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ASSET STOCKS AND THE USE OF MONETARY AND FISCAL POLICIES TO REDUCE INFLATION

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CONTENTS

ABSTRACT

This paper analyzes the dynamic behaviour of a country's economy under different policy regimes, by examining the cyclical effects that occur when certain intermediate macroeconomic targets are adopted. To highlight the differences in the adjustment paths that result, the study deliberately limits policy choice: either money supply or nominal income as targets, and either real interest rates or tax-financed government spending as instruments. Successively more complicated models are considered as the capital stock, government debt outstanding; and net claims on foreigners are introduced. For these models, the main conclusion is that targetting money supply is likely to bring about a more cyclical path for an economy than targetting nominal income. In addition, using the real interest rate as instrument may produce instability, that is, explosive adjustment, when asset stocks are included in the model -- and indeed instability will be present when increases in spending are financed by government debt issue. The government spending instrument, when tax financed, is less likely to result in instability. However, whether a country is a net creditor or a net debtor is shown to be important for open economy models: if the country is a large net debtor, any of the policy choices may imply instability.

RÉSUMÉ

Dans la présente étude, l'auteur examine le comportement dynamique d'une économie en retenant différents types de politiques et en analysant les effets cycliques qui se produisent lorsque les autorités visent certaines cibles intermédiaires de politique macro-économique. L'étude fait ressortir les différences qui existent entre les profils d'ajustement en offrant des choix de politique qui sont intentionnellement limités. Ces choix se font entre la masse monétaire et le revenu nominal, lorsqu'il s'agit des cibles, et entre les taux d'intérêt réels et des dépenses publiques financées par l'impôt, lorsqu'il s'agit des instruments utilisés. Les modèles étudiés sont plus complexes à mesure qu'on incorpore des variables telles que le stock de capital, l'encours de la dette du gouvernement et les créances nettes sur l'étranger. L'étude démontre que, pour ces modèles, la stratégie des taux cibles d'expansion monétaire est susceptible d'engendrer des fluctuations cycliques plus prononcées que celle d'un revenu nominal cible. De plus, l'utilisation du taux d'intérêt réel comme instrument peut provoquer une grande instabilité, c'est-à-dire des fluctuations de plus en plus importantes, lorsqu'on intègre des stocks d'actifs au modèle - et de fait, c'est ce qui se produit lorsque le gouvernement finance l'augmentation de ses dépenses en recourant à l'emprunt. Le financement des dépenses publiques au moyen de recettes fiscales risque moins d'engender de telles fluctuations. L'étude montre toutefois que, pour les modèles d'une économie ouverte, il importe de savoir si la position nette du pays vis-à-vis de l'étranger est créditrice ou débitrice: si le pays est un gros débiteur net, tous les choix de politique peuvent provoquer l'instabilité mentionnée.

INTRODUCTION

Policies intended to reduce steady state inflation will bring about a variety of paths for the economy in the transition to the long run. The purpose of this paper is to analyze the cyclical effects of different policy regimes, examining in particular the behaviour of the economy when either of two intermediate macroeconomic policy targets is adopted, with one of two instruments used to hit the selected target. In each case the intermediate target is a nominal growth rate, and the long-term effects on the inflation rate are the same. In each case, also, it is assumed that real output is unaffected in the long run.

Each policy, however, has different implications for the length and severity of cycles in inflation and in real output. For this reason proper comparison of policies involves looking at not just initial or long-run effects but whole adjustment paths, and it is such a comparison that is attempted here. In order to highlight differences in adjustment paths, this paper offers a menu for policy choice that is deliberately limited: either money supply or nominal income as intermediate targets, and either real interest rates[1] or tax-financed government spending as instruments. These policies will be compared on the basis of the dynamic path that results when the growth rate of a target variable is lowered.

There are several reasons for expecting a cyclical adjustment path. Suppose we consider a lowering of the money supply target by 1%, starting from a point of steady state equilibrium with zero real growth. If one presumes that people's expectations are slow to adjust, and that the general public has to be convinced that inflation will come down by actually observing it fall, a decrease in money growth will require in the first instance opening up output and unemployment gaps, which themselves will put downward pressure on the rate of change of

1 The authorities are assumed to know the structure of the economy and to observe the relevant variables, so that one can speak equivalently of the nominal or the real interest rate as instrument.

prices. After all adjustments have been worked out, the actual (and expected) rate of inflation will be lower by 1%, and output will have returned to its equilibrium level. However, the lower nominal interest rates that are consistent with the lower rate of inflation increase narrowly defined money demand, so that the path for the price level must be correspondingly lower, implying that over some period actual inflation must have been lower by more than 1%. Such overshooting would seem to require cyclical paths for real and nominal variables.

Whether or not money is the intermediate target the pressure of asset stocks leads one to expect a complex adjustment process.[2] First, restraint on aggregate spending will be associated with a lower capital stock, and this lower level may constrain the ability of the economy to meet future demand when it materializes. Consequently, initial success in lowering inflation rates may be reversed later. Second, if government spending is not matched by tax receipts, the government will be accumulating debt and consequently debt service payments will become more and more important. Then, reestablishing equilibrium requires the real value of debt to return to its initial level with respect to output. Third, changes in the stock of indebtedness to foreigners through current account surpluses or deficits will be a source of persistence in the effects of restraint in the growth of total spending which may delay the reestablishment of long-run equilibrium.

It is also clear that differences in the incidence of the policy instruments will bring about different dynamic behaviour. A decrease in tax-financed government expenditures can be expected to lower the capital stock relative to what it would otherwise have been through the familiar accelerator mechanism, but it may produce a lesser fall than when high real interest rates are used to achieve the same intermediate target. The budget deficit effects are of course also different, tending to produce smaller deficits for the fiscal policy instrument than for the

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2 Some of them have been discussed by Clarence Barber and John McCallum, in Unemployment and Inflation, the Canadian Experience (Ottawa: Canadian Institute for Public Policy, 1980).

monetary policy instrument. Finally, it seems likely that hitting either a money or a nominal income target solely through lower tax-financed expenditures would produce a higher price of foreign exchange, and a smaller current account deficit, than the sole use of higher real interest rates.

Clearly there are other criteria that may have at least as large a weight in any policy choice as the dynamic effects considered here.[3] For monetary policy, for example, it is clearly desirable to have as a target a variable that is controllable within the financial system rather than one that results from the interaction of financial and real forces like national income. Another criterion is the availability of data: national accounts data are issued with a lag of several months and are subject to quite large revisions, while financial data are generally more timely. In addition, interest rates and tax-financed government expenditures are not equally flexible instruments and the latter cannot be varied frequently to hit either intermediate target. If the intermediate target is the money supply, the fiscal policy instrument in any case is operating not directly through financial markets but indirectly via aggregate demand, output or labour-market gaps and price changes, and thence on the demand for money. The indirectness of the route, in a world where the exact form of the linkages is uncertain, makes it hard to imagine targetting money using that instrument. Indeed uncertainty, in the sense of the relative variance of shocks hitting financial versus goods markets,[4] is not considered here. According to Charles Freedman's taxonomy,[5] I consider only one of three possible ways of analyzing

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3 See, for instance W.R. White, "Alternative Monetary Targets and Control Instruments in Canada: Criteria for Choice", Canadian Journal of Economics 12 (November 1979), pp. 590-604 and Charles Freedman, Monetary Aggregates as Targets: Some Theoretical Aspects, Technical Report #27, Bank of Canada, Ottawa, 1981.

4 See William Poole, "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model", Quarterly Journal of Economics 84 (May 1970), pp. 197-216.

5 Charles Freedman, Monetary Aggregates as Targets: Some Theoretical Aspects, op. cit.

targetting, namely the feedback-rule approach.

The analysis here presumes that the effects of the instruments are certain and hence that targets can be achieved exactly, and that instruments are on an equal footing in terms of effectiveness. This is not true in reality, and therefore we observe target ranges and the assignment of instruments to particular goals. Nevertheless, the analysis here gives some indication why in the real world one would prefer to put more weight on one target than on another, or on one instrument than on another. It should be stressed that the analysis does not start with any premise as to whether inflation is a monetary phenomenon. On the contrary, in each case long-run price stability is accompanied by money growth equal to the growth of potential output.[6] The policy choice involves how to get to the long-run position, and whether fiscal policy or monetary policy does more to slow the rise in spending during the transition.

The plan of the paper is as follows. The notation and methods used to analyze the system of differential equations are presented in Chapter 1.

Chapter 2 considers the choice of intermediate target in a very simple closed economy model consisting of equations for aggregate demand, price change, inflation expectations and the demand for money. It is clear from the analysis that targetting money and targetting nominal income are not the same thing if money is interest elastic. It is shown that money targetting is more likely than income targetting to induce cyclical behaviour in real variables and in inflation. If both are cyclical, the cycles resulting from money targetting will be less damped and possibly unstable. Stability requires that the Cagan[7] condition hold -- that is, that the product of the speed of adaptation of expectations to

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6 Adjusted by the amount that the income elasticity of money demand differs from unity.

7 Philip Cagan, "The Monetary Dynamics of Hyperinflation", in M. Friedman, ed., Studies in the Quantity Theory of Money (Chicago: University of Chicago Press, 1956).

actual inflation and the interest semi-elasticity of money demand be less than one. This condition recurs in more complicated models as well, and we assume throughout that it does hold.

Chapter 3 expands the model somewhat by allowing for induced effects on the capital stock and hence on capacity output.[8] Here the use of real interest rate movements and of tax-financed government expenditure to hit either target is compared. It is shown that a feedback rule using tax-financed government spending will produce a stable adjustment[9] to a lower inflation rate, while one using real interest rates may imply instability. In particular, for the latter to be stable one must ensure that the effect of a change in the capital stock on aggregate demand dominates its effect on aggregate supply. This in itself is necessary but not sufficient for stability.

Chapter 4 extends the closed economy model developed in Chapter 3 to include government bonds as a component of household wealth, and allows government spending to be financed by bonds. In such a context, the use of government spending to hit, say, a nominal income target, will generally produce a stable economy, while varying real interest rates and leaving government spending and taxes unchanged will generally not do so. The only situation where the use of monetary policy will not be unstable, in a model where bond issues are the financing instrument that is varied endogenously and taxes are not, is when bond holdings are so large that the positive effect of interest payments on disposable income and hence on aggregate demand offsets the negative effect on investment. In this case monetary policy has a perverse effect in that expansive monetary policy is necessary to bring down inflation.[10]

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8 Pierre Duguay has examined the long-run effects of real interest rates on the rate of inflation in "L'influence des taux d'intérêt sur l'offre macro-économique: Conséquences pour le contrôle de l'inflation", a paper presented at the 22nd Annual Congress of the Société canadienne de science économique, Montréal, May 12, 1982.

9 Provided once again that the Cagan condition holds.

10 This result is clearly related to the debate on money-financed vs. bond-financed fiscal policy, started by Allan Blinder and Robert Solow,

Chapter 5 extends the analysis to an open economy, where domestic goods are imperfect substitutes for foreign goods, imports and exports depend on the terms of trade, current account deficits correspond to accumulation of indebtedness to foreigners, and the level of the exchange rate depends on the level of indebtedness.[11] For simplicity, the complications associated with the capital stock and the government debt, discussed in earlier chapters, are ignored here. It is shown that the level of indebtedness will matter for the question of stability: under both monetary and fiscal policy too large a level of indebtedness will cause the economy to respond in an unstable fashion. Assuming indebtedness is within this limit, both policies will be stable provided the current account strengthens in response to an increase in indebtedness. However, the two policies can in principle bring about quite different adjustment paths. For either one, use of the interest rate instrument in this simple model will work initially by appreciating the real exchange rate, possibly worsening the current account in the process. Use of tax-financed government expenditures, on the other hand, will typically improve the current account, at least initially. If the initial position of the economy is not thought to be in equilibrium with respect to the current account or the exchange rate, then a preference for one or the other policy should thus depend, at least in part, on whether the exchange rate is thought overvalued or undervalued. Furthermore, as the effectiveness of policy requires that the indebtedness to foreigners not exceed a certain proportion of income, in some situations one may prefer

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"Does Fiscal Policy Matter?", Journal of Public Economics 2 (November 1973), pp. 319-337, and carried on by a number of authors. Note that fiscal policy is stable here because government spending is allowed to vary endogenously, rather than being fixed as in Blinder-Solow. Note also that stability properties depend crucially on whether bonds are net wealth or not (see Bennett McCallum, "On Macroeconomic Instability from a Monetarist Policy Rule", Economics Letters 1 (1978), pp. 121-124); we assume that they are.

11 This analysis draws on Marcus Miller and Willem Buiter, "Monetary Policy and International Competitiveness", Oxford Economic Papers 33 (Supplement July 1981), pp. 143-175, but goes beyond their work in allowing for accumulations of international indebtedness.

policies that decrease current account deficits over those likely to approach that limiting value.

Finally, Chapter 6 presents a simulation model which incorporates the features of all the previous chapters, and in which bonds, capital and claims on foreigners all appear as components of wealth. This general model is too complicated to be treated analytically. Simulations are performed, for plausible parameter values, that illustrate the conclusions of the simple models, and in some cases qualify them.

Before getting into the body of the paper a few words are in order about the nature of the models that are analyzed and the consequent limitations on the generality of the conclusions. First, and probably most important, is the observation that there is stickiness in the determination of wages and prices, and that expectations are not formed rationally. If one assumes otherwise, then policy questions are very much less difficult. With no price stickiness and with rational expectations, the rate of inflation can be lowered without requiring any real variables to differ from their full employment values. Gradualism would have no justification: the only sensible policy would be to lower the rate of growth of money immediately to its non-inflationary level -- the rate of inflation would respond without any delay. Such a world does not resemble the world we live in. Price determination where the actual rate of inflation depends on people's expectations and on the degree of slack in the economy, and where people have to be shown that inflation is coming down before they lower their expectations of it, seems the most fruitful context for policy analysis.

Second, because the focus of the study is on the process of transition to lower inflation, and on the dynamics of the adjustment from one steady state to another rather than on the steady state properties themselves, I have excluded the possibility that the real variables in steady state equilibrium depend on the rate of growth of the money supply. In other words, money is superneutral in the models of this study. This property has been ensured by the assumption that money balances are not considered by households to be net wealth but are created by transfers from one household to another. Moreover, other reasons for the non-neutrality of inflation -- for instance, distortions due to historical-cost depreciation and deductibility of nominal borrowing costs, as well as taxability of nominal capital gains -- are not captured by the model.

Despite these caveats, the insights provided by the analysis in this paper are thought to be significant since they highlight important differences in the way monetary and fiscal aggregate demand policies operate on the economy. An obvious extension, though not attempted here, is to consider a combination of the two policies to achieve several targets. The author however believes that one should view "optimal" policy choice with considerable scepticism.

Chapter 1

NOTATION AND METHODOLOGY

The analysis in this study is built upon a basic model where there is an equation for aggregate demand, a Phillips curve, an equation for expectations of inflation, a money demand equation, and the government budget constraint. Aggregate demand is composed of consumption, which depends on real disposable income and real wealth, investment, government spending, and, in the open economy version, net exports. Wealth is variously equal to holdings of the physical capital stock, the real value of government bonds, or the real value of net financial claims on foreigners.

