the implications of a gradual, as opposed to an immediate, return to target after a shock.

For these purposes we return to the analysis of individual shocks. Arrays of simultaneous shocks are examined in this context later in this section. In each simulation, the period in which the shock occurs (in the case of a transitory shock) or begins (in the case of a permanent shock) is characterized by a nominal interest rate rule, with the latter held constant at control. Thus, all other variables, including the true target (money, M or nominal income, YN) adjust to equilibrate the system. In the second period the nominal rate of interest once again becomes an endogenous variable while the targeted variable becomes exogenous.[14] The path of the latter is determined as follows. Suppose x was the solution for the targeted variable in Period 1, when the shock took place, and assume that the nominal rate of interest (RN) was held constant during that period. The three different exogenous paths for the targeted variable that are imposed on the system are as shown in Table 9.

These experiments were constructed for both the M-rule and the YN-rule for transitory and permanent IS (U_Y) and AS (UP) shocks. In all, there were twelve simulations for each rule, and these are summarized in Figures 70-93. Each figure gives the path followed by a particular variable after a particular shock, but three times, one for each of the three response speeds. Because the lag structure of the model causes some loss of observations, shocks occur in Period 15 in the figures. In order to highlight the differences between the solutions, we plot them only over a three-year post-shock horizon. After that time the solutions are virtually indistinguishable.

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13. This finding was first documented in White (1976). However, see Lane (1984) for an alternative interpretation based on the well-known Lucas critique.

14. Maintaining a constant nominal rate of interest in subsequent periods results in an explosive path for all shocks, whether transitory or permanent.

Before considering the individual plots, some general observations are in order. First, as we have noted before, the economy returns to control after a shock much more quickly under a YN-rule than under an M-rule. A related feature of the solutions is that the interest rate response to shocks is always more vigorous under the YN-rule than it is under the M-rule. However, the price of smoother interest rates under the M-rule is that rates generally remain away from control much longer than under the YN-rule. With respect to the shock-recognition lag, when we compare these figures with those presented in Section 4 we see that, not surprisingly, putting M or YN back on target in one quarter after a recognition lag of one quarter requires a more vigorous interest rate response than in the absence of a recognition lag under both rules. The additional increase in nominal interest rates that is necessary when there is a recognition lag is on the order of 60 basis points and 10 basis points for the M- and YN-rules, respectively, for a transitory IS shock; for a transitory AS shock the figures are 200 and 300 basis points, respectively. There is no evidence of instrument instability. With respect to the different speeds of response, it is often difficult to discriminate between the solutions under the M-rule, whereas the differences are somewhat more evident under the YN-rule. Often this has more to do with scale than with the absolute size of the differences;

Table 9

	Speeds of response		
	One quarter	Two quarters	Four quarters
Period 0	0	0	0
1 (shock)	x	x	x
2	0	x/2	3x/4
3	0	0	x/2
4	0	0	x/4
5	0	0	0
6	0	0	0

x = simulated value of M or YN, as the case may be, when RN is held at control during first period of the shock.

frequently, the movements in the variables are larger under the M-rule so on the same size of graph the differences will appear smaller.

In Table 10 we present the first two years of the interest rate paths for the two transitory shocks. For comparison we also present the results from the case considered previously, where there is no information lag. We can see that there are nontrivial differences in interest rate settings between the sudden and more gradual policies. For a transitory IS shock the cost of getting the targeted variable back on target in the first quarter after the shock rather than after one year is in the first instance about 40 basis points and 200 basis points for the M- and YN-rules, respectively. However, over the two-year period after the shock, nominal rates are on average 30 basis points and 120 basis points higher, respectively, under the more gradual policy. In other words, under the M-rule one can have interest rates 40 basis points lower in the first period, but an average 30 basis points higher during the next two years, under the gradual policy compared to the one-quarter lag case, whereas under the YN-rule the comparable figures are 200 and 120 basis points, respectively. Qualitatively similar comparisons are true for the temporary AS shock. This means that gradualism may be more beneficial in terms of less interest rate variance under a YN target than it is under an M target. Thus, it would seem that, for a given rate of time preference, a gradual response to a shock may be more easily justified under a YN-rule than under an M-rule.

With respect to the individual results, we can see in the figures for the transitory IS shock (Figures 70-75) that the introduction of gradualism into the M-rule tends to increase the amplitude of all cycles. Under the YN-rule, real output remains above control longer, and subsequently overshoots control more when a four-quarter response lag is in place, and prices rise about three times as much as under the one-quarter response rule.[15] In the case of a permanent IS shock (Figures 76-81) there is no apparent difference between the three different M-rules. However, under the YN-rule, the four-quarter speed of response reduces the amplitude of

15. With both the M-rule and the YN-rule, the short-run appreciation of the exchange rate is reduced through the use of gradualism -- by about 60 per cent in the case of the YN-rule, for example.

Table 10

NOMINAL INTEREST RATE PATHS ADOPTED DURING FIRST TWO YEARS

Transitory IS Shock

	M-Rule				YN-Rule			
Period	No-lag case	One-quarter	Two-quarters	Four-quarters	No-lag case	One-quarter	Two-quarters	Four-quarters
0	0	0	0	0	0	0	0	0
1 (shock)	0.554	0	0	0	2.720	0	0	0
2	0.302	1.129	0.852	0.713	-1.268	2.808	1.448	0.768
3	0.283	0.077	0.491	0.420	-0.174	-0.916	1.121	0.780
4	0.127	0.228	0.125	0.348	-0.647	-0.229	-0.601	0.571
5	-0.102	-0.102	-0.052	0.077	-0.791	-0.782	-0.572	0.179
6	-0.342	-0.332	-0.332	-0.333	-0.771	-1.018	-1.013	-0.988
7	-0.391	-0.429	-0.424	-0.396	-0.524	-0.663	-0.786	-0.739
8	-0.322	-0.300	-0.319	-0.323	-0.231	-0.237	-0.307	-0.463
9	-0.241	-0.252	-0.240	-0.251	0.027	-0.059	-0.062	-0.195
Transitory AS Shock								
0	0	0	0	0	0	0	0	0
1 (shock)	2.890	0	0	0	3.524	0	0	0
2	0.476	4.789	3.344	2.622	1.078	6.357	4.595	3.714
3	1.317	0.244	2.400	2.034	1.391	0.429	3.068	2.626
4	0.864	1.387	0.851	2.017	0.443	0.984	0.503	2.021
5	0.394	0.392	0.654	1.326	-0.744	-0.732	-0.461	0.572
6	-0.415	-0.360	-0.362	-0.369	-1.363	-1.683	-1.677	-1.644
7	-0.453	-0.652	-0.625	-0.482	-1.075	-1.255	-1.415	-1.354
8	-0.397	-0.281	-0.381	-0.404	-0.593	-0.601	-0.691	-0.893
9	-0.247	-0.303	-0.245	-0.302	-0.230	-0.342	-0.346	-0.518

all subsequent cycles in real output and prices. Inspection of the numerical solutions indicates that the same is true, but to a lesser extent, under the M-rule.

Turning now to the transitory AS shock (Figures 82-87), we find that, for both the M-rule and the YN-rule, the amplitude of the cycles is increased by slowing the speed of response. In addition, although it is difficult to see in Figures 88-93, the comments that were made above in respect of the permanent IS shock apply equally well to the permanent AS shock.

We argued above that, in contrast with the M- and YN-rules, under the S-rule there would be no policy-response lag. It therefore seems appropriate to compare the performance of the M- and YN-rules with a shock-recognition lag with that of the exchange rate rule under the assumption of no shock-recognition lag. Comparison of the plots presented here in Figures 70-93 with those in Figures 1-32 (Section 4) reveals that changing the information assumptions underlying the M- and YN-rules has affected the conclusions in terms of policy rule rankings presented in Section 4 only very slightly. When one compares the no-lag responses of Section 4 with the one-quarter lag, one-quarter response scenarios presented here, one sees some tendency for interest rate responses to be more vigorous in the presence of lags, as mentioned above. But the ranking of the policies remains the same in terms of interest rate variability, and in all other respects as well.

In summary, the above analysis has shown that our small dynamic model can simulate one-quarter recognition lags, with nominal interest rates held constant during this learning period, without encountering problems of instrument instability when targeted variables are brought back to control immediately after recognition. Bringing money or nominal income back to target in the next quarter, of course, generally requires a larger interest rate response than in the simple, no-recognition-lag case. The interest rate response becomes less vigorous as the authorities choose to lengthen the amount of time taken to return to target. The effect of this slower policy response is to increase the amplitude of cycles for transitory shocks but to reduce this amplitude for permanent shocks. Furthermore, gradualism implies that interest rates remain away from control for a longer period of time after the shock. However, no important conclusions regarding the comparisons between money supply, exchange rate and nominal income targeting policies have been altered by these changes in information assumptions.

In contrast with analyses of static theoretical models, the analysis presented here indicates that a policy response is required even for transitory shocks that are over before a response can possibly be implemented. This is because the dynamics of the model imply that the effects of such shocks on the economy die out only over time.[16]

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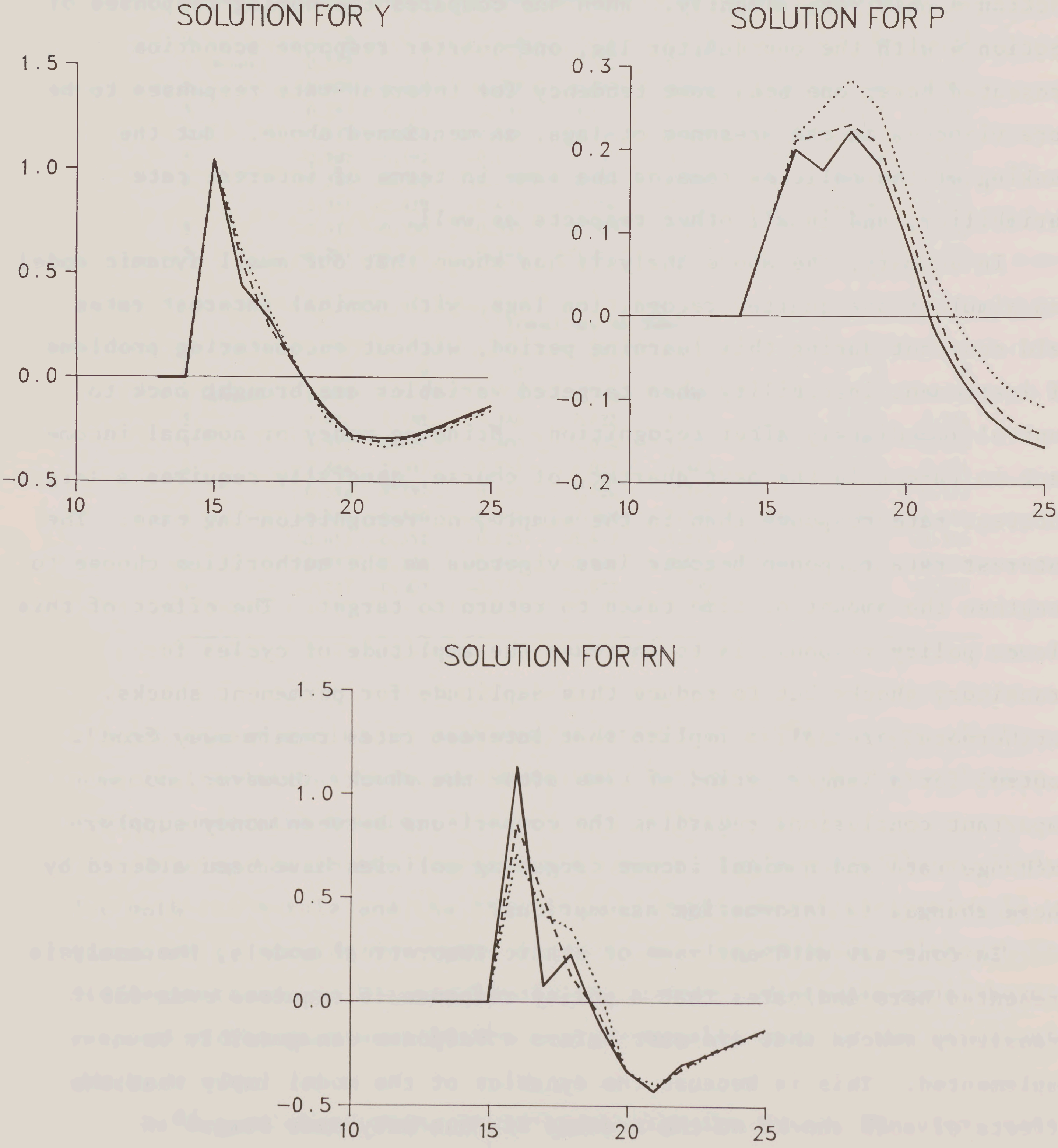
16. In effect, our "transitory" shocks are transitory in only a limited sense because of the presence of lagged dependent variables in the model. A similar scenario could be produced by imposing autocorrelated shocks on an otherwise static model.

Figures 70-72

TRANSITORY UY SHOCK: M RULE

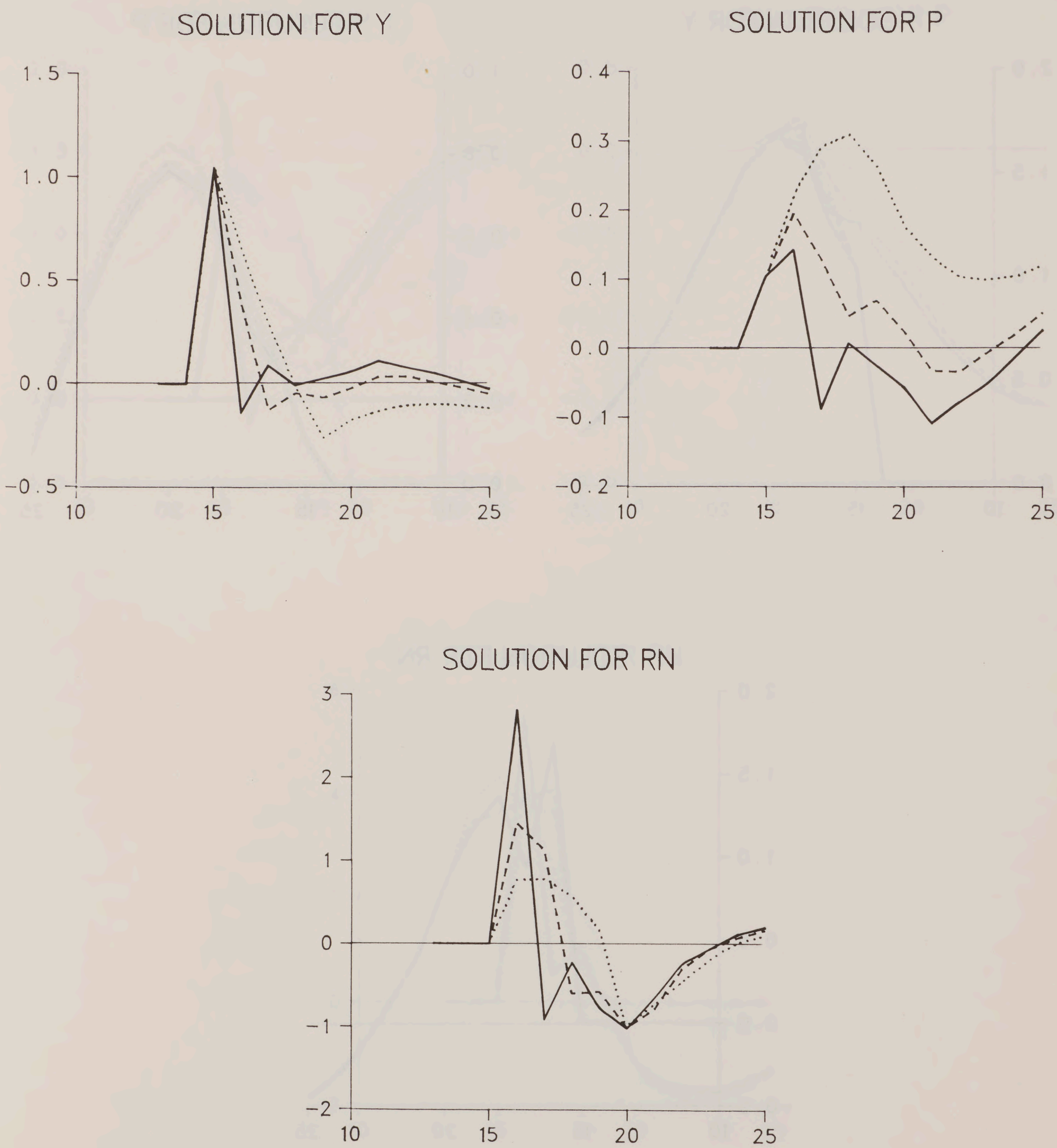
POLICY-RESPONSE LAG: 1 Quarter ——— 2 Quarters - - - - - - 4 Quarters ....