The basic model is generalized in steps, and is augmented by various equations. When capacity is endogenous, there are additional equations for the capital stock and for aggregate supply, the latter a function of the actual capital stock and (exogenous) labour supply. In the open economy model, there are additional equations for the exchange rate, exchange rate expectations, and the change in claims on foreigners.

The dynamics of the model result from three sources: expectations allied with sticky prices, asset accumulations, and costs of adjustment that make instantaneous adjustment of factors of production uneconomic. One could also make policy instruments move gradually to hit intermediate targets; for instance, the interest rate could move only part of the way towards the level needed to achieve desired money growth.[12] This was not done, however; intermediate targets are exactly achieved, though they themselves may be moving or allowed to move only gradually to the path consistent with price stability.

The gradual adjustment is modelled as a system of differential equations. The dynamic analysis is performed by calculating the eigenvalues of the system of differential equations; however, since

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12 Such policies were considered by Charles Freedman, "Gradualism -- Theory and Simulations", paper presented at Conference on Issues in Public Policy (I), Queen's University, April 16-18, 1978.

the system of equations is non-linear, it is necessary first to linearize the model. The non-linear model behaves locally in the same way as its linearized version.[13]

Following the methodology of Aoki,[14] we linearize around a reference path, which we take to be the final steady state solution of the model. In this steady state, output is at capacity and actual and expected inflation are both equal to the money growth rate, assumed constant. For convenience, we take the growth rate of real variables to be zero, both in the initial and in the new steady state, the latter exhibiting lower inflation. Following Aoki's notation, ${}_rX$ stands for the deviation of X from its reference path value $\bar{X}$:

$${}_rX = X - \bar{X}$$

For small deviations, the $_r$ operator can be treated as a derivative, so the rules for manipulating it are as follows:

$${}_r(X + Y) = {}_rX + {}_rY$$

$${}_r(XY) = \bar{X}{}_rY + \bar{Y}{}_rX$$

$${}_r\ln X = {}_rX/\bar{X}$$

In what follows, we will drop the use of a bar over a level to indicate the final steady state; it will be understood that in all discussions of linearized models, $_r$ refers to deviations from steady state, the variable itself to its steady state value.

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13 Such models are now commonplace in economics. Some useful references are A.W. Phillips, "Stabilization in a Closed Economy", Economic Journal, 64 June (1954), pp. 290-323, A.R. Bergstrom, The Construction and Use of Economic Models, (London: English Universities Press, 1967), and S. Turnovsky, Macroeconomic Analysis and Stabilization Policy (Cambridge: Cambridge University Press, 1977).

14 Masanao Aoki, Optimal Control and System Theory in Dynamic Economic Analysis, (Amsterdam: North-Holland, 1976), pp. 59-68.

Another notational convention will be the use of D to stand for the time derivative operator, so

$$DX(t) = \frac{d}{dt} X(t)$$

It should be noted that the D and δ operators commute, so

$$\delta DX = D\,\delta X$$

The models we consider can be written

$$DX = f(X,Y) \tag{1}$$

where X,Y are vectors of endogenous and exogenous variables, respectively, and f is a vector-valued function. We linearize (1) to obtain

$$D\,\delta X = F\,\delta X + G\,\delta Y \tag{2}$$

where $F = (\delta f_i / \delta X_j)$ and $G = (\delta f_i / \delta Y_j)$, the derivatives being evaluated at new steady state levels. Now the stability of (2) will depend on the sign of the eigenvalues of F, where the eigenvalues are the values of λ satisfying the determinantal equation

$$|F - \lambda I| = 0 \tag{3}$$

In fact, it is easier to rearrange (2) to give

$$(D.I - F)\,\delta X = G\,\delta Y \tag{2'}$$

and treat D as the unknown λ, so that solving for the eigenvalues involves taking the determinant of the matrix on the left of (3). This will give a polynomial in D of the same order as the number of first order differential equations in the model.

The experiment that interests us is causing a step change in one of the Y's, namely, the rate of growth of money (or nominal income). Since we linearize around the new steady state path, the value of each of the δY's after the shock is zero. We therefore have a homogeneous system of differential equations, and the difference in the rate of growth of nominal magnitudes between the two steady states shows up in the initial conditions. The path for the δX's will be described by the sum of exponentials, corresponding to the roots of (3). If, for instance, the system of differential equations is of the third order, the path of each endogenous variable will be described by[15]

$$\delta X_i(t) = C_1 H_{1i} e^{\lambda_1 t} + C_2 H_{2i} e^{\lambda_2 t} + C_3 H_{3i} e^{\lambda_3 t} \qquad (4)$$

where C_1, C_2 and C_3 are constants determined by initial conditions and H_{ij} is the j'th element of the eigenvector associated with λ_i. Now the λ_i can be real or complex. If there are complex roots, then they come in pairs, and each pair has the form

$$a \pm bi$$

where $i = (-1)^{\frac{1}{2}}$. If the system described by (4) has a pair of complex eigenvalues, say

$$\lambda_1 = a + bi \text{ and}$$

$$\lambda_2 = a - bi$$

then (4) can be written

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15 See a textbook on differential equations, for instance, Lyman M. Kells, Differential Equations: A Brief Course with Applications (Toronto: McGraw Hill, 1968), Chapter 6.

$$\delta X_i(t) = A_i e^{at} \sin(bt+B_i) + C_3 H_{3i} e^{\lambda_3 t} \tag{5}$$

where A_i and B_i are real numbers that are functions of C_1, C_2, H_{1i} and H_{2i}.

Since we have linearized the system about the new steady state, we require for stability that

$$\lim_{t \to \infty} \delta X_i(t) = 0$$

and it can be seen from (5) that this requires that a and λ_3 be less than zero, that is, the real part of each eigenvalue has to be negative. We will examine the response of the model with either monetary or fiscal policy to see if it is stable.

In addition, even if the dynamics are stable, it is of interest to discover whether, after an initial peak or trough, the adjustment is monotonic (all roots are real), or cyclical (some roots are complex), and the nature of the cycles, if they exist. The real and imaginary parts of the complex eigenvalues give information about the nature of the cycles. From (5), one can see that the period of the cycles will be $2\pi/b$, that is, the smaller $|b|$ is, the longer are the cycles, and the larger $|a|$ is, the faster they die out. Clearly one wants the cycles to be heavily damped. Note that the amplitude of the cycles will also depend on the interaction of initial conditions and the parameters of the model. This then is the method of analysis that we use: to look at what different policies imply for the eigenvalues of the model.

Chapter 2

MONEY STOCK VERSUS NOMINAL INCOME AS INTERMEDIATE TARGETS

The first question we will address is whether there is a reason to prefer either the money stock or nominal income as intermediate target in feedback control rules. The two are sometimes not distinguished, at least not if the income elasticity of the demand for real balances is unity. If nominal income is growing "too fast", then at unchanged interest rates so is money demand, provided the targets for the two are compatible. Therefore use of money as an intermediate target will force the central bank to raise interest rates, which in turn will decrease aggregate demand and bring down nominal income growth. Thus targetting on money or nominal income will have much the same effect, or so the argument goes. However, we will show below that, other things being equal, targetting nominal income and targetting money give in fact quite different dynamic responses.

We start by specifying our basic model. Aggregate demand is equal to consumption, which depends on real disposable income, Y-TAX, and wealth, V; on investment, which is negatively related to the real interest rate, r; and on real government spending, GOV:

$$Y = c(Y\text{-}TAX,\ V) + I(r) + GOV \qquad (1)$$

For our purposes here V is exogenous; later chapters will endogenize it. We can also substitute out for TAX using the government budget constraint, since here government expenditures are financed entirely by taxes and there are no government bonds outstanding so TAX = GOV. We assume, for now, that capacity output is exogenous at the value Y^c, and that unused capacity causes actual inflation DlnP to differ from expected inflation:

$$DlnP = \pi + \phi\ (lnY/Y^c) \qquad (2)$$

Expected inflation adapts to actual inflation as follows:

$$D\pi = \beta(D\ln P - \pi) \quad (3)$$

Finally, there is a demand for narrow money equation of the form

$$\ln M = \ln P + \rho \ln Y - \gamma(r+\pi) \quad (4)$$

All parameters ϕ, β, γ, and ρ are positive; the derivatives of functions have the following signs:

$c_1>0$, $c_2>0$, $I'<0$

In addition, $c_1<1$

The model, as we have formulated it, has two possible intermediate targets, money or nominal income growth, and two possible instruments, the real interest rate[16] or tax-financed government spending. We can clarify the model somewhat by describing what happens in each of the four cases.

If money is being targetted and the real (or nominal) interest rate is the instrument, then r moves so that the demand for money, equation (4), yields the desired money stock. Taxes and government spending are exogenous, but movements in r will change aggregate demand as described by (1) and this will open up a gap between actual output and potential output, affecting price changes and expected inflation via (2) and (3). All three variables will feed back on to money demand, inducing further movements in r.

If money is being targetted using real government spending, the linkage operates solely through aggregate demand, as neither GOV nor TAX appears directly in the demand for money function. In order to lower money growth, government spending (and taxes) are cut back, lowering real income and prices. These lower the demand for money over time. What is

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16 The model could just as well have been written in terms of the nominal rate R, in which case $R-\pi$ would appear in the investment function.

assumed about monetary policy in this case is that the central bank passively lowers nominal interest rates in line with expected inflation so that r stays constant.

If nominal income is targetted then the demand for money equation can be ignored, as M does not appear in the other equations. However, money demand would eventually grow at the same lower growth rate as nominal income. Both instruments r and GOV would operate on real aggregate demand via (1), and the rate of inflation would vary as a function of the output gap and inflation expectations. Thus a lowering of the nominal income target would initially be achieved primarily through lower real income; the slack that was generated would put downward pressure on the inflation rate until a point was reached where it was equal to the targetted nominal income growth (recall that real income growth is assumed zero in steady state).

In order to examine formally the behaviour of the system under the different policy rules, we linearize it around the new steady state and express it in terms of logs of all variables except the interest rate and the rate of inflation. The system of equations corresponding to (1)-(4) in deviation form is as follows:

$$\delta \ln Y = \eta \delta \ln GOV - i \varepsilon \delta r/(1-c_1) \qquad (1')$$

$$D \delta \ln P = \delta \pi + \phi(\delta \ln Y - \delta \ln Y^c) \qquad (2')$$

$$D \delta \pi = \beta(D \delta \ln P - \delta \pi) \qquad (3')$$

$$\delta \mu = D \delta \ln P + \rho D \delta \ln Y - \gamma(D \delta r + D \delta \pi) \qquad (4')$$

Equation (1') results from differentiating (1) and imposing $\delta V=0$, $\delta GOV = \delta TAX$, and dividing through by Y:

$$\delta Y/Y = c_1(\delta Y/Y - \delta GOV/Y) + I' \delta r/Y + \delta GOV$$

If we write the ratios of government spending and investment to output as η and i, respectively, and let $\varepsilon = -I'/I$ be the proportionate change in investment due to a change in the interest rate, in absolute value, then (1') results. Equations (2') and (3') are straightforward; the term in $\delta \ln Y^c$ is of course zero here since capacity is assumed given (it is made endogenous in the next chapter). Equation (4') results from time-differentiating (4) first, making the left-hand side variable the rate of growth of money, μ. Now it is convenient to add another equation to describe the rate of growth of nominal income ν:

$$\delta\nu = D\delta \ln Y + D\delta \ln P \tag{5}$$

It is worth noting here that provided $\delta\mu = \delta\nu$, nominal income targetting can also be nested in equation (4'), since if $\rho = 1$ and $\gamma = 0$ then (4') is equivalent to (5). We will use this property below. It is assumed that we start from a position in steady state where the previous target was achieved, so that new targets for the path of either money or nominal income can be expressed equivalently as rate of growth targets.

We thus suppose that initially inflation, nominal income growth and money growth are equal to some common value. The authorities lower their target for either μ or ν by, say, one per cent, so that $D\delta \ln P$ and $\delta\pi$ equal .01 initially. How does the system converge to its new equilibrium where $\delta\pi = 0$, under either choice of intermediate target?

Suppose we use real interest rates as the instrument to hit either target. The system of equations can in each case be reduced to two equations in two unknowns, r and π. Substitution of (2') into (3') expresses the change in expectations of inflation solely in terms of the output gap:

$$D\delta\pi = \beta\phi(\delta \ln Y - \delta \ln Y^c) \tag{6}$$

Substitution of the equation (1'), eliminating Y, and setting $\delta \ln Y^c = 0$ yields

$$D\delta\pi = -[\beta\phi i\varepsilon/(1-c_1)]\,\delta r \qquad (7)$$

Now if money is being targetted, equation (4') is relevant, and we can substitute (1'), (2') and (6) into it to yield an equation for the change in interest rates required to hit the money target (note that $\delta\mu = 0$ since our deviations are taken with respect to the new steady state, and we assume that the target is exactly achieved period by period):

$$[\rho i\varepsilon/(1-c_1) + \gamma]\,D\delta r = \delta\pi - [\phi(1-\beta\gamma)i\varepsilon/(1-c_1)]\,\delta r \qquad (8)$$

If on the other hand nominal income is targetted, substitution into (5) yields an alternative feedback rule for the real interest rate:

$$[i\varepsilon/(1-c_1)]\,D\delta r = \delta\pi - [\phi i\varepsilon/(1-c_1)]\,\delta r \qquad (9)$$

Figures 1 and 2 present phase diagrams for money and nominal income targetting, respectively. The line $D\delta\pi = 0$ gives the locus of points for which expectations of inflation are unchanged. Only if the real interest rate is at its equilibrium level can this be so; if it is above, then there is slack in output markets and this puts continuing downward pressure on observed $D\delta \ln P$ and hence on expectations. The line $D\delta r = 0$ gives the combinations of δr and $\delta\pi$ values for which the real interest rate need not change in order to keep to target. In the money case, for a given value of π the effect of r on the target growth rate comes through two channels: higher r means more slack which lowers $D\ln P$ and money growth, but also downward pressure on expectations of inflation and on the nominal interest rate which raises money growth. If the latter effect dominates then the locus of (δr, $\delta\pi$) points will be downward sloping as in Figure 1b. This case is unstable, as we shall see below. It corresponds to the case where $1-\beta\gamma < 0$, that is, where the Cagan stability condition is not satisfied. Money demand responds too strongly to the nominal interest rate and hence to inflation expectations, which are themselves too sensitive to the output gap. Even if the Cagan condition is satisfied, money targetting may well produce cycles in r, π

Figure 1

MONEY STOCK CONTROL

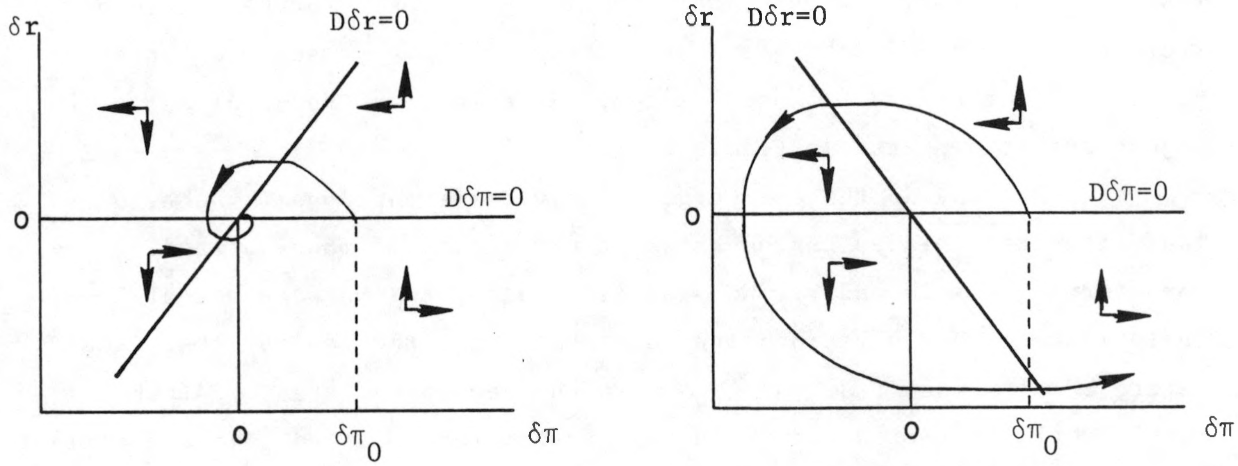
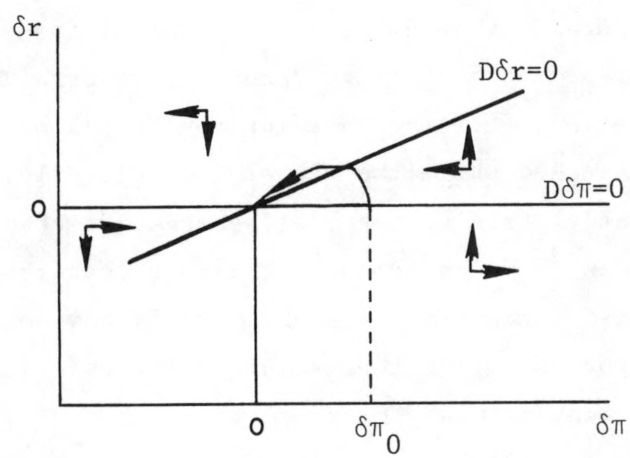


Figure 1a: $1-\beta\gamma>0$

Figure 1b: $1-\beta\gamma<0$

Figure 2

NOMINAL INCOME CONTROL



and the other endogenous variables: this is the situation depicted in Figure 1a.

If nominal income is targetted, on the other hand, the system is necessarily stable, as we shall see below. Furthermore, the slope of the $D\delta r = 0$ locus is less than in the money case, and hence the possibility of cycles is more limited. An example of non-cyclical adjustment is depicted in Figure 2.

Why do the two targets imply different dynamic behaviour? The difference relates to the interest elasticity of money demand. Consider Figures 1a and 2. We start from a gap $\delta\pi_0$ between actual inflation and the inflation rate consistent with the new targets. Real interest rates must rise so as to meet the new money target. After a little while inflation and inflation expectations respond. In the nominal income case, real interest rates initially rise more sharply, but they rise only so long as the rate of inflation is greater than what it will be in steady state. At the point where they are equal, output is below potential by just enough to offset the gap between inflation expectations, which lag behind, and steady state inflation. Maintaining this gap puts continuing downward pressure on actual inflation, and in fact from this point on the gap (and hence real interest rates) can be gradually reduced.

When money is targetted the reversal of direction of real interest rate movements must wait longer. At the point where actual inflation has reached its steady state level (the intersection with the $D\delta r = 0$ curve in Figure 2) the expected rate of inflation is still falling, hence money demand is rising, ceteris paribus. This must be offset by a continuation of rising real interest rates as in Figure 1a, until the point is reached where the actual rate of inflation is far enough below its steady state value to offset the effect of decelerating inflation expectations on money demand. The fact that real interest rates have continued to rise further means that there is now downward pressure on inflation and an overshooting that requires reversal later. Hence the likelihood of a cyclical path as in Figure 1a.

We can formally analyze the dynamic behaviour of the use of interest rates to hit either of the targets by calculating the characteristic equations in the two cases. We have a system of two first order differential equations in each case, equation (7) and either equation (8) or equation (9), for money and nominal income targetting respectively. Money targetting produces the following characteristic equation:

$$[\rho+\gamma(1-c_1)/i\varepsilon]\, D^2 + \phi(1-\beta\gamma)\, D + \beta\phi = 0 \tag{10}$$

while nominal income targetting yields

$$D^2 + \phi D + \beta\phi = 0 \tag{11}$$

It is necessary and sufficient for stability that all three coefficients have the same sign. Since all parameters are positive, it is easy to see that in the case of (10) this will follow if and only if

$$(1-\beta\gamma) > 0$$

the Cagan stability condition. From (11), it is clear that nominal income targetting is always stable in this simple model.

Assuming that money targetting is stable, we can go on to examine the possibility of cycles. They will exist if the discriminant of the characteristic equation is negative. It is clear from the equations above that if $\rho = 1$, money targetting must produce cycles if nominal income targetting does, but not vice versa, and only if money demand is not a function of the interest rate are the two equivalent, as mentioned above. Nominal income targetting using the interest rate as instrument is cyclical provided

$$\phi^2 - 4\beta\phi < 0$$

It is more likely to be cyclical the greater the response of expectations to actual inflation and the less the response of actual inflation to the output gap. In the case of money targetting, cyclicality only requires

$$\phi^2(1-\beta\gamma)^2 - 4\,[\rho+\gamma(1-c_1)/i\varepsilon]\beta\phi < 0$$

a much weaker requirement given likely values of the parameters.

Supposing both are cyclical it can be shown that cycles are more damped in the case of nominal income targetting. Given a negative discriminant in both cases, the real parts of the roots corresponding to (10) and (11) are

$$-\phi(1-\beta\gamma)/2[\rho+\gamma(1-c_1)/i\varepsilon] \qquad (12)$$

and

$$-\phi/2 \qquad (13)$$

respectively. If the income elasticity of the demand for money is unity, then clearly (13) is a more negative number than (12), meaning a larger damping factor in the case of nominal income targetting. Only in the implausible case of a very low income elasticity would this result be reversed. To see this, assume $\rho = 0$. Then (12) could be more negative than (13), but only if

$$i\varepsilon(1-\beta\gamma)/\gamma(1-c_1) > 1 \qquad (14)$$

Hence one would be controlling money without any direct income effects on money demand, income operating only through the effect of slack on prices. If income moved strongly in response to interest rate changes because investment was very sensitive to interest rates and the multiplier was large, then such a situation might result in an offset to the tendencies toward undamped fluctuations resulting from interest-sensitive money demand.

It is thus clear that using real interest rates to target money has some undesirable effects compared to nominal income targetting. What is the situation when tax-financed government spending is the instrument used to hit either target? To answer this question we go back to equations (1')-(4') and (5). Here the solution is easier because GOV only appears in one equation, that for aggregate demand, and its effect only works through the output variable. Since $\delta r = 0$ by assumption the path for $\delta \ln Y$ gives a unique path for $\delta \ln GOV$, so we need not use (1') in analyzing the dynamics; we can condense the system to two differential equations in $\delta \ln Y$ and $\delta \dot{\pi}$. The characteristic equations that result are the following, for <u>money targetting</u>

$$\rho D^2 + \phi(1-\beta\gamma)D + \beta\phi = 0 \tag{15}$$

and for <u>nominal income targetting</u>

$$D^2 + \phi D + \beta\phi = 0 \tag{16}$$

Interestingly enough (16) is identical to (11). In this simple model, if nominal income is targetted, interest rates and government spending produce the same dynamics because each enters the model solely by influencing demand, and the path for output must be the same in the two cases. This equivalence will not hold when we expand the model to include asset stocks, as we will see below, because the two instruments will affect them differently.

Getting back to a comparison of the dynamics of different targetting rules using government spending as instrument, it is clear from (15) and (16) that once again nominal income targetting is stable while money targetting is so only if the Cagan condition holds. Similarly, if money demand is unit elastic with respect to income, money targetting is more likely to exhibit cycles and, if so, the cycles will be less damped.

In conclusion, in this world without lags in reporting, data problems, or lags in the aggregate demand effects of monetary and fiscal policy instruments, one should prefer nominal income over money as a target. Aside from the caveats implicit in the preceding sentence, it is also true that targets are not rigidly adhered to, and it is target ranges, and not point estimates, that are specified, leaving some flexibility in the movement of the instrument. These considerations imply that flexibility is certainly useful in money targetting, and that one should allow some overshoot of money targets during downturns of real activity and undershoots during booms. They also imply that there should be explicit consideration when fixing the width of target bands of the optimal amount of "give" that would be desirable. Operation of monetary targetting in this fashion would help give some of the advantages of nominal income targetting, though retaining the controllability and observability advantages of narrow money.

Chapter 3

A COMPARISON OF THE DYNAMIC EFFECTS OF DIFFERENT INSTRUMENTS IN A CLOSED ECONOMY WITH CAPACITY ENDOGENOUS

A natural extension of the discussion in the previous chapter would be to ask, suppose we choose either money or nominal income as intermediate target, which instrument should we use, real interest rates or real government spending? Is there a clear preference for one or the other, in terms of a more stable dynamic adjustment or more damped cycles?

From the simple model of the last chapter, we recall that for nominal income targetting both instruments would give the same dynamic adjustment, and real income and inflation would be identical in the two cases. The paths of the money supply would not be the same, except when steady state was reached, but this would not matter as money supply would not be a target variable. If money were targetted, however, the dynamics would differ.[17] If we define a symbol q as follows:

$$q = \gamma(1-c_1)/i\varepsilon \qquad (1)$$

which is a positive quantity, then the pair of roots for either instrument can be written as

$$\lambda = \frac{-\phi(1-\beta\gamma) \pm (\phi^2(1-\beta\gamma)^2 - 4(\rho+q)\beta\phi)^{\frac{1}{2}}}{2(\rho+q)} \qquad (2)$$

where q = 0 when tax-financed government spending is used, but q takes the value given by (1) when real interest rates are used. Now all parameters are positive, so it is clear that the roots have negative real parts provided the Cagan stability condition holds. We assume that this is the case.

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17 Compare equations 10 and 16 in Chapter 2.

What is the effect of a non-zero q? Suppose that use of government spending produces a smooth adjustment, that is, the roots are real, so the discriminant is positive. A non-zero q will decrease the value of the discriminant, and may therefore make it negative. Thus real interest rates are more likely than government spending to produce a cyclical adjustment towards equilibrium when used to lower the demand for money.

What if government spending itself produces a cyclical adjustment? It can be seen that in this case, when there are complex roots, a higher q will give a smaller real part. Therefore, cyclical fluctuations will be less damped and hence will persist longer for interest rates than for government spending.

The model used so far has made investment flows respond only to interest rates, and not to income as in the standard accelerator framework. Furthermore, the link between investment and the capital stock has been ignored. How does accounting for the endogeneity of the capital stock and, through it, of capacity output change the story? To answer this question we have to expand the model. Let K be the capital stock, which is both a component of wealth and an argument in the production function. Capacity output is assumed equal to the output that can be produced with the actual capital stock and the supply of labour (N), with the latter exogenous:

$$\ln Y^{c} = \alpha \ln K + (1-\alpha)\ \ln N \tag{3}$$

where $0<\alpha<1$.

We also have to specify in more detail the investment function. We posit a simple lagged adjustment of K to the profit-maximizing capital stock:

$$DK = \theta(\alpha Y/(r+d) - K) \tag{4}$$

where d is the depreciation rate, and gross investment is given by

$$I = DK + d.K \tag{5}$$

Now, substituting (5) into our aggregate demand equation, and linearizing, we get

$$(1-c_1)\delta \ln Y = (1-c_1)\eta\delta \ln GOV + (c_2+D+d)k\delta \ln K \tag{6}$$

where $k = K/Y$. Similarly, (4) can be approximated by

$$D\delta \ln K = \theta\delta \ln Y - \varepsilon\delta r - \theta\delta \ln K \tag{7}$$

where $\varepsilon = \theta/(r+d) > 0$.

Suppose we are targetting money. Our system of equations is then, after using (3) to substitute out for Y^c in (2) from Chapter 2,

$$\begin{vmatrix} (1-c_1)\eta & 0 & -(1-c_1) & 0 & (c_2+D+d)k \\ 0 & 0 & -\beta\phi & D & \beta\phi\alpha \\ 0 & -\gamma D & [\rho D+\phi(1-\beta\gamma)] & 1 & -\phi(1-\beta\gamma)\alpha \\ 0 & \varepsilon & -\theta & 0 & (D+\theta) \end{vmatrix} \begin{vmatrix} \delta \ln GOV \\ \delta r \\ \delta \ln Y \\ \delta \pi \\ \delta \ln K \end{vmatrix} = 0 \tag{8}$$

It can be shown that here the characteristic equations for (i) government spending and (ii) interest rate instruments respectively, are

$$A(D)[D+\theta(1-\alpha)] + \theta\alpha\rho D^2 = 0 \tag{9i}$$

and

$$\gamma D^2[(1-c_1)(D+\theta)-\theta(c_2+D+d)k] - \varepsilon\{(1-c_1)[\beta\phi\alpha+\phi(1-\beta\gamma)\alpha D]$$

$$- A(D)(c_2+D+d)k\} = 0 \tag{9ii}$$

where $A(D) = \rho D^2 + \phi(1-\beta\gamma)D + \beta\phi$, which, as we saw in Chapter 2, has roots with negative real parts as long as $1-\beta\gamma > 0$, which is assumed.