Figures 73–75

TRANSITORY UY SHOCK: YN RULE

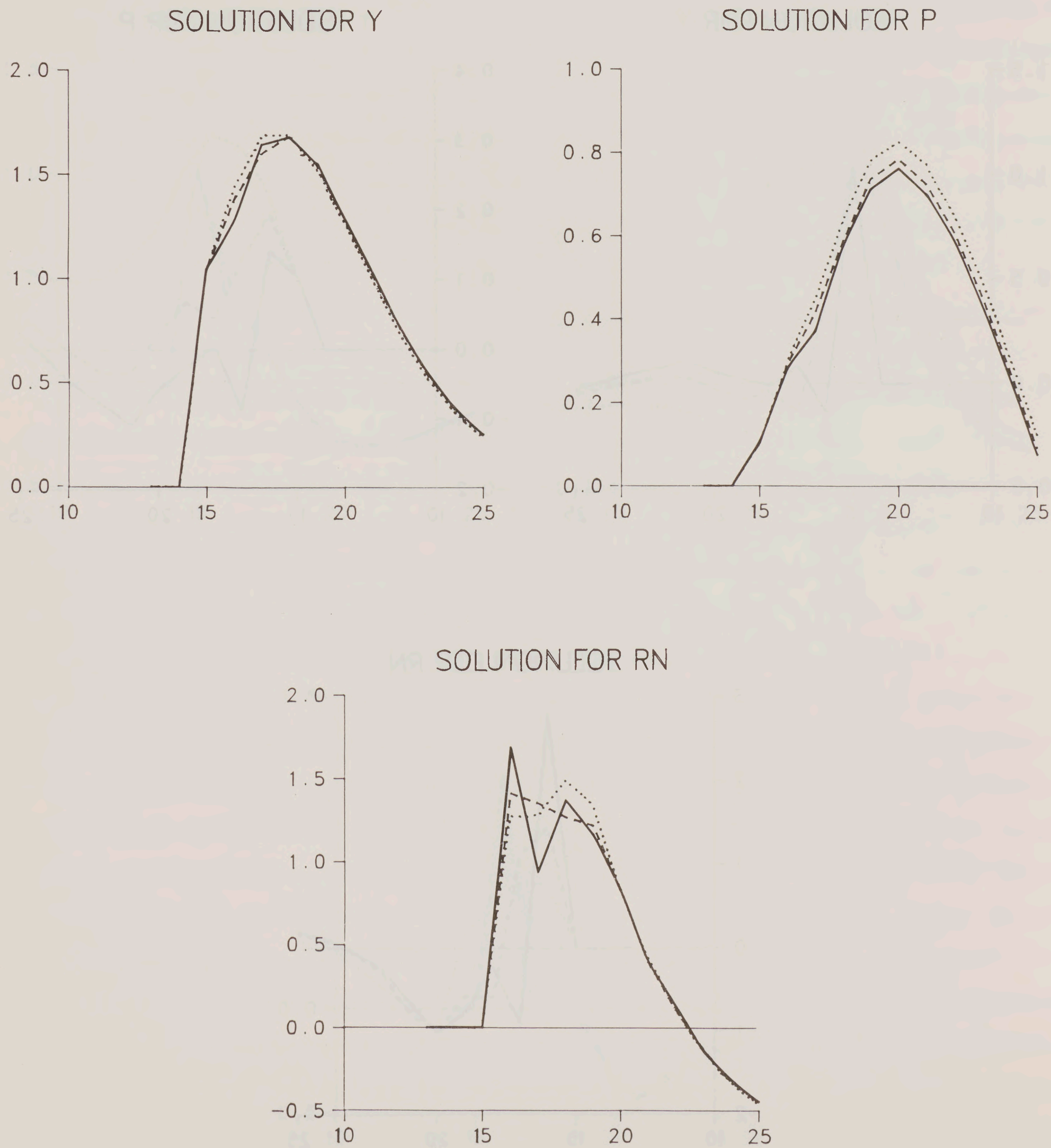
POLICY-RESPONSE LAG: 1 Quarter ——— 2 Quarters - - - - - - 4 Quarters ....



Figures 76-78

PERMANENT UY SHOCK: M RULE

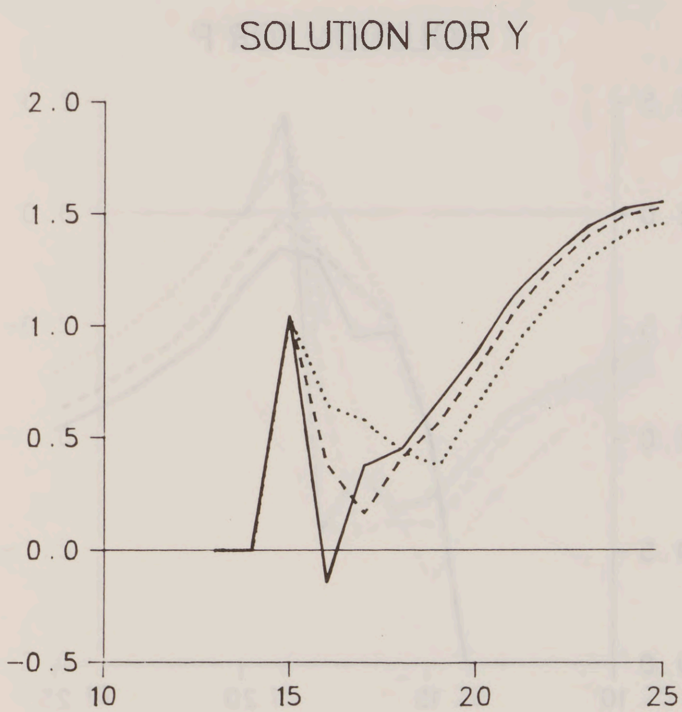
POLICY-RESPONSE LAG: 1 Quarter ——— 2 Quarters - - - - - - 4 Quarters ....

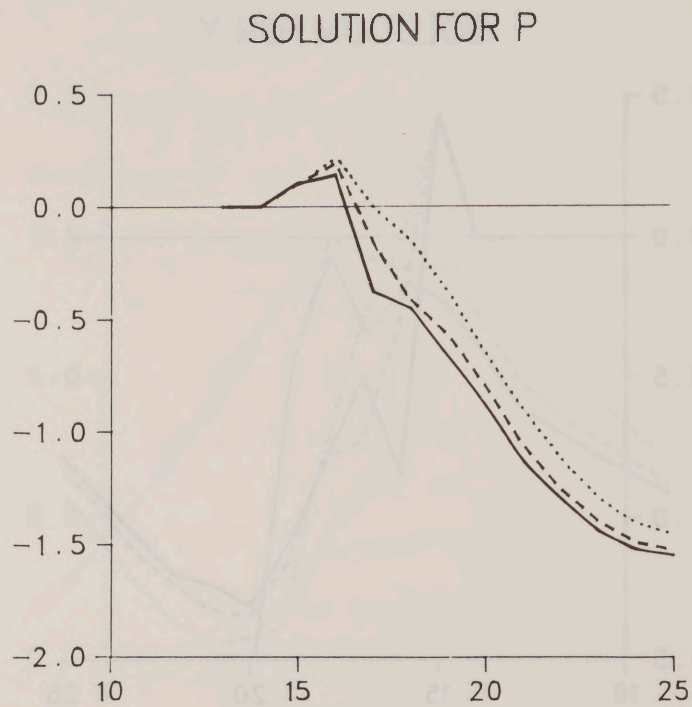


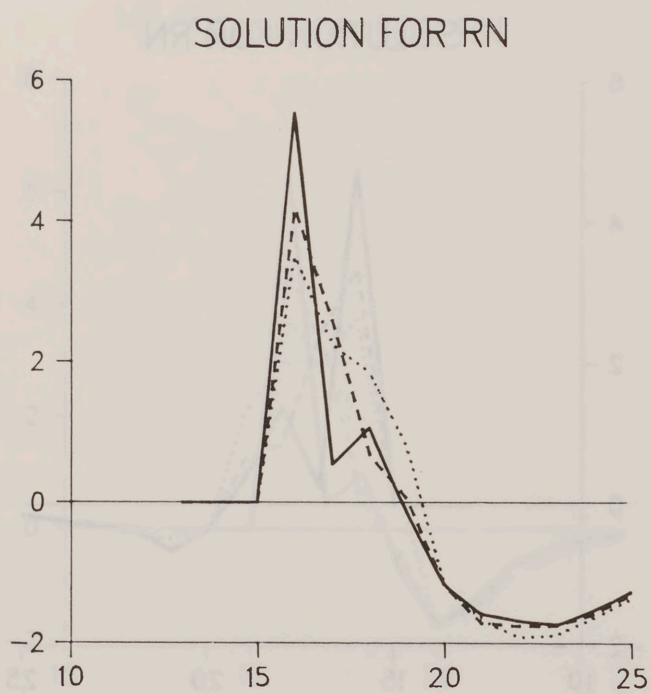
Figures 79-81

PERMANENT UY SHOCK: YN RULE

POLICY-RESPONSE LAG: 1 Quarter ——— 2 Quarters - - - - 4 Quarters ....