Now we showed above that if $\gamma = 0$, the two give the same pattern of dynamic adjustment when the endogeneity of capital is not taken into account. Let us examine (9i) and (9ii) in the light of this special case. Each of these characteristic equations is a cubic. Now necessary and sufficient conditions for all the roots of a polynomial of the form

$$a_0X^3 + a_1X^2 + a_2X + a_3 = 0$$

to have negative real parts are as follows, provided a_0 is arbitrarily chosen to be positive:[18]

$$a_1 > 0,\ a_2 > 0,\ a_3 > 0 \text{ and } a_1a_2 > a_0a_3 \tag{10}$$

These conditions will always be satisfied for (9i).
For (9ii), on the other hand, if we set $\gamma = 0$ and group terms, we have

$$\rho D^3+[\phi+\rho(c_2+d)]D^2 + \phi[\beta+(c_2+d) - \alpha(1-c_1)/k]D$$

$$+ \beta\phi[(c_2+d) - \alpha(1-c_1)/k] = 0 \tag{9ii'}$$

The coefficients of the terms in D^3 and D^2 are clearly positive, but the remaining two are ambiguous. From inspection of the last term, it is clearly necessary for stability that

$$(c_2+d)k/(1-c_1) > \alpha \tag{11}$$

It can only be shown to be sufficient as well for satisfying (10), hence necessary and sufficient for stability, when $\gamma = 0$. The condition (11)

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18 See Paul Samuelson, Foundations of Economic Analysis, (Cambridge, Mass.: Harvard University Press, 1947), p. 432.

can be given the following interpretation: when money is not interest elastic, interest rates operate solely by decreasing the capital stock. Condition (11) ensures that this decrease, via wealth effects on consumption and lower replacement demand, will decrease aggregate demand, after the multiplier has been taken into account, by more than aggregate supply has been decreased. If this condition is not satisfied then an output gap in a perverse direction opens up, and there is upward, not downward, pressure on price changes.[19]

Thus introducing the capital stock, and letting it respond endogenously, but only gradually, to interest rates and income, is a possible independent source of instability when the money supply is being targetted and interest rates rather than government spending are varied to hit the target. What is the situation when nominal income is targetted? In this case the third equation in the system of equations (8) above would be replaced by

$$D\delta \ln Y = (\delta\mu - \delta\pi) - \phi(\delta \ln Y - \alpha\delta \ln K) \tag{12}$$

and the characteristic equations are now, for government spending

$$D^3 + (\phi+\theta)D^2 + \phi[(1-\alpha)\theta+\beta]D + \beta\theta\phi(1-\alpha) = 0 \tag{13i}$$

and for interest rates,

$$D^3 + (\phi+c_2+d)D^2 + \phi[(c_2+d) + \beta - \alpha(1-c_1)/k]D$$

$$+ \beta\phi[(c_2+d) - \alpha(1-c_1)/k] = 0 \tag{13ii}$$

It can be shown that (13i) always has stable roots, while the necessary

19 See Pierre Duguay, "L'influence des taux d'intérêts sur l'offre macro-économique: conséquences pour le contrôle de l'inflation", op. cit.

and sufficient condition for (13ii) to be stable is once again condition (11).

If we assume that the model is stable for use of both interest rates and government spending, what can we say about the nature of the adjustment path as a function of α? Suppose we start with equations (9i) and (9ii) with $\gamma = 0$. The characteristic equations are still complicated, so we shall make further simplifying assumptions. Starting from case i, use of the tax-financed government spending instrument, assume $\theta = \beta$. Then (9i) can be factored as follows:

$$(D+\beta)\ (\rho D^2 + \phi D + \beta\phi\ (1-\alpha)) = 0 \qquad (14i)$$

Since $\alpha < 1$, the quadratic always yields stable roots, as noted above. An increase in α however raises the value of the discriminant,

$$\phi^2 - 4\rho\beta\phi + 4\rho\beta\phi\alpha$$

and hence decreases the likelihood of cyclicality, or, if there are cycles, increases their length. Other things equal, the length of the adjustment period will be longer, however.

Turning to case ii, the active use of the interest rate as policy instrument, we start from (9ii) with $\alpha = 0$ and again make a simplifying assumption, that $\beta = c_2 + d$. The characteristic equation that results is

$$\rho D^2 + \phi D + \phi\ \left[c_2 + d - (1-c_1)\ \alpha/k\right] = 0 \qquad (14ii)$$

It can be seen that condition (11) is necessary and sufficient for stability. An increase in α will increase the possibility of instability if roots are real and lower the speed of adjustment. If roots are complex and there is cyclical adjustment, an increase in α will lengthen the cycles and perhaps make adjustment non-cyclical.

To sum up, the endogenization of capacity does introduce the possibility that use of interest rates to hit a money or nominal income target may be unstable if the linkages between interest rates and aggregate demand are weak compared to aggregate supply effects. The problem does not arise for the tax-financed government spending instrument that we consider here because it operates on aggregate demand only.[20] It is not the case however that the endogenization of capacity tends to introduce cycles; on the contrary, the possibility of cyclicality diminishes as α increases, for both policy instruments, and cycles lengthen.

It is clear that the Cagan stability condition is quite important for the stability of the economy's response to aggregate demand policy when money is targetted, and that even if the condition is satisfied, the interaction of an interest-elastic money demand and expectations that adapt rather quickly to actual inflation will likely produce cycles. This result suggests that a criterion for preferring one monetary aggregate over alternatives, i.e., that it has a high interest elasticity of demand and is thus controllable through moderate variations in interest rates, may have received too much emphasis. Such an aggregate will be controllable in a static sense, when only the LM curve is considered, but it may not be dynamically controllable in the context of a full IS-LM system with an expectations-augmented Phillips curve.

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20 An interesting generalization would allow labour supply and hence capacity output to react to tax changes. Such effects are likely to be secular rather than cyclical, however.

Chapter 4

A CLOSED ECONOMY MODEL WITH BOND FINANCING OF GOVERNMENT DEFICITS

The previous chapter considered a model where the only form of wealth was physical capital, and where the real value of wealth did not depend on the price level. Here we examine a quite different model where wealth is given by bond holdings, which were issued to finance government deficits. Furthermore, we suppose that governments, instead of balancing their budgets by raising taxes, finance any deficits by issuing new short-term bonds paying a nominal interest rate $r + \pi$. In this chapter, we ignore the endogeneity of the capital stock. The objective here is to compare the use of interest rates and real government spending as instruments to lower either nominal income growth or money growth, in the context of deficit financing on the part of governments.[21]

Suppose we revert to the model of Chapter 2 but add government bonds. Then our model would comprise the following equations:

$$Y = c(Y\text{-}TAX + r\,B/P,\ B/P) + I(r) + GOV \quad (1)$$

$$DlnP = \pi + \phi(lnY/Y^{c}) \quad (2)$$

$$D\pi = \beta\,(DlnP-\pi) = \beta\phi lnY/Y^{c} \quad (3)$$

$$DB/P + TAX = GOV + (r+\pi)\,B/P \quad (4)$$

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21 The stability of models incorporating a government budget constraint has given rise to an extensive literature, which I will not attempt to survey here. Recent articles examining various monetary and fiscal policies include "Flexible Policies and IS-LM Dynamics" by Gary Smith, in the Journal of Macroeconomics 4 (Spring 1982), pp. 155-178, and "Bond Financed Fiscal Policy and the Problem of Instrument Instability" by William Scarth, in the Journal of Macroeconomics 1 (Winter 1979), pp. 107-117. However, neither article compares alternative instruments for hitting a money target, nor do they consider a nominal income target.

plus either an equation corresponding to hitting a nominal income target,

$$\upsilon = D\ln P + D\ln Y \tag{5a}$$

or a money supply target

$$\mu = D\ln P + \rho D\ln Y - \gamma(Dr + D\pi) \tag{5b}$$

Let us start for simplicity with a nominal income target, so our model corresponds to (1)-(4) and (5a). We assume that TAX and Y^c are exogenous at their steady-state values. We also assume that in steady state real income is constant and that the inflation rate is equal to the rate of nominal income growth, which is also the rate of growth of bonds.

We proceed to linearize the model around a reference path of steady state growth in all nominal variables, as before, with an additional variable lnB, on the assumption that B is strictly positive. The steady state ratio of bonds to income, (B/P)/Y, is denoted λ in what follows. Only the government budget constraint requires explanation. First, it should be noted that provided $B \neq 0$

$$D\ln(B/P) = D\ln B - D\ln P = (DB/P)/(B/P) - D\ln P.$$

Hence, from (4),

$$D\ln(B/P) = (GOV - TAX)/(B/P) + (r+\pi) - D\ln P$$

$$= (GOV - TAX)/(B/P) + r - \phi \ln Y/Y^c \tag{6}$$

from (2). Proceeding to linearize (6) by taking deviations around the steady state,

$$D\delta\ln(B/P) = -\frac{GOV-TAX}{B/P}\delta\ln(B/P) + \frac{\delta GOV-\delta TAX}{(B/P)} + \delta r - \phi\,\delta\ln Y \tag{7}$$

Now, in steady state $D\ln B = \pi$ so from (4)

$$\frac{GOV-TAX}{B/P} = -r$$

Furthermore, we note that with taxes exogenous at their steady state value,[22] $\delta TAX = 0$; also we write (7) in terms of the logarithm of government spending:

$$D\delta \ln(B/P) = r\delta \ln(B/P) + \frac{GOV}{B/P}\delta \ln GOV + \delta r - \phi\delta \ln Y \tag{8}$$

Now

$$\frac{GOV}{B/P} = \frac{GOV}{Y} / \frac{B/P}{Y} = \eta/\lambda \quad \text{so}$$

$$D\delta \ln(B/P) = r\delta \ln(B/P) + (\eta/\lambda)\delta \ln GOV + \delta r - \phi\delta \ln Y \tag{9}$$

Using (9) and the linearized versions of (1), (3) and (5a) (having eliminated (2)) we write the system of equations as follows:[23]

$$\begin{vmatrix} \eta & (c_1\lambda - i\varepsilon) & -(1-c_1) & 0 & (c_1 r + c_2)\lambda \\ 0 & 0 & -\beta\phi & D & 0 \\ 0 & 0 & (D+\phi) & 1 & 0 \\ -\eta/\lambda & -1 & \phi & 0 & (D-r) \end{vmatrix} \begin{vmatrix} \delta \ln GOV \\ \delta r \\ \delta \ln Y \\ \delta\pi \\ \delta \ln(B/P) \end{vmatrix} = 0 \tag{10}$$

It can readily be verified, by deleting the second and first columns respectively, and expanding the determinant of the matrix on the left-hand side of (10), that the characteristic equation for use of government expenditures is

$$(D^2 + \phi D + \beta\phi)\,[D - r(1 - c_1) + c_2] = 0 \tag{11i}$$

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22 It can be readily verified that the dynamics are not affected by making taxes proportional to income.

23 Cf. equation (8) in Chapter 3.

and for use of interest rates is

$$(D^2 + \phi D + \beta\phi)\,[(c_1\lambda - i\varepsilon)(D-r) + (c_1 r + c_2)\lambda] = 0 \qquad \text{(11ii)}$$

Both have as one of their factors a second-order polynomial which, as we saw in Chapter 2, has roots whose real parts are necessarily negative. The remaining root differs between the two policies. To hit the nominal income target using GOV, the third root is

$$D = r(1-c_1) - c_2 \qquad \text{(12i)}$$

The reasoning behind this result is as follows. Suppose the government debt were to be arbitrarily increased by one dollar, all other things being held fixed, so that on the instant it occurred the government budget constraint did not hold. What changes would this induce in the deficit? Recall that the government is varying its expenditures GOV to hit a nominal income target. Nominal income growth will be unaffected as long as GOV offsets the real demand effects of the debt increase, since π depends on the output gap as in (3). Therefore, to keep nominal income at target, GOV must decrease by the amount that consumption increases, namely $c_1 r + c_2$. Consumption is stimulated by both higher interest flows and greater wealth. The net effect on the government deficit results from both the fall in spending and higher debt service (in real terms, adjusted for the inflation premium, it is $-c_1 r - c_2 + r$). Provided this is negative, an increase in the stock of debt will be self-correcting as it will generate a budget surplus bringing the debt back down.

When interest rates are used the third root is

$$D = -(c_2\lambda + ri\varepsilon)/(c_1\lambda - i\varepsilon) \qquad \text{(12ii)}$$

Since all parameters are positive, the root can only be stable if

$$c_1\lambda > i\varepsilon \tag{13}$$

However, what (13) means is that if the real interest rate is increased holding other variables fixed, its positive effect on consumption through interest earnings will exceed in magnitude the negative effect on investment. That is, tight monetary policy will be stimulative, not restrictive. Otherwise, use of monetary policy will be destabilizing. We can see the reason for this most easily if we start from a position where λ is quite close to zero, so that interest payments are a negligible component of disposable income initially. Tight monetary policy lowers aggregate demand in the usual fashion, tending to depress nominal income. The government deficit however increases because interest payments are higher.[24] Given real government spending and taxes, these interest payments are financed by debt issue. But since government debt is assumed to be a component of household net wealth, the higher debt tends to increase consumption, offsetting partially the required restraint on aggregate demand. As a result, real interest rates must rise further, further increasing the government deficit, and so on.

Clearly use of monetary policy here puts us squarely in a Blinder-Solow world. Fiscal policy, because it forces expenditure to adjust, does not lead to debt feeding upon itself. Consider the effect of restrictive fiscal policy starting from a position where λ is close to zero. Here cutbacks in spending decrease aggregate demand but improve the government balance. Less government debt issue means less net wealth, reinforcing the restrictive aggregate demand effects, and this will be stable provided condition (12i) holds.

The case where the money supply is targetted need not be treated in detail. When fiscal policy is used, the characteristic equation is identical to (11i), except that the leading polynomial is

$$\rho D^2 + \phi(1-\beta\gamma)\, D + \beta\phi \tag{14}$$

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24 This effect is accentuated if taxes depend on income.

This is familiar from Chapter 2, and is stable if the Cagan stability condition holds. For monetary policy, if $\gamma = 0$, then the characteristic equation is identical to (11ii), again with the first polynomial replaced by (14). Hence the same problem of instability arises as when targetting nominal income, unless bond holdings are so large that the effect of raising interest rates is perverse. If γ is greater than zero, there is an even greater possibility of instability, as extra terms make the coefficients D^3 and D^2 in the characteristic equation less positive (or more negative).

Chapter 5

AN OPEN ECONOMY MODEL WITH EXTERNAL DEBT

Going beyond a closed economy model to one where the country in question is small compared to the rest of the world, and where the rest of the world is following different monetary and fiscal policies or is subject to different shocks, means that the nature of the adjustment process is fundamentally changed. The exchange rate becomes a crucial linkage between macro policy settings and both real output and inflation, and monetary and fiscal policies can have quite different effects on the exchange rate. Since the exchange rate is influenced by relative interest rates, monetary tightness may tend to produce a stronger value of the domestic currency, and hence a weaker current account balance, than a reduction in tax-financed government expenditures. These flow effects will tend to accumulate and persist because current account deficits mean accumulation of debt vis-à-vis foreigners and will involve continuing debt service abroad. Increased foreign debt will lower national income and depress aggregate demand, other things being equal. By operating more directly on aggregate demand, fiscal policy should not involve as much movement in the terms of trade or accumulation of indebtedness to foreigners.

The model developed below is a simple open economy model in which the domestically produced good is an imperfect substitute for foreign goods, so that exports and imports depend on the terms of trade, or rather on its reciprocal the real exchange rate, which we shall write as

$$t = P^* e/P$$

where P^* is the foreign price level, and e is the price of foreign exchange. As above in the closed economy models, output will be taken as being demand determined, equal to the sum of consumption, net exports and government expenditure, with the capital stock and investment being ignored here:

$$Y = C + X - t.IM + GOV \quad (1)$$

where IM is measured in units of foreign goods and hence is multiplied by t. Consumption is assumed to be a function of inflation-corrected real disposable income and real wealth, the latter consisting solely of foreign-currency claims (or liabilities, if negative), on foreigners, F:

$$C = c(Y - TAX + r^*(eF/P), \; eF/P) \quad (2)$$

Imports and exports are homogeneous in domestic and foreign income respectively, as well as both depending on the terms of trade:

$$IM = i(t)\, Y \qquad i' < 0 \quad (3)$$

$$X = x(t)\, Y^* \qquad x' > 0 \quad (4)$$

Now the change in claims on foreigners is just the current account, so

$$DF = (P/e)\, X - P^*\, IM + (r^* + \pi^*)\, F \quad (5)$$

The exchange rate is assumed to be governed by interest rate parity, plus a term giving an effect to the net claim position vis-à-vis foreigners, divided by income:

$$(D\ln e)^e = r + \pi - r^* - \pi^* + a_0\, ((eF/P)/Y - a_{00}) \quad (6)$$

This equation can be interpreted as either a demand curve for foreign assets on the part of Canadians where a larger holding of foreign assets results from a higher rate paid abroad than domestically, or as a lending supply curve on the part of foreigners. On the latter interpretation, foreigners would demand a higher rate for assuming a greater risk, as measured by the Canadian indebtedness to output ratio, and the effective borrowing rate would be r* plus this risk premium.[25] Under either interpretation, a_{00} is the equilibrium net foreign asset proportion, and reflects the optimum amount given the stochastic structure of the economy and utility functions of borrowers or lenders.

Exchange rate expectations are assumed to reflect a gradual return to an equilibrium real exchange rate $\bar{t}$ (which may itself be a function of exogenous variables):

$$(Dlnt)^e = a_1 (ln\bar{t} - lnt)$$

But since

$$Dlnt = Dlne + DlnP^* - DlnP$$

$$(Dlne)^e = a_1 (ln\bar{t} - lnt) + \pi - \pi^* \qquad (7)$$

From (6) and (7), we have

$$lnt = (1/a_1)(r^*-r) - (a_0/a_1)((eF/P)/Y - a_{00}) + ln\bar{t} \qquad (8)$$

Finally, the model is rounded out by an open economy Phillips curve where, because of importables, exchange rate movements and foreign inflation enter directly:

$$DlnP = b(\pi + \phi lnY/Y^c) + (1-b)(Dlne + DlnP^*) \qquad (9)$$

a demand for money equation,

$$lnM = lnP + \rho lnY - \gamma(r+\pi) \qquad (10)$$

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25 In this case disposable income would be lower by the amount of the risk premium times the amount of indebtedness, and the stability conditions presented below would be modified.

adaptive expectations on inflation,

$$D\pi = \beta(D\ln P - \pi) \quad (11)$$

and the government budget constraint,

$$GOV = TAX \quad (12)$$

where it is assumed for simplicity that governments do not issue bonds but finance their expenditures solely by taxes. The money supply is created by transfers, so it does not affect the budget constraint.

Now, it is convenient to define a new variable, real net claims on foreigners:

$$f = e\ F/P$$

and to note that

$$Df = f(D\ln t - D\ln P^*) + (eDF)/P$$

We further assume that for the foreign country,

$$D\ln P^* = \pi^*,$$

that is, foreign inflation is correctly anticipated. This permits us to write (5) as

$$Df = (r^* + D\ln t)\ f + x(t)\ Y^* - t\ i(t)Y \quad (13)$$

Next, substituting (12) into (2), and (2)-(4) into (1), we have

$$Y = c(Y - GOV + r^*\ f, f) + x(t)\ Y^* - t\ i(t)\ Y + GOV \quad (14)$$

Now, substituting the expression for Dlnt into (9) and solving for

DlnP, we get

$$DlnP = \pi + \phi \ln(Y/Y^c) + h\ Dlnt \tag{15}$$

where h = (1-b)/b. Hence, from (15) and (11),

$$D\pi = \beta\phi \ln(Y/Y^c) + \beta h\ Dlnt \tag{16}$$

Differentiating (10) and substituting (15) for DlnP and (16) for Dπ, we have

$$\mu = DlnM = \pi + \phi \ln(Y/Y^c) + h\ Dlnt + \rho\ DlnY$$
$$- \gamma(Dr + \beta\phi \ln(Y/Y^c) + \beta h\ Dlnt) \tag{17}$$

Finally, we rewrite (8) as

$$\ln t = (1/a_1)(r^* - r) - (a_0/a_1)(f/Y - a_{00}) + \ln\bar{t} \tag{18}$$

Equations (13), (14), (16)-(18) make up our system of five equations in five endogenous variables f, Y, π and t, plus either r or GOV, one of which is set in order to achieve a target growth rate for money or for nominal income. The latter intermediate target can in fact be treated formally in the same way by using (17) and supposing that $\rho = 1$ and $\gamma = 0$. As we saw in Chapter 2 then money and nominal income targetting procedures are identical.

The model is non-linear, and we once again linearize around a reference path, in particular the new steady state, where all real variables are constant, the price level grows at μ, and the nominal exchange rate grows at the difference between money growth rates at home and abroad so the real exchange rate is constant.

It can be shown that in the new steady state with lower money growth and inflation, all real variables except real money balances return to their previous equilibrium values. Suppose that the real interest rate is held constant at the world rate. In equilibrium $t = \bar{t}$, and f/Y must be equal to a_{00} as implied by (18). Output itself is equal to capacity output, which is exogenous; if it were endogenous, it would equal its previous steady state value since r is unchanged. Now in the absence of growth in steady state all real variables must be constant, so $Df = 0$ and $D\ln t = 0$. Equation (13) then forces the real exchange rate t to be such that interest and dividend inflows are offset by net merchandise imports. Since each of these is unchanged from its previous steady state value, so is the real exchange rate. Looking now at aggregate demand, equation (14), it must equal aggregate supply, implying that real government spending must equal its previous equilibrium value. Alternatively, if GOV is kept at its previous value, a similar argument shows that r and other real variables must be unchanged from one steady state to another. On the other hand, there is a linkage in this model between a higher real interest rate and larger real government spending, and if both are allowed to change, other real variables also change from one steady state to another.

The linear model and its characteristic equations, when either the real interest rate or tax-financed government spending is the policy instrument, are presented in Appendix A. Although the dynamics are in fact quite complicated, the nature of the adjustment paths can be clarified by considering a few special cases.

First, consider an economy in which the effect of foreign goods prices in the determination of the domestic output price is negligible, so that $h = 0$ and where money demand is interest inelastic, so that $\gamma = 0$. In such a world monetary and fiscal policy operate in diametrically opposite ways when used to bring down inflation. Lower government expenditure initially works by decreasing domestic demand, while higher interest rates work by diverting domestic and foreign demand toward foreign goods.[26] In other words, fiscal policy is expenditure

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26 There are subsequent feedbacks that blur the distinction somewhat.

reducing, while monetary policy is expenditure switching, to use Harry Johnson's useful distinction. This is so because, in order to decrease money demand by a given amount, both policies must reduce real output Y. Monetary policy does so by raising the real exchange rate, which lowers net exports; fiscal policy does so by reducing government spending, which through the multiplier also reduces consumption. The two policies have opposite effects on the current account, as the former reduces net exports, while the latter, because output falls less than absorption at unchanged terms of trade, actually improves the current account.

Turning now to a formal derivation of the dynamics of adjustment of this model with $h = \gamma = 0$, it can be seen from Appendix A that the characteristic equations corresponding to both instruments have a factor in the following polynomial,

$$A(D) = \rho D^2 + \phi(1-\beta\gamma)D + \beta\phi \qquad (19)$$

which has two stable roots provided the Cagan stability condition holds, that is,

$$1 - \beta\gamma > 0 \qquad (20)$$

Since $\gamma = 0$ this is necessarily true. The remaining characteristic root differs in the two cases, however. For use of government spending (case i), it is

$$D = \frac{r^* - a_2 a_0/a_1}{1 + g a_0/a_1} \qquad (21ii)$$

while for use of real interest rates (case ii), it is

$$D = \frac{r^*(1-c_1)-c_2}{1+g(c_1 r^* + c_2)/a_2} \qquad (21ii)$$

Recall that all of the parameters a_0, a_1, a_2, c_1 and c_2 are positive; g, which is the ratio of foreign assets to domestic output, can be either positive, for a net creditor country, or negative, for a net debtor country. We shall presume, however, that if the country is a net debtor then its indebtedness is not large enough to make the denominator negative in (21i) or (21ii), i.e.,

$$g > - a_1/a_0 \tag{22i}$$

or $$g > - a_2/(c_1 r^* + c_2) \tag{22ii}$$

If the denominators are indeed positive then the following conditions must hold in order for the characteristic roots to be stable:

$$r^* < a_2 a_0/a_1 \tag{23i}$$

or $$r^* < c_2/(1-c_1) \tag{23ii}$$

The difference in behaviour can better be understood when we account for the following considerations. First note that with $\gamma = 0$, the path of money growth is uniquely determined by the path of output Y, capacity being fixed. This can be shown by differentiating (17) and substituting out for $D\pi$ using (16), yielding

$$\begin{aligned} D\mu &= D\pi + \phi(D\ln Y - D\ln Y^c) + \rho D^2 \ln Y \\ &= \beta\phi(\ln Y - \ln Y^c) + \phi(D\ln Y - D\ln Y^c) + \rho D^2 \ln Y \\ &= (\rho D^2 + \phi D + \beta\phi)\ln Y - \phi(D+\beta)\ln Y^c \end{aligned} \tag{24}$$

In the case of active fiscal policy, since the path of Y is determined by (24) and GOV is used to keep Y on its path, the aggregate demand equation (14) merely determines GOV. Equations (13) and (18) constitute a dynamic system describing the current account and the real exchange rate.

Linearizing each equation and substituting out for lnt yields

$$(1+ga_0/a_1)D\delta f/Y = (r^*-a_0a_2/a_1)\delta f/Y+(g/a_1)D\delta r^* + (g+a_2/a_1)\delta r^*$$

$$+ (g^2a_0/a_1)D\delta \ln Y + (ga_0a_2/a_1-i)\delta \ln Y+xa_y\delta \ln Y^* \quad (25)$$

The root to (25) corresponds to equation (21i).

Active monetary policy, case ii, must attain the same real income path, but using r, which operates solely by affecting the real exchange rate. Therefore equation (18) just serves to describe the path of r necessary to adhere to the real income path. The values of f and lnt consistent with this path are given by (13) and (14). Again linearizing and substituting out for lnt gives

$$(1+g(c_1r^*+c_2)/a_2)D\delta f/Y = ((1-c_1)r^*-c_2)\delta f/Y$$

$$- (c_1g^2/a_2)D\delta r^* + g(1-c_1)\delta r^*$$

$$+ (g/a_2)(1-c_1+i)D\delta \ln Y$$

$$+ (1-c_1)\delta \ln Y - (g/a_2)xa_yD\delta \ln Y^* \quad (26)$$

The root to (26) corresponds to equation (21ii).

The response to a foreign interest rate shock is also quite different. The linkages between an increase in the foreign interest rate δr^* and the domestic economy are twofold:[27] downward pressure on the real exchange rate; and an increase in our interest earnings from abroad, if we are a net creditor, or in our debt service if we are a net debtor.

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27 Assuming that $D\delta r^* = 0$ and $D\delta \ln Y^* = \delta \ln Y^* = 0$, so that real foreign demand effects are offset by other policies, e.g. fiscal stimulation.

These income changes, like the terms of trade effects, induce a change in aggregate demand, the income change by changing consumption (reducing it if we are a net debtor, increasing it if we are a net creditor), and the terms of trade effects by increasing net exports. Now the use of fiscal policy to hit a money target involves counteracting the aggregate demand effects through changes in government spending, leaving the real exchange rate to seek its own level. It will typically depreciate and the current account will improve, as can be seen from equation (25): the coefficient on δr^* will be positive unless we are such a large net debtor that the effect of the depreciation on net exports is offset by increased debt service and

$$(g+a_2/a_1) < 0.$$

The use of the domestic interest rate as the policy tool, on the other hand, requires the real exchange rate effects of higher foreign interest rates to be offset in order to keep real income on target (recall that in this model real interest rates only operate through net exports, not domestic demand). Therefore the rate only affects the current account via changes in net interest payments from abroad. Consequently, in (26), the coefficient of δr^* will have the same sign as g, and the current account will deteriorate if the country is a net debtor.

It is also instructive to examine the effects on the dynamics of adjustment of a non-zero stock of net claims on foreigners, that is, a non-zero g. It can be shown that if $h = \gamma = 0$, the effect of greater net indebtedness is to increase the speed of return to a steady-state equilibrium, while the effect of greater net claims is to slow that adjustment. Provided conditions (22i) and (22ii) hold and hence the roots are stable for normal values of the parameters, a more negative g makes the roots given by (21i) and (21ii) more negative. The cyclical properties of the dynamic adjustment process are not affected; they are described by equation (19).

Why is there this dampening role for net indebtedness? A positive shock to the stock of net claims on foreigners will feed back onto the domestic currency value of those claims in three ways: 1) interest earnings will increase; 2) the exchange rate will appreciate, and its new level will give a lower value for the flow of net exports; and 3) the change in the exchange rate will induce a change in the stock of claims valued in domestic currency. For a normal range of values for f, including both a net claim position and moderate values of net indebtedness, effect 2) must dominate effect 1) for stability. The role of valuation effects is to attenuate or amplify the net effect of the first two channels; valuation effects cannot dominate because they correspond to a level shift rather than a continuing flow. When the country is a net debtor, a shock to the stock of net claims stimulates domestic demand via the third channel because the value of net claims has been reduced: the increased excess demand that results will tend to reverse the shock to f and bring about a quicker adjustment. The opposite is true for net creditor countries.[28] The exact form of the linkages depends on the policy instrument used, hence the difference in (25) and (26).

The above discussion is also applicable to nominal income targetting, and the same divergent behaviour from use of real interest rates or real government spending will occur. If however we relax the assumption that money demand is inelastic, the adjustment process is more complex, and money and nominal income targetting are not necessarily qualitatively similar. No longer is the path for real income determined solely by the target path for money. Nevertheless, monetary and fiscal policies continue to have potentially quite different incidence on the real exchange rate and on the current account. Equations (25) and (26) still hold for fiscal and monetary policy, respectively, for a given path for real income, but now the latter is no longer recursive if a money target is followed. Nevertheless, use of restrictive monetary policy will tend to give a higher real exchange rate and a lower value of the current

28 Paul Krugman and Lance Taylor, "Contractionary Effects of Devaluation", Journal of International Economics 8 (August 1978), pp. 445-456.

account than fiscal policy. Thus the preference for one policy over the other may well depend in part upon whether, for instance, the exchange rate is thought overvalued or undervalued initially.

When the domestic price level depends directly on foreign prices and the exchange rate, the situation changes considerably. Since interest rates affect the level of the exchange rate, there is a linkage between monetary policy -- that is, the interest rate instrument -- and the rate of inflation that is not open to fiscal policy. It can be shown that conditions (23i) and (23ii) are still necessary for the stability of the model when either government spending or the real interest rate is used to control nominal income or money. The dynamics are considerably more complicated, however, and we return to the nature of these dynamics in the next chapter when discussing the simulation model.