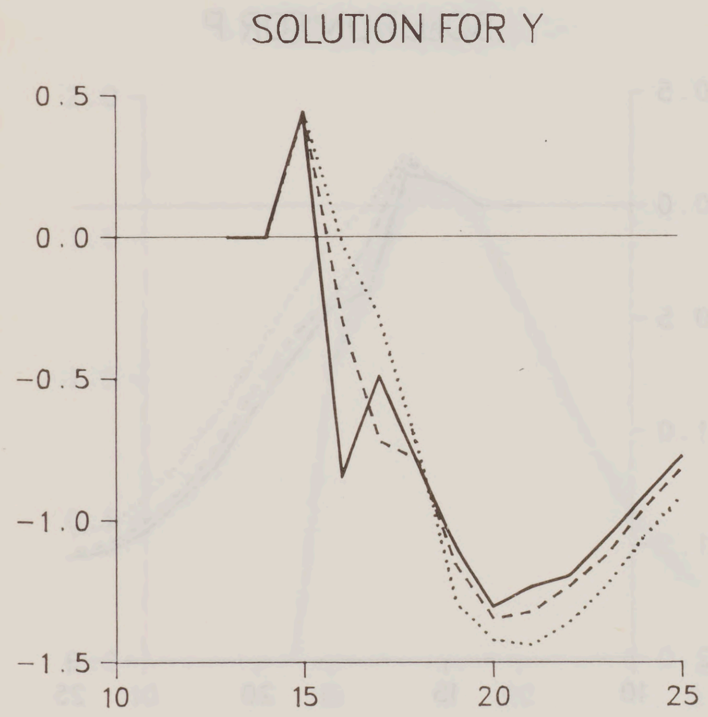


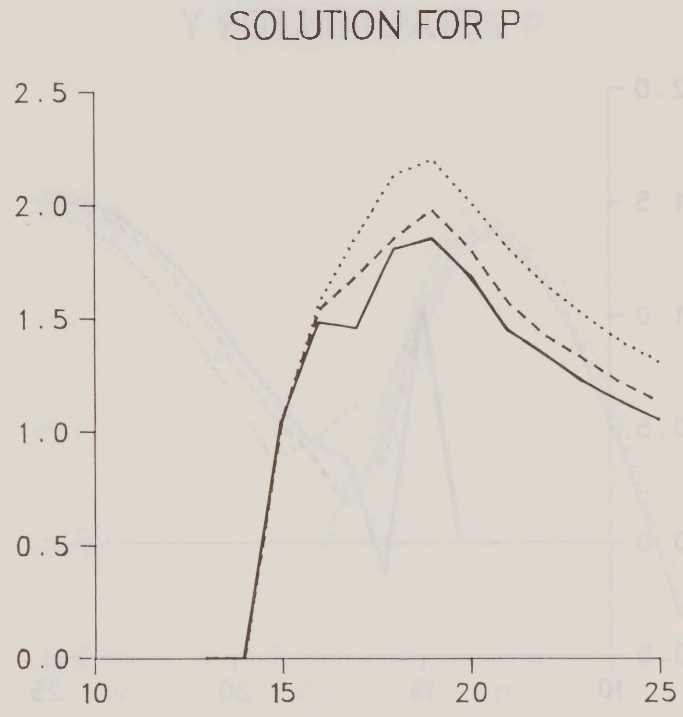


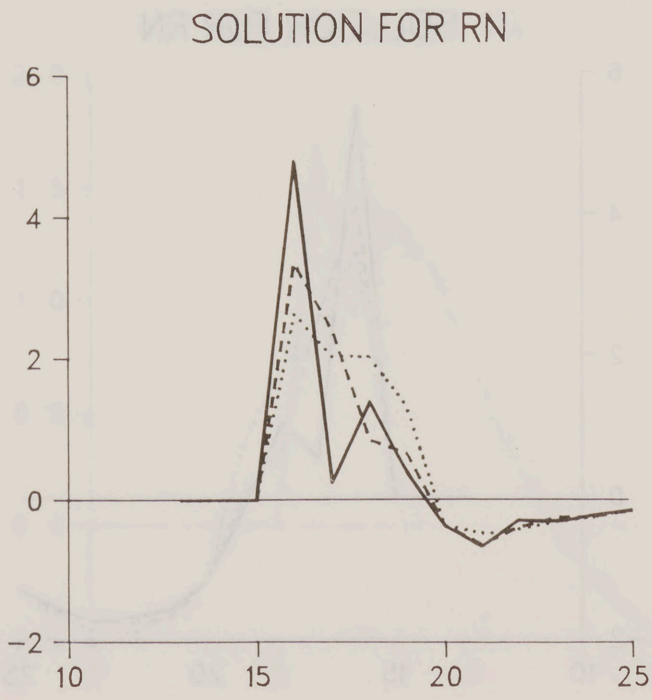
Figures 82-84

TRANSITORY UP SHOCK: M RULE

POLICY-RESPONSE LAG: 1 Quarter (solid line) 2 Quarters (dashed line) 4 Quarters (dotted line)



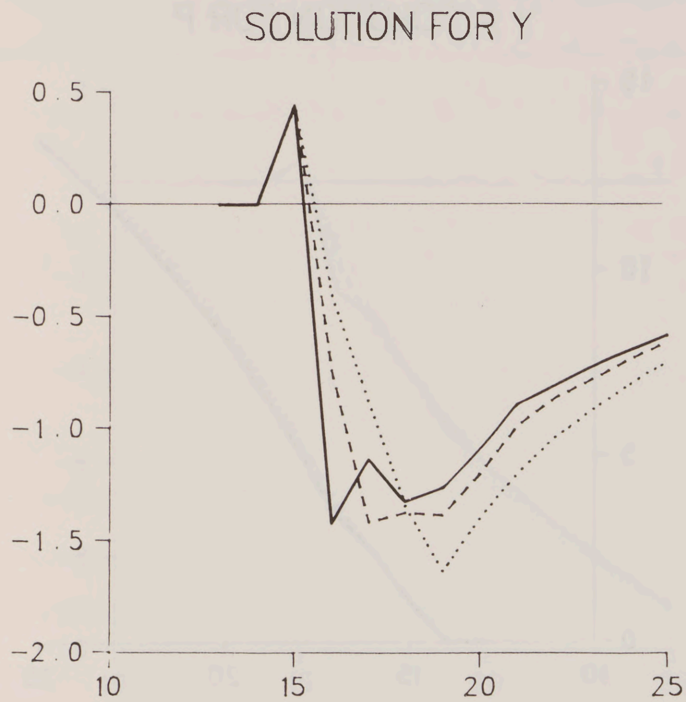


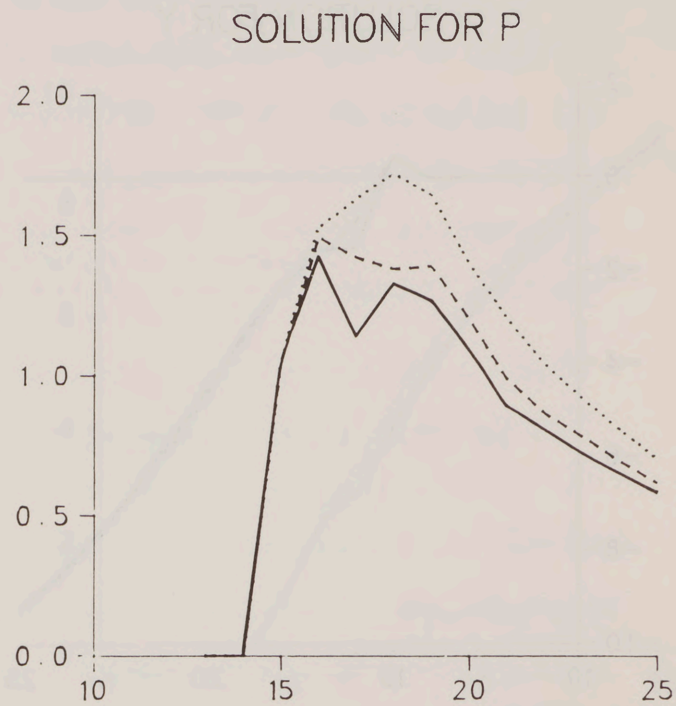


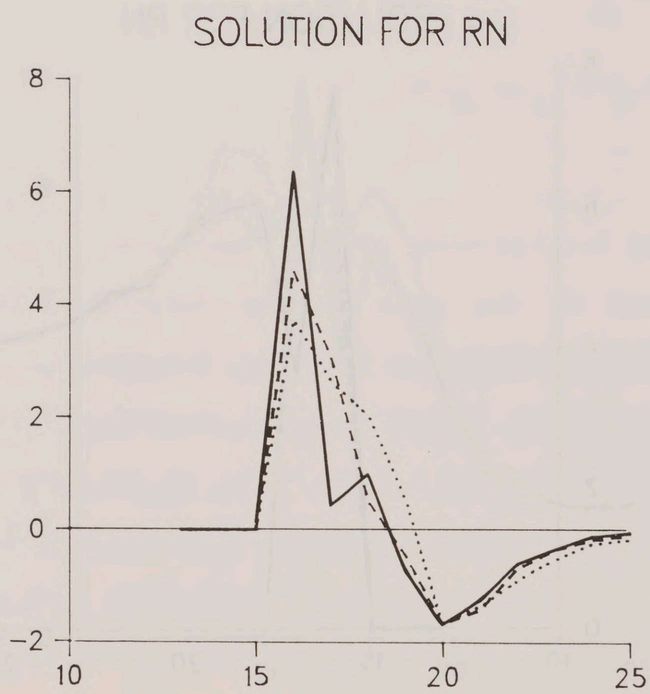
Figures 85-87

TRANSITORY UP SHOCK : YN RULE

POLICY-RESPONSE LAG: 1 Quarter (solid) 2 Quarters (dashed) 4 Quarters (dotted)





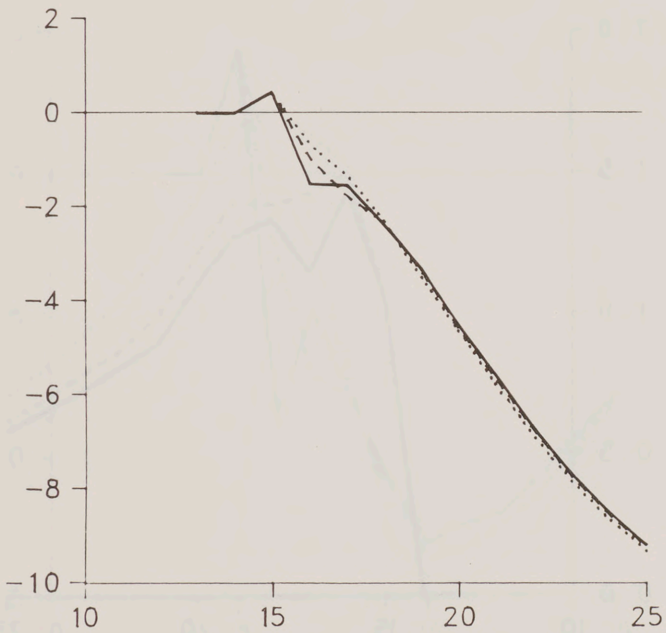


Figures 88-90

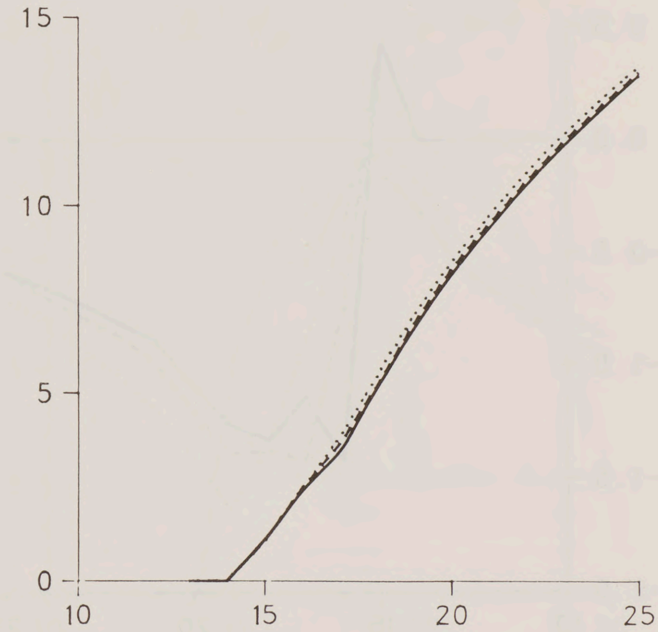
PERMANENT UP SHOCK: M RULE

POLICY-RESPONSE LAG: 1 Quarter ——— 2 Quarters - - - - - - 4 Quarters ....

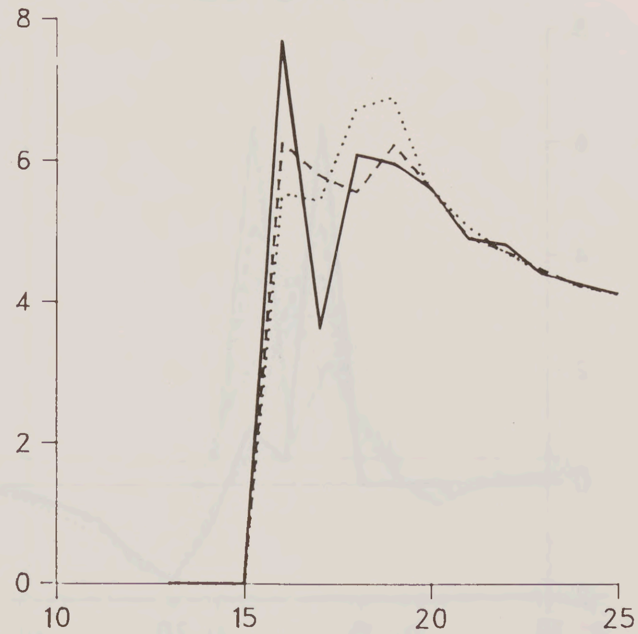
SOLUTION FOR Y



SOLUTION FOR P



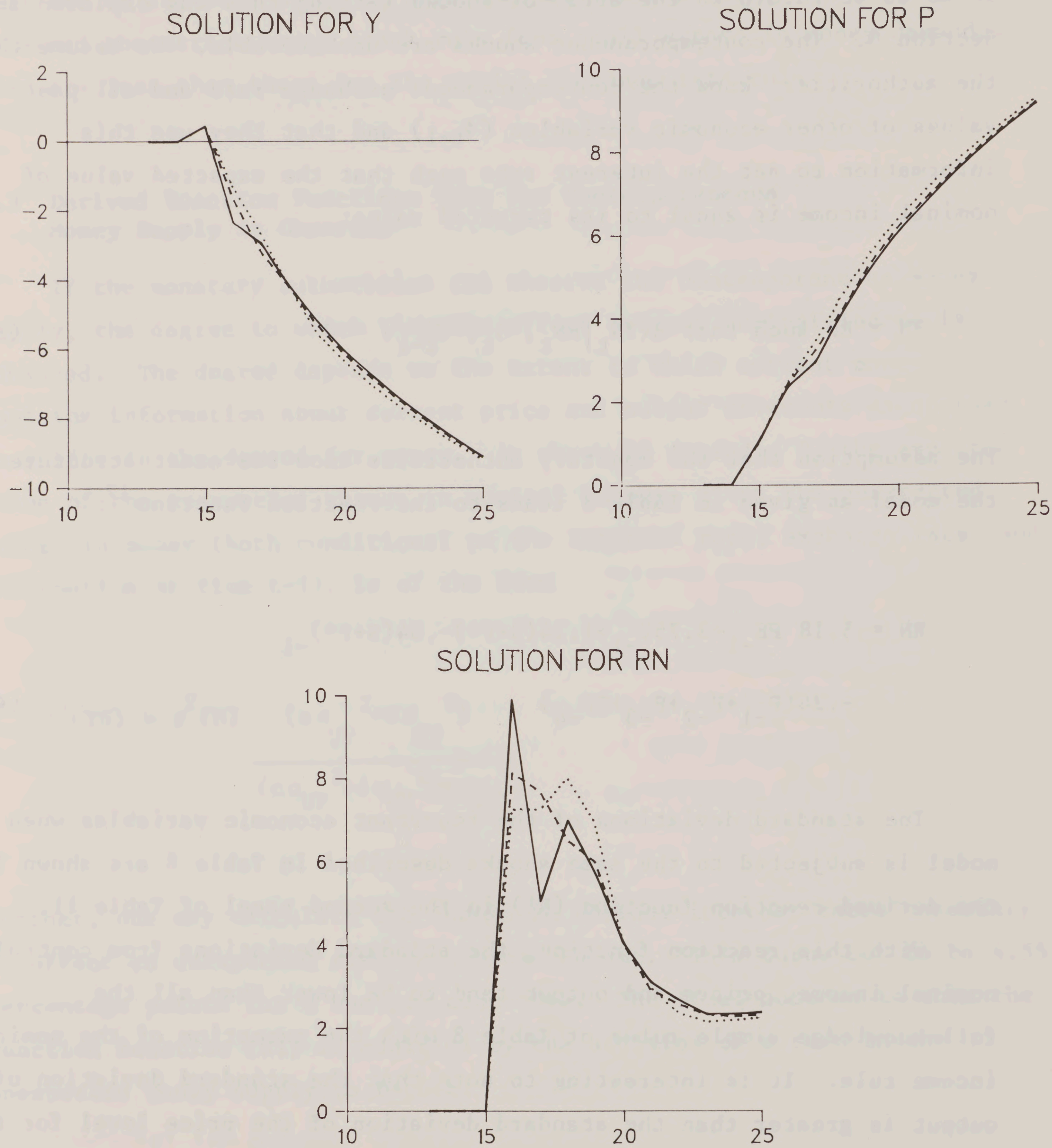
SOLUTION FOR RN



Figures 91-93

PERMANENT UP SHOCK: YN RULE

POLICY-RESPONSE LAG: 1 Quarter 2 Quarters 4 Quarters



7.2 Derived Reaction Functions and the Full Array of Shocks

In this section we again relax (but in a different way) the assumption that the monetary authorities can observe all contemporaneous variables and can achieve their desired target exactly period by period. To do so we return to the array-of-shocks setting that was explored in Section 6. The contemporaneous shocks are unobservable. We assume that the authorities know the contemporaneous exchange rate and all past values of other economic variables (Φ_{t-1}) and that they use this information to set the interest rate such that the expected value of nominal income is equal to its targeted value:

$$RN = RN \text{ such that } E(YN_t | RN_t, S_t, \Phi_{t-1}) = 0 \quad (22)$$

The assumption that the monetary authorities know the exact structure of the model as given in Table 3 leads to the reaction function[17]:

$$RN = 5.18\ PE_{-1} + 3.75Y_{-1} + 1.14(S+P^*) - .64(S+P^*)_{-1}$$
$$- .24(P_{-1} + P_{-2} + P_{-3} + 2P_{-4}) \quad (R1)$$

The standard deviations of the important economic variables when the model is subjected to the same shocks described in Table 8 are shown for the derived reaction function (R1) in the second panel of Table 11.