It is interesting to note in passing that the analysis here has pointed to a reason for being concerned about the current account. If the level of indebtedness approaches the region where the response to policies is unstable, then one would want to avoid adding to indebtedness by running deficits. The situation of a net debtor country is analogous to the situation of a firm with a high debt/equity ratio. In a world of perfect certainty, it is clearly desirable to borrow further as long as the interest rate is less than the return on the asset acquired. However, a high debt/equity ratio leaves the company exposed to unexpected shocks, and it may be forced to pursue undesirable policies, e.g., to liquidate assets, if there is an unfavourable demand shift or interest rates rise -- assuming its debt is at a floating rate. The next chapter attempts to quantify the level of indebtedness that could lead to instability and compares it to the level that prevails for Canada.

Chapter 6
SIMULATION MODEL RESULTS

Previous chapters have treated closed economy models with endogenous capacity and with government bond indebtedness, and an open economy model with external debt. Since dynamic analysis was already quite complicated, a model combining all the above features would not have been tractable analytically. Nevertheless, important insights into the roles of monetary and fiscal policy were gained by the limited analysis. The purpose of this chapter is to illustrate them in a more general context incorporating all of the features of the above models. In the process, we shed some light on features of the partial models that are crucial to their interaction in the more general model. As well, we compare money and nominal income control via simulation.

The equations of the model are given in TROLL notation[29] in Appendix Table B-1. The variable names correspond as closely as possible to those in earlier chapters. The basic model is as in Chapter 5, except that capacity output is made endogenous, capital and bonds as well as foreign assets are part of wealth, investment depends on a long rate (RL) which responds with a lag reflecting expectations to the short rate (R), and the government budget constraint allows for bond issue. Variants of the model make either GOV or r exogenous, and make either bonds B or TAX the residual form of financing.

The models considered above have assumed that adjustments take place continuously, so that dynamics are described by differential equations. The usual econometric models work with data that are sampled at discrete intervals, and for which differentials are not observable. This model can be expressed in terms of observables by integration.[30] For instance, if we have an equation of the form

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29 See TROLL Reference Manual, (Cambridge, Mass.: MIT Center for Computational Research in Economic and Management Science, 1978).

30 Procedures for deriving the discrete time analogue to a continuous time model are discussed by A.R. Bergstrom in "Non Recursive Models as

$$DX(t) = aX(t) + bY(t) \qquad (1)$$

integration over some unit interval, say a year, produces

$$\int_{T-1}^{T} DX(t)dt = a\int_{T-1}^{T} X(t)dt + b\int_{T-1}^{T} Y(t)dt \qquad (2)$$

Suppose we have discrete observations at the end of each time period, which we denote as X_T^*, etc. Now the term on the left side of (2) is exactly equal to the change in end-of-year observations on X, $X_T^* - X_{T-1}^*$. Each of the integrals on the right-hand side can be approximated as the average of end-of-year observations, i.e.,

$$\int_{T-1}^{T} X(t)dt = (X_T^* + X_{T-1}^*)/2$$

So (2) can be approximated by

$$X_T^* - X_{T-1}^* = a(X_T^* + X_{T-1}^*)/2 + b(Y_T^* + Y_{T-1}^*)/2 \qquad (3)$$

At this point we make a notational change to make the simulation more easily interpretable in terms of the original differential equations model. We now define

$$DX_T = X_T^* - X_{T-1}^* \qquad (4)$$

and $X_T = (X_T^* + X_{T-1}^*)/2$ (5)

Then our discrete time version of (2) is

$$DX_T = a\ X_T + b\ Y_T \qquad (6)$$

Discrete Approximation to Systems of Stochastic Differential Equations", Econometrica 34 (January 1966), pp. 173-182, and by C.R. Wymer in "Econometric Estimation of Stochastic Differential Equation Systems", Econometrica 40 (May 1972), pp. 565-577.

In addition, we need a link between DX_T and X_T, which, from (4) and (5) we can see takes the form

$$X_T = X_{T-1} + (DX_T + DX_{T-1})/2 \tag{7}$$

These links appear as equations (19)-(26) in Appendix Table B-1. The time unit here is a year, and the simulations were performed over 50 periods.

The model is only illustrative, and we have not tried to estimate the parameters. Rather they were chosen with a vague eye to previous empirical work, and also to be such that the basic model was not unstable when either instrument, real interest rates or tax-financed government spending, was used to hit either a money or a nominal income target. In particular, the importance of the Cagan stability condition must be stressed here. The interest semi-elasticity of money demand, usually estimated for M1 to be around 2, could easily combine with a speed of adaptation of expectations greater than .5 to produce instability when money is targetted. These parameter values would not however involve instability when nominal income was targetted. Our choice of parameters gives $\beta\gamma = 0.8$, so the Cagan condition is satisfied. Appendix Table B-2 gives the parameter values, the steady state values of real variables, and the starting values of nominal variables. The magnitudes here were chosen mainly with an eye for convenience, though such things as ratios of capital and imports to output were intended to be within the bounds of reasonable experience. Their sizes are less crucial; what is important is that real net claims on foreigners (F, which corresponds to small "f" in Chapter 5), not take on too large negative values. We use a value of zero in most simulations, but we illustrate the effect of a substantially negative F in some of the simulations reported below.

Use of the TROLL LIMO feature confirms that the basic model is stable with the choice of parameters and starting values mentioned above. For instance, when money is targetted using the interest rate, there are six non-trivial characteristic roots, two complex pairs and two real roots. Each root has a magnitude less than unity. Of the complex roots, one pair has a very long period, 235 years, and this does not contribute significantly to the fluctuations in the plots presented below. The second pair is the interesting one; it has a period of 14.6 years. A sensitivity analysis performed by LIMO confirms that this root pair depends crucially on the values assigned to β and γ. Only these two parameters affect the magnitude of the roots with an elasticity greater than 0.1.

Figures 3 and 4 illustrate the proposition developed in Chapter 2 that choosing money or nominal income as an intermediate target makes a considerable difference to the dynamics of adjustment. Both simulations start from an equilibrium where real variables are constant and the money supply and prices grow at 10% per annum; the model implies that, in the absence of shocks, these values would prevail forever. The figures depict the behaviour of real output Y and the rate of inflation DNP in response to a permanent lowering of the rate of money or nominal income growth to 9%. Since real income is constant in steady state, each policy produces an inflation of 9% after all variables have settled down. However, use of nominal income as intermediate target gives a much smoother path. Money control gives a quicker deceleration of inflation, but because of the large cycles initial gains are later reversed. The lags, in years, seem very long between the implementation of a lower money growth rate and permanently lower inflation; though the parameter values chosen are necessarily somewhat arbitrary, the results do suggest that success in lowering inflation in this way is likely to be long in coming, and to be subject to reversals. Nominal income control clearly gives a smoother and quicker adjustment in this model. However, even here the rate of inflation does not decelerate smoothly but rather overshoots its long-run value and subsequently rises.

Figures 5 to 8 compare the response of selected endogenous variables to the use of different instruments for controlling money, assuming that the the latter is the intermediate target. It can be seen that the use of interest rates to lower money growth produces over the medium term a smaller value for the current account balance (DF)[31] and

31 More precisely, this is the first difference in the stock of indebtedness to foreigners divided by the domestic price level.

Figure 3

NOMINAL INCOME VERSUS MONEY AS INTERMEDIATE TARGET

Using Interest Rate Instrument

RESPONSE OF REAL OUTPUT (Y)

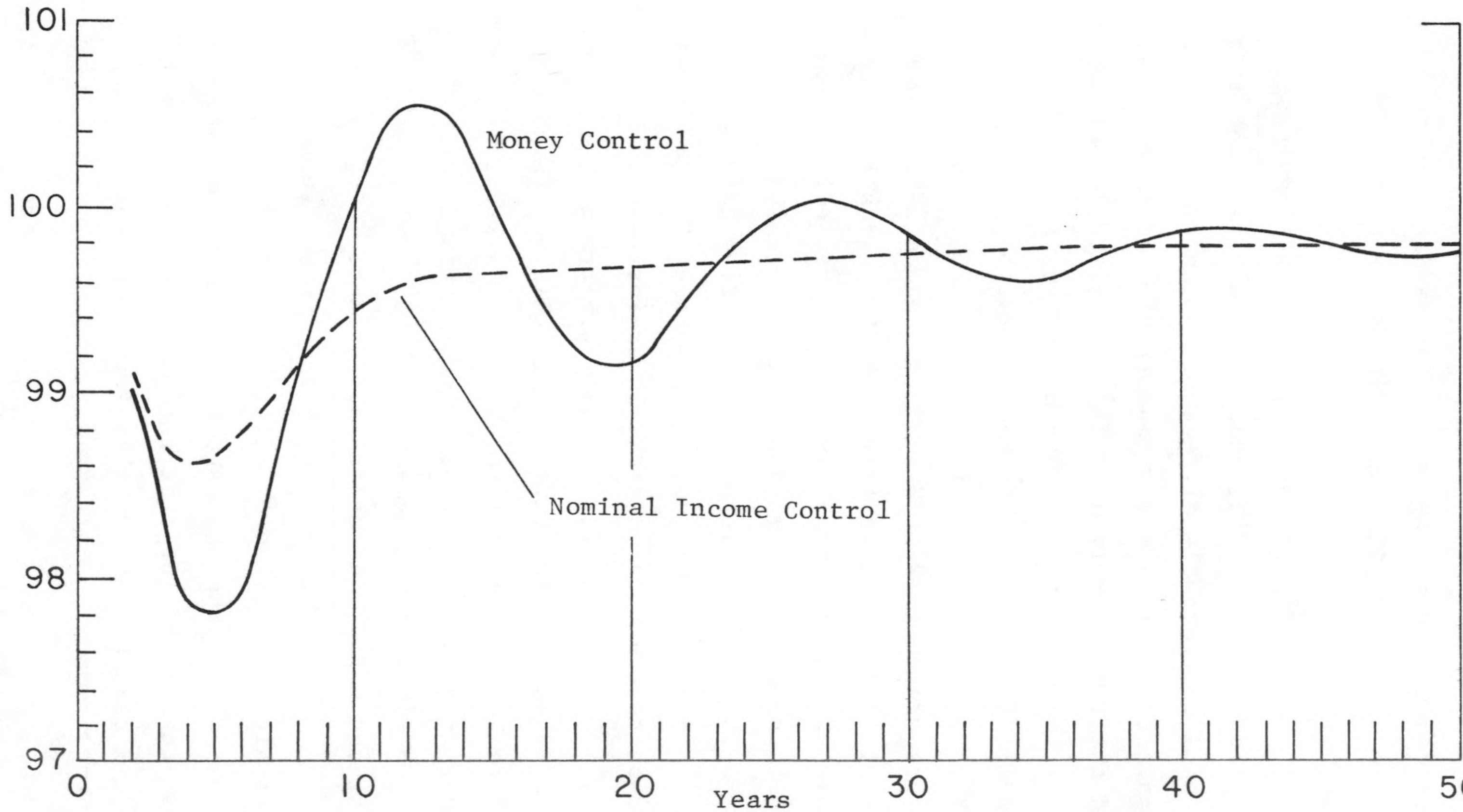


Figure 4

NOMINAL INCOME VERSUS MONEY AS INTERMEDIATE TARGET

Using Interest Rate Instrument

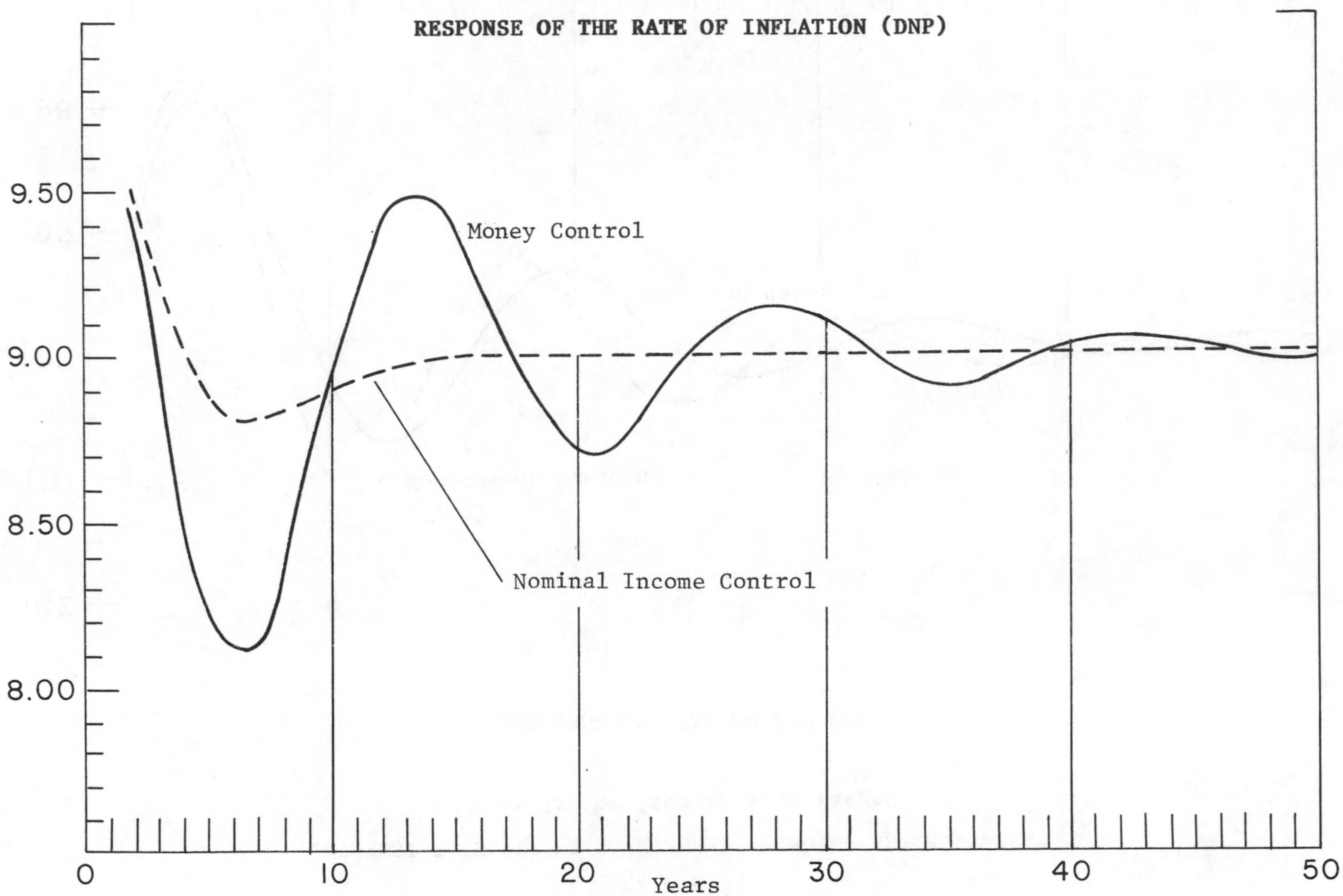


Figure 5

INTEREST RATE VERSUS GOVERNMENT SPENDING AS INSTRUMENT

Money Is the Intermediate Target

RESPONSE OF REAL OUTPUT (Y)