With this reaction function, the standard deviations from control of nominal income, prices and output tend to be lower than all the full-knowledge simple rules of Table 8 with the exception of the nominal income rule. It is interesting to note that the standard deviation of output is greater than the standard deviation of the price level for the

17. See Appendix B for the derivation.

reaction function, whereas under the strict YN-rule, the standard deviations of these two variables are identical. This is related to the unpredictability of contemporaneous shocks to aggregate demand relative to other shocks (see the top part of Table 11).

The standard deviations of the nominal and real interest rate for (R1) are of the same order of magnitude as for the strict nominal income rule and the strict money stock rule (for small errors in money demand). They are less than those for the strict exchange rate rule.

7.3 Derived Reaction Functions When the Contemporaneous Money Supply is Observed

If the monetary authorities can observe the contemporaneous money supply, the degree to which they can affect current nominal income is enhanced. The degree depends on the extent to which current money contains information about current price and output movements as opposed to shifts in the demand for money. As shown in Appendix B, the expected value of the unexpected change in nominal income, given the unexpected change in money (both conditional on the interest rate, exchange rate, and information at time t-1), is of the form

$$\Delta^U(YN) = \Delta^U(M) \quad \frac{(a\sigma_{UP}^2+b\sigma_{UY}^2)}{(c\sigma_{UP}^2+d\sigma_{UY}^2+e\sigma_{UM}^2)}$$

Further, one may calculate the change in nominal interest rates necessary to offset an unexpected rise in nominal income. This turns out to be 4.55 percentage points for a one per cent rise. The final outcome is that the reaction function (R1) is modified by the addition of a term in the unexpected money supply to become:

$$RN = 5.18\ PE_{-1} + 3.75Y_{-1} + 1.14(S+P^*)$$
$$- .64(S+P^*)_{-1} - .24(P_{-1} + P_{-2} + P_{-3} + 2P_{-4})$$
$$+ 4.55\Delta^{U}M \quad \frac{(1.048\sigma_{UP}^{2} + .11\sigma_{UY}^{2})}{(1.028\sigma_{UP}^{2} + .02\sigma_{UY}^{2} + \sigma_{UM}^{2})} \qquad (R2)$$

As shown in the final panel of Table 11, the gains from this additional information are not great, given our parameter assumptions. In the most favourable cases (5 and 7), the standard deviation of nominal income is reduced by about 0.14 and the standard deviation of the nominal interest rate is reduced by about 0.35. In Cases 2 and 4, when the variance of money demand is high, the standard deviation of nominal income is reduced by less than 0.01 and the standard deviation of nominal interest rates is reduced by less than 0.05.

Even given the strongly optimistic assumption here that the authorities can observe the money stock contemporaneously without error, the results of our simulations show that, under the joint assumptions of complete instrument control and full knowledge of the structure of the economy, contemporaneous money stock numbers are not significantly helpful in stabilizing nominal income in the context of our model. However, in the real world, where these two assumptions are unlikely to hold, there may be an important role for contemporaneous money stock data in this sense. This is essentially an empirical issue.

7.4 Response of the Model to Permanent Shocks

In this section we discuss briefly the properties of the model when faced with permanent U_Y, U_S and R* shocks with policy characterized by R1, or R2. In each case, we compare the solutions with the unattainable benchmark of those generated under a perfect nominal income rule where the information set of the monetary authorities includes all information, including the current values of the shocks. Since the results may be summarized easily, we do not present graphs of these solutions.

Table 11

SIMULATED STANDARD DEVIATIONS UNDER ALTERNATIVE SHOCK ASSUMPTIONS AND PARTIAL INFORMATION (250 observations, %)

Exogenous shocks	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
U_Y	0.989	0.989	0.989	0.989	0.841	0.841	0.841	0.841
U_P	0.410	0.410	0.410	0.410	0.615	0.615	0.615	0.615
U_M	0.797	1.992	0.797	1.992	0.797	1.992	0.797	1.992
U_S	0.792	0.792	1.485	1.485	0.792	0.792	1.485	1.485
Derived reaction function (R1)								
Y	1.764	1.759	1.763	1.757	2.177	2.169	2.173	2.164
P	1.373	1.378	1.381	1.384	1.950	1.949	1.951	1.949
YN	1.212	1.213	1.212	1.213	1.198	1.198	1.198	1.198
RN	4.106	4.127	4.348	4.359	4.667	4.666	4.858	4.849
RR	4.827	4.839	4.945	4.952	5.179	5.183	5.276	5.276
M	1.733	3.517	1.873	3.600	1.793	3.519	1.921	3.598
S	3.906	3.948	4.092	4.126	4.518	4.567	4.660	4.702
SR	4.648	4.680	4.788	4.815	5.888	5.922	5.984	6.014
Derived reaction function (R2)								
Y	1.703	1.747	1.703	1.746	2.073	2.146	2.071	2.142
P	1.323	1.361	1.330	1.368	1.853	1.920	1.855	1.920
YN	1.144	1.205	1.143	1.205	1.062	1.178	1.061	1.178
RN	3.916	4.086	4.170	4.321	4.311	4.581	4.514	4.769
RR	4.559	4.806	4.692	4.923	4.661	5.090	4.774	5.189
M	1.663	4.746	1.812	3.560	1.676	3.451	1.819	3.533
S	3.852	3.948	4.040	4.124	4.402	4.555	4.550	4.689
SR	4.560	4.675	4.701	4.809	5.714	5.899	5.814	5.990

We can begin by making some general observations which apply to all three sets of permanent shocks. First, both reaction functions considered give about the same speed of convergence to control as does the pure YN-rule. Second, the two derived reaction functions -- R1, which ignores the information contained in current money, and R2, which includes this information -- give solutions that are only occasionally distinguishable from one another.

The reaction functions considered generally cope well with permanent shocks, in the sense that no explosive oscillations arise. Furthermore, in many cases the dynamic solutions are very similar to those obtained under the pure, full-information nominal income rule. Where the reaction functions fail is in allowing nominal income to deviate permanently from target in some cases. Thus, these rules would allow base drift in the presence of permanent shocks. Of course, such rules are designed with transitory or temporary shocks in mind; permanent shocks, by their very nature, are eventually recognized as such, and should therefore prompt the appropriate adjustment to policy. The close correspondence of the reaction function solutions to the pure YN-rule solutions, especially in terms of the interest rate response, is an indication that these rules can at least cope with permanent shocks in the early stages while the authorities establish whether or not a shock is permanent.

8 CONCLUSIONS

We have built a very small, dynamic, open-economy IS-LM-AS model with expectations that are autoregressive in the short run but consistent or rational in the long run, selected its parameters on the basis of previous empirical work, and examined its properties under a variety of policy rules, information assumptions, and stochastic specifications. Our central purpose was to learn more about the differences between targeting nominal income, the money stock and the exchange rate in the context of a particular open-economy model.

Previous theoretical work on static models with rational expectations had demonstrated a clear dominance, in terms of minimizing the variance of real output and the price level, of the nominal income rule over money supply and exchange rate targeting, for all except aggregate supply shocks, which produced ambiguous rankings. Our purpose was to investigate these results for a specific dynamic model that did not impose short-run "rationality" and whose structure was sufficiently simple that one could effectively trace the linkages that produced a particular result.

The first sections examined the relative efficiency of three simple rules, each fixing exogenously one of the money stock, the exchange rate, or nominal income, in the face of transitory and permanent aggregate demand, aggregate supply, money demand, and exchange rate, foreign price level and foreign interest rate shocks. Fixed nominal or real interest rate targets were untenable because they produced explosive solutions to most shocks, and the price-level rule was eliminated from consideration because it gave rise to dramatic and explosive oscillations in the solutions. Summary measures of variability around control were calculated over various horizons, and we concluded that, as with the theoretical work, the nominal income rule tended to dominate in terms of real output and price-level variance for all shocks to our model. The latter result was reassuring, since it indicated that it is possible that the conclusion may be independent of the modelling of expectations. We also noted that in terms of speed of convergence, the nominal income rule produced the most desirable dynamic properties for the major economic variables when the model was subjected to shocks, including shocks to the growth rates of the targeted variables. Finally, we were able to see directly that the cost of this improved performance was an increase in the variances of financial variables in general.

The special case of foreign interest rate shocks was considered in some detail, given their special relevance for Canada. We saw in our model that the nominal income rule suggested a domestic interest rate response which lay between that called for under the exchange rate rule, which was more vigorous, and that for the money supply rule, which was less so. The model demonstrated clearly that the less vigorous response dictated by the money stock rule was paid for later with a more prolonged period of above-control domestic interest rates, while the overly vigorous exchange rate rule response generated a series of enduring, secondary cycles in the economy. We also considered briefly the continua of policy rules whose solutions lay intermediate between those of the pure nominal income, money stock and exchange rate rules. This analysis indicated that for loss functions that place a weight on the variance of financial variables a pure rule would be an unlikely choice, for in most cases the trade-offs traced by the model revealed that one or another of the conditional rules would dominate. However, for loss functions containing only the variances of real output and prices, the nominal income rule clearly dominated, for this particular model.

The remainder of our study dealt with the issue of the information set which is assumed to be available to the monetary authorities. To this point in the discussion, complete and instantaneous information and the possibility of an immediate policy response had been assumed. We began by supposing that there was a one-quarter shock-recognition lag, after which full information was once again assumed. Because our model is dynamic and expectations autoregressive, even transitory one-period shocks produce enduring cycles in the economy, so that despite the fact that the shock has disappeared by the time it is recognized, policy has a role to play in subsequent periods. Because of the lag, the targeted variable is thrown off-target by the shock, and we experimented with different speeds of bringing the variable back to target. The model experienced no instability problems during these experiments. We found that slowing the speed of return to target had the effect of increasing the amplitude of subsequent cycles for transitory shocks, but decreasing this amplitude for permanent shocks. Furthermore, invoking this less restrictive information assumption had no effect on earlier conclusions regarding the ranking of the alternative policies.

Even so, the one-quarter recognition lag was felt to be too generous, and the assumed measure of control over variables such as nominal income and the money supply was believed to be excessive. Therefore, we modified our assumptions so that the authorities have no knowledge of the individual contemporaneous shocks. In addition, rather than examining shocks in isolation, we created a stochastic environment for our model, with variances chosen on the basis of previous empirical work, and simulated the model over long periods of time. The simple exogenous policy rules were replaced by interest rate reaction functions, each expressed only in terms of observable variables such as the current exchange rate, current and lagged money stock, and lags on all other variables. Two reaction functions designed to stabilize nominal income were considered: a rule derived from the model that ignores current money, but which requires full knowledge of the structure and parameters of the model, and a rule that also uses the current information contained in the money stock. Both rules provided dynamic properties not unlike the simple nominal income rule. No instability problems were encountered, although we found that these rules typically failed to anchor the level of the nominal variables of the economy when faced with permanent shocks; the model did not explode but some amount of base drift usually occurred. Finally, very little was gained in terms of variance reduction by utilizing information contained in the contemporaneous money stock, given our other assumptions. Thus, the preferred interest rate reaction function places high weights on lagged real output and a distributed lag of the price level, approximately unit weight on the current exchange rate, and a partially offsetting negative weight on the lagged exchange rate. The parameters of this rule (R1) are only as reliable as those imposed on the model structure, and it will be interesting to see if empirical work on reduced-form or structural models can generate similar estimates.

A potentially important problem that has not been addressed in this study is the problem of parameter uncertainty. Knowledge of the underlying parameters of our economic system may be unevenly allocated across the various equations, giving rise to an a priori bias in favour of one policy rule or another. In particular, the nominal income rule may require more complete knowledge of the parameters of the economy than do the other two rules, but this sort of differential uncertainty has not been examined here. The importance of this issue might be analyzed either by repeating the above analysis for a range of parameter values, or by using a data-based version of the model to construct confidence intervals around the various solutions. The latter approach would seem to be the more efficient of the two, for it would reduce substantially the number of simulations that would be necessary.

APPENDIX A

A Digression on Base Drift in a Conditional Rule

Within the context of a deterministic model, or of an economic projection where errors have expected value zero and policy rules are rigidly followed, following the rate of change equation (A1) is equivalent to following the levels equation (A2) independently of whether the rule depends on money, nominal income or both. (In a more general conditional rule the exchange rate could be added as well.)

$$\dot{M} = \dot{M}* + \beta(\dot{YN}* - \dot{YN}) \tag{A1}$$

$$M = M* + \beta(YN* - YN) \tag{A2}$$

However, as the world unfolds one can always write

$$\dot{M}_t = \dot{M}^*_t + \beta(\dot{YN}^*_t - \dot{YN}_t) + u_t \tag{A1'}$$

where u_t is the error made in reaching the composite target $\dot{M}^*_t + \beta\dot{YN}^*_t$. If target growth rates are always calculated on last period's actual values, base drift will occur as the integral of (A1') gives

$$M_t + \beta YN_t - (M^*_t + \beta YN^*_t) = \int_o^t u_s \, ds \tag{A1''}$$

The stochastic version of equation (A2) on the other hand would be

$$M_t + \beta YN_t - (M^*_t + \beta YN^*_t) = v_t, \tag{A2'}$$

which shows that deviations from targets depend only on a current quarter error. Furthermore, although rule (A2') will not prevent changes in the price level from those anticipated when the rule was established, it will prevent cumulative error in the sense that inflation has to converge to a steady-state rate consistent with the trend rate of real output growth and the growth rates of the target. This can be shown by writing the long-run demand for money as

$$M_t - P_t = aY_t - bRN_t + U_t^M \qquad (A3)$$

plugging it into (A2'), and using the definition $YN_t = P_t + Y_t$ yields:

$$P_t + aY_t - bRN_t + U_t^M + \beta P_t + \beta Y_t - M_t^* - \beta YN_t^* - v_t = 0 \qquad (A4)$$

Rearranging, we have

$$P_t = \frac{M_t^*}{1+\beta} + \frac{\beta}{1+\beta} YN_t^* - \left(\frac{a+\beta}{1+\beta}\right) Y_t + \frac{b}{1+\beta} RN_t - \frac{U_t^M}{1+\beta} + \frac{v_t}{1+\beta} \qquad (A5)$$

In the long run, Y_t is given by supply output (YS_t^*) and RN_t is given by the foreign real rate RR_t^* plus the steady-state rate of inflation, which, by differentiation of equation (A5) is:

$$\dot{P} = \frac{\dot{M}^*}{1+\beta} + \frac{\beta}{1+\beta} \dot{YN}^* - \left(\frac{a+\beta}{1+\beta}\right) \dot{YS}^* \qquad (A6)$$

Using $Y = YS^*$, $RN = RR^* + \dot{P}$ and equation (A6) in equation (A5) yields:

$$P = \frac{M_t^*}{1+\beta} + \frac{\beta}{1+\beta} YN_t^* - \left(\frac{a+\beta}{1+\beta}\right) YS_t^* + \frac{b}{1+\beta} \left[RR_t^* + \frac{\dot{M}^*}{1+\beta} + \frac{\beta}{1+\beta} \dot{YN}^*\right.$$

$$\left. - \left(\frac{a+\beta}{1+\beta}\right) \dot{YS}_t^*\right] - \frac{U^M}{1+\beta} + \frac{v_t}{1+\beta} \qquad (A7)$$

This shows that cumulative error is avoided.

One additional implication of equations (A6) and (A7) is that there are no problems caused <u>in the steady state</u> by a target growth rate $\dot{M}^*$ which is inconsistent with YN*, or a target level M* which is inconsistent with YN*; all that will happen is that neither M* ($\dot{M}^*$) nor YN* ($\dot{YN}^*$) will be achieved -- one will be surpassed and the other undershot. Choosing targets that are inconsistent in the short run will of course affect the pattern of adjustment in the economy. As well, if the authorities are ultimately more interested in YN than in M, then placing a high weight on an M* which is inconsistent with YN* may cause severe welfare losses.

APPENDIX B

The Derivation of Theoretical Reaction Functions

Reaction function (R1) may be derived as follows from equations (1), (2) and (5) in Table 1:

$$Y = -\alpha_1 RN+4\alpha_1(PE-P)+\alpha_2(P^*+S)-\alpha_2 P+\alpha_3 Y_{-1}+U_Y \tag{B1}$$

$$P = PE_{-1}+\alpha_4 Y+U_p \tag{B2}$$

$$PE-P = \alpha_{11}P+\Sigma W_i P_{-i}+(1-\alpha_{11}-\dots-\alpha_{14})(S+P^*)$$

$$-(1-\alpha_{11}-\dots-\alpha_{14})(S+P^*)_{-1} \tag{B3}$$

where $\Sigma W_i P_{-i} = -.06(P_{-4}+P_{-2}+P_{-3}+2P_{-4})$

By inserting (B3) and (B1) into (B2) we have:

$$P = PE_{-1}-\alpha_1\alpha_4 RN+4\alpha_1\alpha_4\alpha_{11}P+4\alpha_1\alpha_4\Sigma W_i P_{-i}$$

$$+ 4\alpha_1\alpha_4(1-\alpha_{11}-\dots-\alpha_{14})(S+P^*)$$

$$- 4\alpha_1\alpha_4(1-\alpha_{11}-\dots-\alpha_{14})(S+P^*)_{-1}$$

$$+ \alpha_2\alpha_4(S+P^*)-\alpha_2\alpha_4 P+\alpha_3\alpha_4 Y_{-1}$$

$$+ U_P+\alpha_4 U_Y \tag{B4}$$

$$P = \frac{1}{1+\alpha_2\alpha_4-4\alpha_1\alpha_4\alpha_{11}}\Big[PE_{-1}+\alpha_3\alpha_4 Y_{-1}+(4\alpha_1\alpha_4(1-\alpha_{11}-\ldots-\alpha_{14})+\alpha_2\alpha_4)$$

$$(S+P^*)-4\alpha_1\alpha_4(1-\alpha_{11}-\ldots-\alpha_{14})(S+P^*)_{-1}$$

$$+\,4\alpha_1\alpha_4\Sigma W_i P_{-i}-\alpha_1\alpha_4 RN+U_P+\alpha_4 U_Y\Big] \qquad \text{(B5)}$$