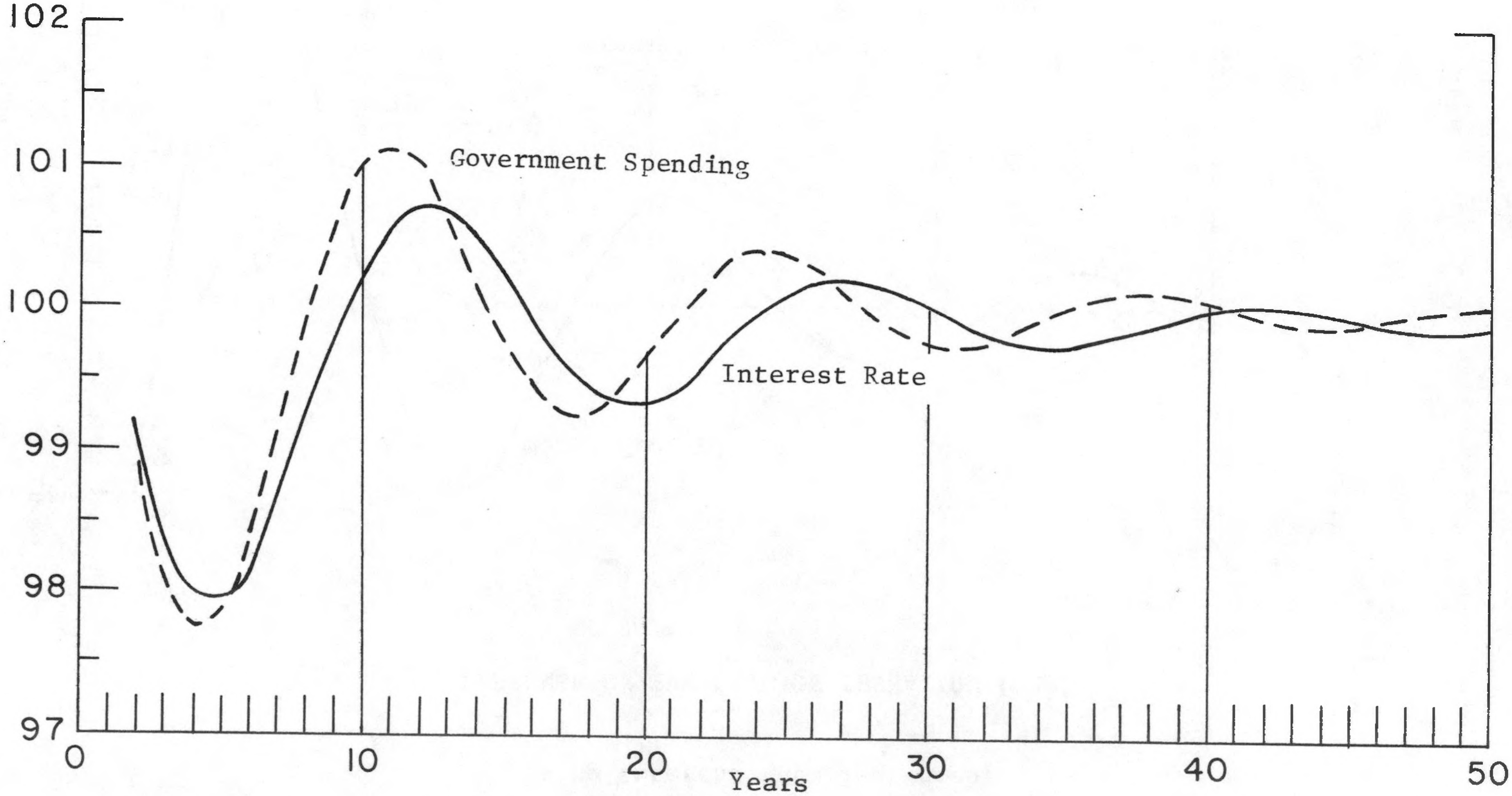


Figure 6

INTEREST RATE VERSUS GOVERNMENT SPENDING AS INSTRUMENT

Money Is the Intermediate Target

RESPONSE OF THE RATE OF INFLATION (DNP)

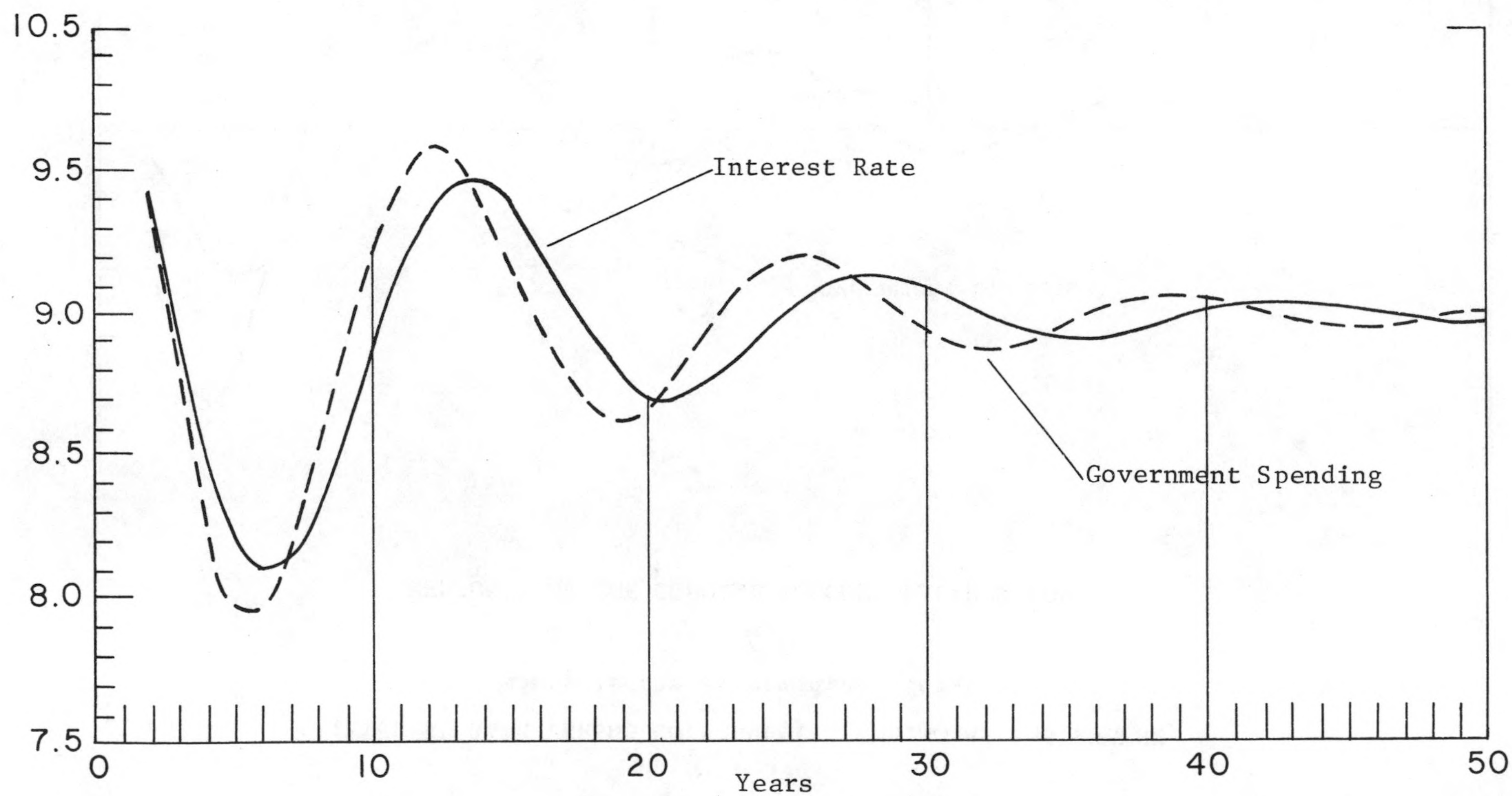


Figure 7

INTEREST RATE VERSUS GOVERNMENT SPENDING AS INSTRUMENT

Money Is the Intermediate Target

RESPONSE OF THE CURRENT ACCOUNT BALANCE (DF)

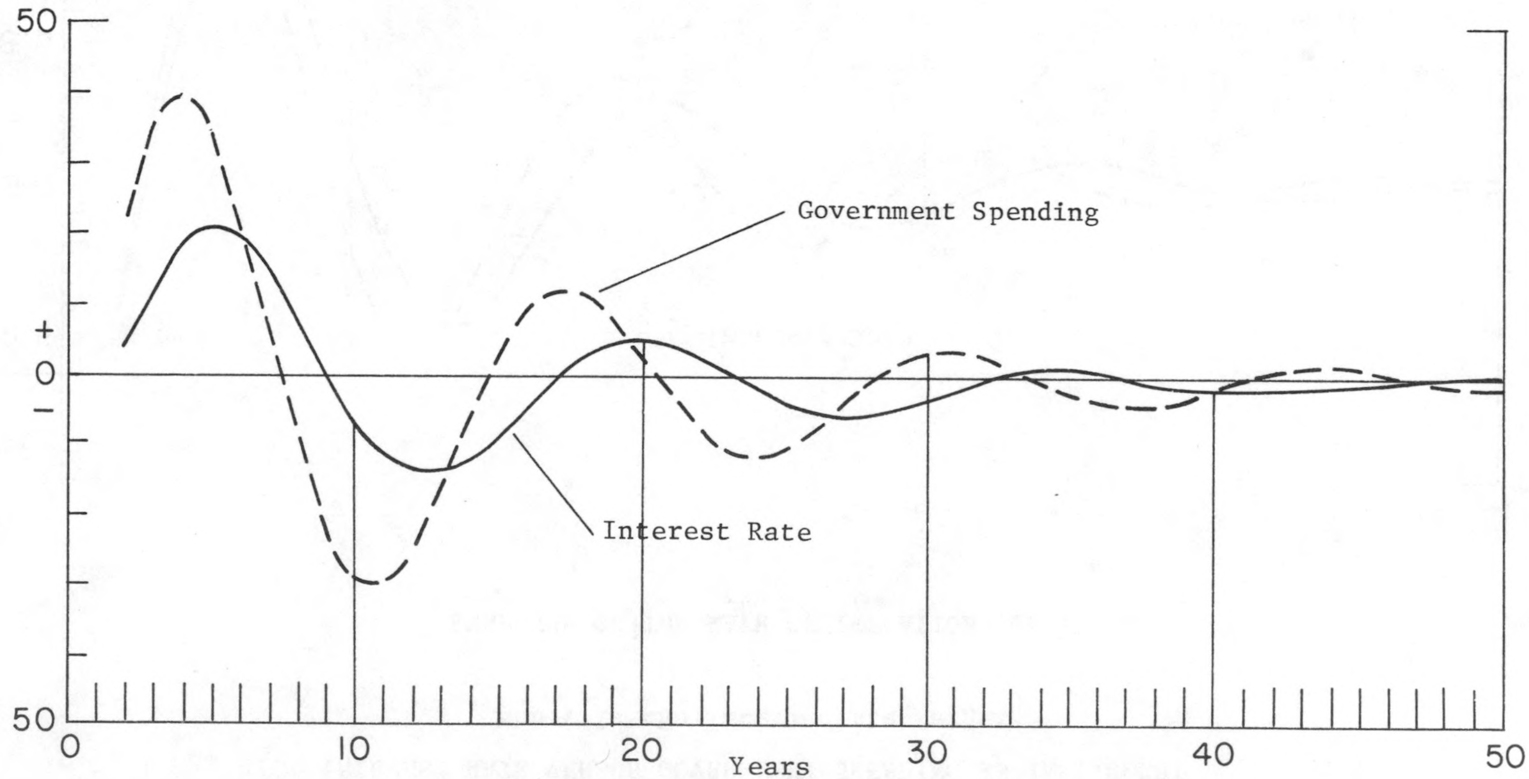
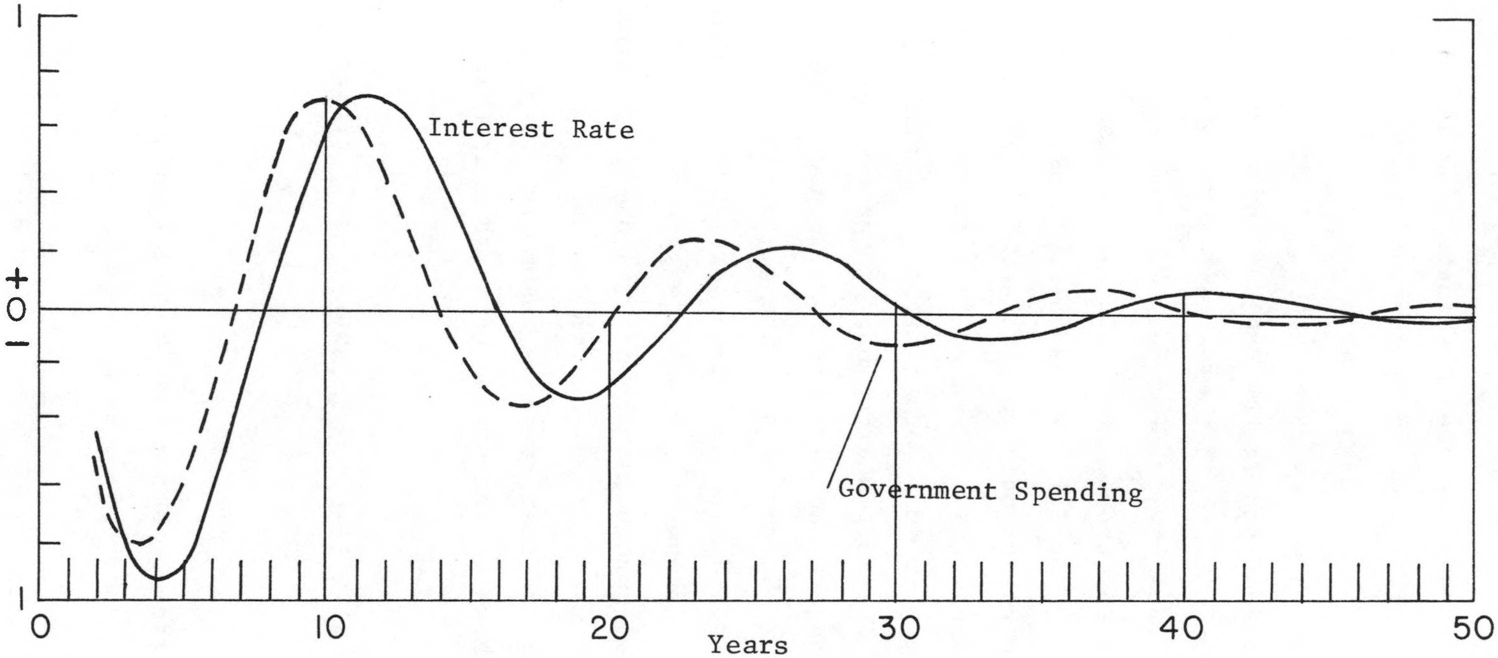


Figure 8

INTEREST RATE VERSUS GOVERNMENT SPENDING AS INSTRUMENT

Money Is the Intermediate Target

RESPONSE OF NET INVESTMENT (DK)



less net investment (DK) than the use of government spending matched by tax receipts. Nevertheless, one is struck by the similarity of the cyclical response whatever the instrument used. The different incidence of the two policies on investment and on financial markets is dominated by the interaction of an interest-elastic money demand, sluggish adjustment of prices and adaptive expectations of inflation, which produces long cycles in output and in the rate of inflation.