So from (B1) and (B5)

$$P+Y = -\,\alpha_1 RN+\alpha_2(P^*+S)+\alpha_3 Y_{-1}+U_Y+4\alpha_1\Sigma W_i P_{-i}$$

$$+\,4\alpha_1(1-\alpha_{11}-\ldots-\alpha_{14})(S+P^*)-4\alpha_1(1-\alpha_{11}-\ldots-\alpha_{14})(S+P^*)_{-1}$$

$$+\,\frac{(1+4\alpha_1\alpha_{11}-\alpha_2)}{(1+\alpha_2\alpha_4-4\alpha_1\alpha_4\alpha_{11}}\Big[PE_{-1}+\alpha_3\alpha_4 Y_{-1}+(4\alpha_1\alpha_4(1-\alpha_{11}-\ldots-\alpha_{14})+\alpha_2\alpha_4)$$

$$(S+P^*)-4\alpha_1\alpha_4(1-\alpha_{11}-\ldots-\alpha_{14})(S+P^*)_{-1}$$

$$+\,4\alpha_1\alpha_4\Sigma W_i P_{-i}-\alpha_1\alpha_4 RN+U_P+\alpha_4 U_Y\Big] \qquad \text{(B6)}$$

If one denotes information by Φ, then solving $E(P+Y|RN,S,\ \Phi_{t-1} = 0$ for RN:

$$RN = \frac{1}{\alpha_1(1+\lambda\alpha_4)}\Big[\lambda PE_{-1}+\alpha_3(1+\alpha_4\lambda)Y_{-1}$$

$$+\,(4\alpha_1(1-\alpha_{11}-\ldots-\alpha_{14})(1+\alpha_4\lambda)+\alpha_2(1+\lambda\alpha_4))(S+P^*)$$

$$-\,4\alpha_1(1-\alpha_{11}-\ldots-\alpha_{14})(1+\alpha_4\lambda)(S+P^*)_{-1}$$

$$+\,4\alpha_1\Sigma W_i P_{-i}\,(1+\lambda\alpha_4)\Big] \qquad \text{(B7)}$$

$$\text{where } \lambda \equiv \frac{(1+4\alpha_1\alpha_{11}-\alpha_2)}{(1-4\alpha_1\alpha_4\alpha_{11}+\alpha_2\alpha_4)} \approx 1.156 \qquad \text{(B8)}$$

$$\text{So } \frac{1}{\alpha_1(1+\lambda\alpha_4)} \approx 4.48$$

$$\text{and } \frac{\lambda}{\alpha_1(1+\lambda\alpha_4)} \approx 5.18 \text{ (See Table 2)}$$

$$RN = 5.18PE_{-1} + 3.75Y_{-1} + 1.14(S+P^*) - .64(S+P^*)_{-1}$$

$$- .24(P_{-1}+P_{-2}+P_{-3}+2P_{-4}) \qquad \text{(B9=R1)}$$

The unexpected change in $M(\Delta^U M)$ is equal to

$$M - E(M|R,S,\Phi_{t-1}) =$$

$$M - E(P|R,S,\Phi_{t-1}) - \alpha_8 E(Y|R,S,\Phi_{t-1}) + \alpha_9 RN - \alpha_{10}(M_{-1}-P_{-1}) \qquad \text{(B10)}$$

$$= M - \left[\frac{1+4\alpha_8\alpha_1\alpha_{11}-\alpha_8\alpha_2}{1+\alpha_2\alpha_4-4\alpha_1\alpha_4\alpha_{11}}\right][PE-1+\alpha_3\alpha_4 Y-1$$

$$+ (4\alpha_1\alpha_4(1-\alpha_{11}-\ldots-\alpha_{14})+\alpha_2\alpha_4)(S+P^*)$$

$$- 4\alpha_1\alpha_4(1-\alpha_{11}-\ldots-\alpha_{14})(S+P^*)_{-1}$$

$$+ 4\alpha_1\alpha_4\Sigma W_i P_{-i} - \alpha_1\alpha_4 RN]$$

$$- \left[-\alpha_8\alpha_1 RN + \alpha_8\alpha_2(P^*+S) + \alpha_8\alpha_3 Y_{-1} + 4\alpha_8\alpha_1\Sigma W_i P_{-i}\right.$$

$$\left. + 4\alpha_8\alpha_1(1-\alpha_{11}-\ldots-\alpha_{14})(S+P^*) - 4\alpha_8\alpha_1(1-\alpha_{11}-\ldots-\alpha_{14})(S+P^*)_{-1}\right]$$

$$+ \alpha_9 RN - \alpha_{10}(M-P)_{-1} \qquad \text{(B11)}$$

If we evaluate using the parameters in Table 2,

$$\Delta^U M = M-1.028PE_{-1}-.152\ Y_{-1}+.33RN-.9(M_{-1}-P_{-1})$$

$$-.037(S+P^*)+.026(S+P^*)_{-1}+(.162)(.06)$$

$$(P_{-1}+P_{-2}+P_{-3}+2P_{-4}) \qquad \text{(B12)}$$

$$E-E(M|RN,S,\Phi_{t-1}) = P-E(P|RN,S,\Phi_{t-1})$$

$$+\ \alpha_8(Y-E(Y|RN,S,\Phi_{t-1}))+U_M \qquad \text{(B13)}$$

Now

$$P-E(P|RN,S,\Phi_{t-1}) = .1(Y-E(Y|RN,S,\Phi_{t-1}))+U_P$$

$$Y-E(Y|RN,S,\Phi_{t-1}) = .8(.3(P-E(P|RN,S,\Phi_{t-1})))$$

$$-.1(P-E(P|RN,S,\Phi_{t-1}))+U_Y$$

$$= .14U_P+U_Y$$

$$P-E(P|RN,S,\Phi_{t-1}) = 1.014\ U_P\ +.1\ U_Y$$

If U_P, U_Y, U_M are uncorrelated then the expected value given M of:

$$P-E(P|RN,S,\Phi_{t-1}) \text{ is } \frac{\Delta^U M(1.014)^2\,\sigma_{UP}{}^2+(0.1)^2\,\sigma_{UY}{}^2)}{D}$$

$$-\ E(Y|RN,S,\Phi_{t-1}) \text{ is } \frac{\Delta^U M((0.1)^2((0.14)^2\,\sigma_{UP}{}^2+\sigma_{UY}{}^2))}{D}$$

where $D = (1.014)^2\,\sigma^2_{UP}+(0.1)^2\,\sigma^2_{UY}+(0.1)^2((0.14)^2\,\sigma^2_{UP}+\sigma^2_{UY})+\sigma^2_{UM}$

$$\text{So}\quad E(P+Y|RN,S,M,\Phi_{t-1})-E(P+Y|RN,S,\Phi_{t-1})=\frac{\Delta^{U}M(1.048\,\sigma_{UP}^{2}+0.11\,\sigma_{UY}^{2})}{1.028\,\sigma_{UP}^{2}+.02\,\sigma_{UY}^{2}+\sigma_{UM}^{2}}\equiv X \tag{B14}$$

To reduce P+Y by the amount of the right-hand side of the above equation (X), we need to increase

$$RN \text{ by } \frac{-1}{\left.\frac{\partial(P+Y)}{\partial(RN)}\right|_{S_t,\Phi_{t-1}}} \text{ times } X.$$

To calculate $\left.\frac{\partial(P+Y)}{\partial(RN)}\right|_{S_t,\Phi_{t-1}}$ we note that

$$Y = .8(.3P+\ldots)-.1P+U_Y-.2RN+\ldots.$$

$$P = .1Y+U_P+\ldots.$$

$$\text{So}\quad Y = .14P+U_Y-.2RN+\ldots.$$

$$P = .1Y+U_P+\ldots.$$

$$Y = .014Y+.14U_P+U_Y-.2RN+\ldots.$$

$$Y \approx .14U_P+U_Y-.2RN+\ldots.$$

$$P = .1Y+U_P \approx 1.014U_P+.1U_Y-.02RN+\ldots.$$

$$P+Y = 1.154U_P+1.1U_Y-.22RN+\ldots.$$

$$\left.\frac{\partial(P+Y)}{\partial RN}\right|_{S_t,\Phi_{t-1}} = -.22 = \frac{-1}{4.55} \tag{B15}$$

If we bring together the various pieces (B9, B12, B14, B15), the reaction function (R2) is:

$$RN = 5.18PE_{-1}+3.75Y_{-1}+1.14(S+P^*)$$

$$- .64(S+P^*)_{-1}-.24(P_{-1}+P_{-2}+P_{-3}+2P_{-4})$$

$$+ 4.55 \frac{(1.048\sigma_{UP}^{2}+0.11\sigma_{UY}^{2})}{(1.028\sigma_{UP}^{2}+.02\sigma_{UY}^{2}+\sigma_{UM}^{2})} *$$

$$(M-1.028PE_{-1}-.152Y_{-1}-.037(S+P^*)$$

$$+ .026(S+P^*)_{-1}+.33RN-.9(M-P)_{-1}$$

$$+ (.162)(.06)(P_{-1}+P_{-2}+P_{-3}+2P_{-4})) \qquad (B16=R2)$$

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