Chapter 3 suggested that in a closed economy model without endogenous capacity, use of real interest rate variations to hit money was more likely to produce cyclical results than use of government spending, provided the demand for money was interest elastic. Figure 9 compares the path for output associated with each of the two policies in the absence of asset stock effects. In order to highlight the differences only 20 periods are plotted. Both exhibit similar cycles but the period of the cycles for Y resulting from use of the interest rate instrument is somewhat longer than that for tax-financed government spending.

We now turn to the significance of asset stock effects. Chapter 3 showed that making capacity output endogenous could be a source of instability when interest rates are the policy instrument, but not when tax-financed government spending is used. Our analysis there implied that aggregate demand effects had to dominate aggregate supply effects for the use of real interest rates as instrument to yield a stable adjustment path. We have chosen parameters such that this condition is satisfied, since

$$(c_2 + d)\ k/(1-c_1) = (0.02 + 0.13)\ 2.5/(1-0.5) = 0.75$$

and $\alpha = 0.375$

It can in fact be shown that a corresponding necessary, but not sufficient, condition[32] in an open economy model is that

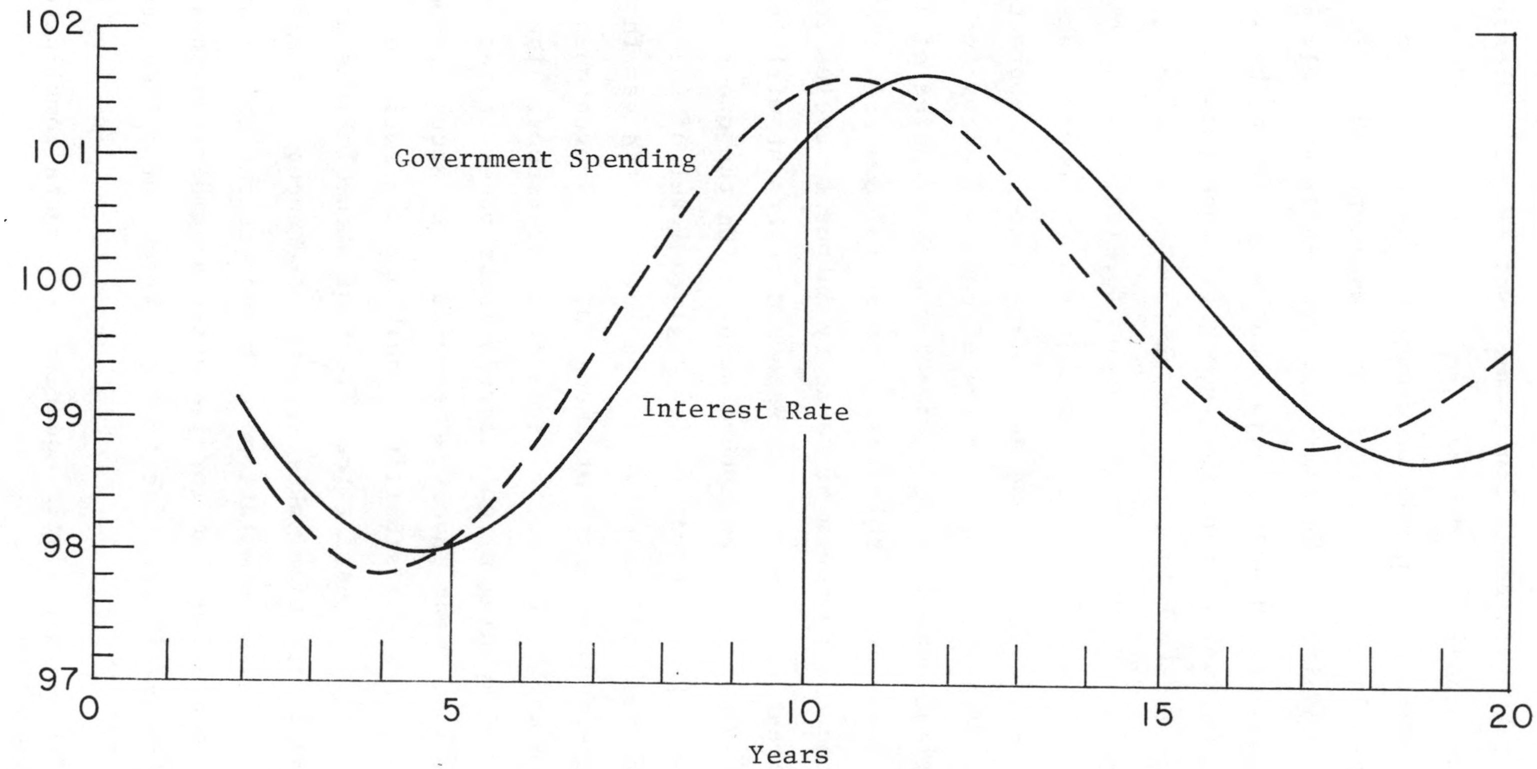
32 This did not arise in Chapter 5, since K was exogenous there.

Figure 9

INTEREST RATE VERSUS GOVERNMENT SPENDING INSTRUMENTS IN THE ABSENCE OF ASSET STOCK EFFECTS

Money Is the Intermediate Target

RESPONSE OF REAL OUTPUT (Y)



$$(c_2 + d)\ k/(1-c_1 + i) > \alpha$$

and that this condition is also satisfied.[33] Clearly if $\alpha = 0$ aggregate demand conditions dominate and stability is assured. Can one presume that the greater is α, the more cyclical is the response of income to money targetting using interest rates as instrument? In the model of Chapter 3, the opposite is the case: cycles are less likely when capacity is endogenous. However, adjustment will tend to be slower. Figure 10 gives the cyclical pattern for income (a) when capacity is exogenous (this path is common to both money and government spending instruments in the case considered, where the weight of foreign goods in the CPI is zero and where money is not interest elastic); and (b) when α takes on the value 0.375, for interest rates and government spending separately. It can be seen that here the endogenization of capacity makes for less, not more, cyclical behaviour of income for either choice of instrument. The economics behind this result is as follows. On the one hand the endogenous response of capacity output to cycles in income and the interest rate means that economic restraint will lead to earlier pressure on capacity when an upturn occurs. On the other hand, the gap will not be as wide in the downturn for a given level of income as when capacity is exogenous, and since it is the level of the gap that puts downward pressure on the rate of change of prices even when income is increasing, there may be less of a tendency to overshoot. The possibility that such cycle-dampening effects may dominate makes it less clear that the interest rate instrument involves perverse asset stock movements, provided of course that the stability conditions are satisfied.

As for the effect of accumulations of claims on foreigners, allowing for them makes little differences to the dynamics, and a comparison of simulations with and without this linkage is not reported. The conclusion that foreign asset accumulation does not make a great difference in practice for the effects of policy instruments is largely

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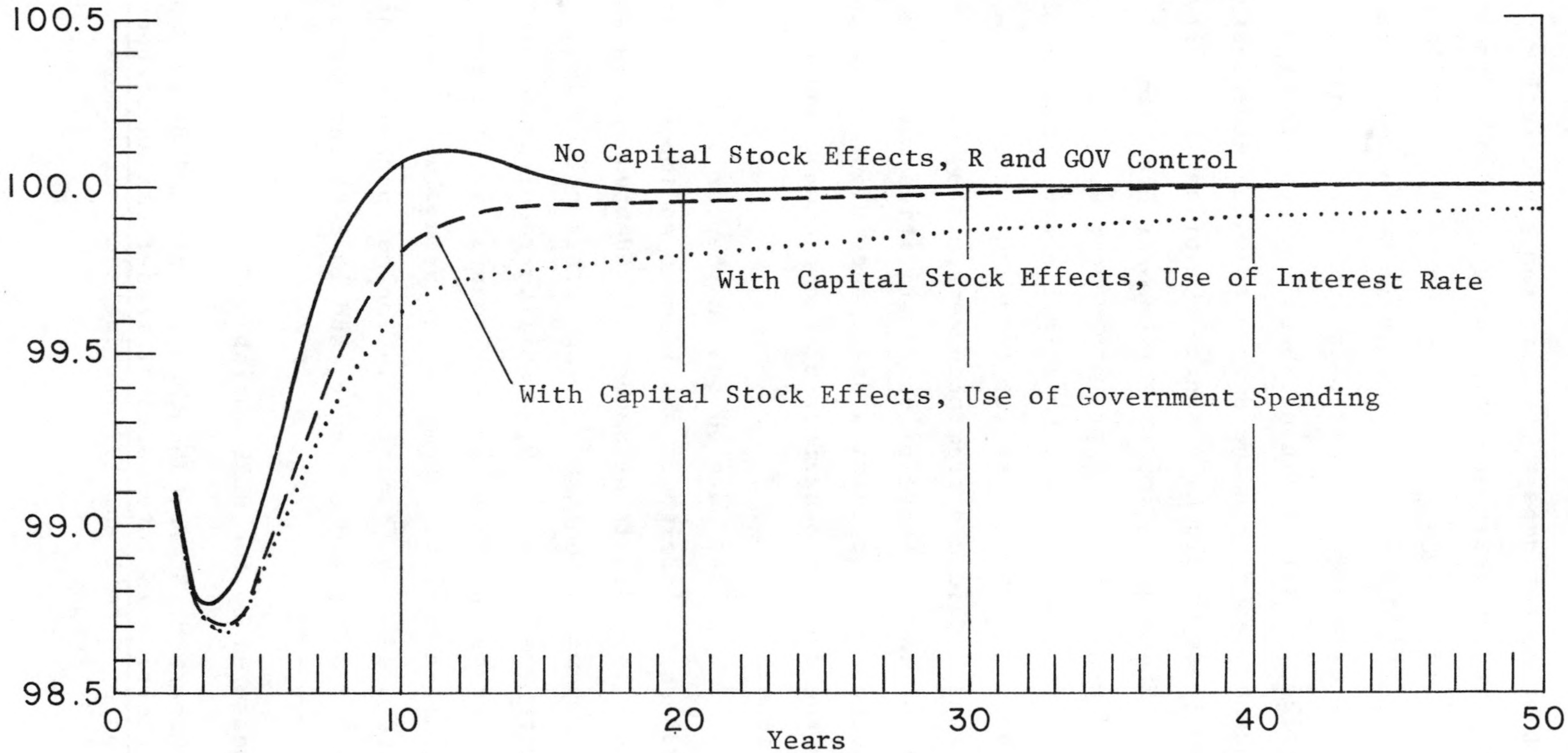
33 The characteristic equations are quite complicated and are not presented here.

Figure 10

THE IMPORTANCE OF CAPITAL STOCK EFFECTS

Money Not Interest Elastic

RESPONSE OF REAL OUTPUT



independent of the choice of parameter values, provided variables take on values that do not depart greatly from historical experience. Variations in both interest rates and government spending affect imports through variations in aggregate demand, which is the important source of fluctuations. The reason for this is as follows. Interest rate movements affect the exchange rate directly, which in turn through variations of competitiveness will influence both imports and exports. The extent of this channel depends on the degree to which expectations of the exchange rate level are weakly held and on the price elasticity of the trade balance. Since goods imports or exports are each only some 30 per cent of GNP and 7 per cent of net private sector wealth,[34] the trade balance must respond very substantially to relative price changes for it to contribute enough variation to wealth for feedbacks onto aggregate demand to be significant. Evidence from econometric studies suggests that neither exports nor imports have price elasticities much superior to unity.[35] Hence accounting for foreign asset accumulations is unlikely to change greatly the dynamic behaviour of the economy, and this is borne out by our simulations.

It was pointed out in Chapter 5 that the initial <u>level</u> of indebtedness to foreigners should make an important difference to the dynamics. Indeed, if the amount of indebtedness is sufficiently large, the economy may be unstable using either policy instrument. In a simple model with $\gamma = 0$ and $h = 0$, stability requires the ratio of net foreign liabilities to income to be less than a_1/a_0 when government spending is used to hit a nominal income target or $a_2/(c_1r^* + c_2)$ if real interest rates are used (see Chapter 5). For our parameter choices, these ratios are 2.5 and 8.33, respectively, and in 1981 the ratio of overall Canadian

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34 As measured by the RDXF variable V.

35 See Robert M. Stern, Jonathan Francis and Bruce Schumacher, <u>Price Elasticities in International Trade: An Annotated Bibliography</u> (Toronto: Macmillan, 1976). The simulation model uses values of 1.25 and 1.0, respectively.

net international indebtedness to GNP[36] was 0.272. Though the relevant parameter values may well differ from those quoted in Appendix Table B-2, the size of the stable region seems to indicate that the net indebtedness ratio would be well inside.

Even within the region of stability, the question remains as to whether the dynamics are importantly influenced by large differences in the level of indebtedness. Figure 11 presents paths of real income when money is controlled using variations in real interest rates, for two alternatives for steady state net indebtedness: 0 and 100 (both relative to a steady state income level of 100). All parameters are the same, except that in the F = -100 case parameters X0 = 0.022 and A00 = -1, and all other variables are the same, except that GOV = 25. The larger indebtedness is accompanied by a services deficit, which is offset by a larger proportion of foreign income spent on our exports. The aggregate demand effects of lower wealth are offset by somewhat higher tax-financed government spending, leaving output and all other nominal magnitudes unchanged. As Figure 11 indicates, with our choice of parameter values a large change in the level of net indebtedness does not appreciably change the dynamics. Similar cycles are present in both paths. It is interesting to note in addition that net indebtedness speeds the adjustment, as was suggested in Chapter 5.

Finally Figures 12 and 13 consider the question of accumulations of another asset, government bonds. We recall from Chapter 4 that using real interest rates to target money or nominal income produces instability when government spending and taxes are exogenous, and bond issues are the residual form of finance. Figure 13 illustrates how real bond holdings, BR, increase without limit because higher debt service costs, associated with the higher interest rates used to bring down money demand, tend to feed upon themselves. Since government bonds are a component of net wealth in the model, and since net wealth affects consumption, the continual increase in real bond holdings requires a

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36 Bank of Canada Review, July 1982, Tables 52 and A15.

Figure 11

THE IMPORTANCE OF STEADY STATE NET INDEBTEDNESS

Money Is the Intermediate Target and Interest Rates the Instrument

RESPONSE OF REAL OUTPUT (Y)

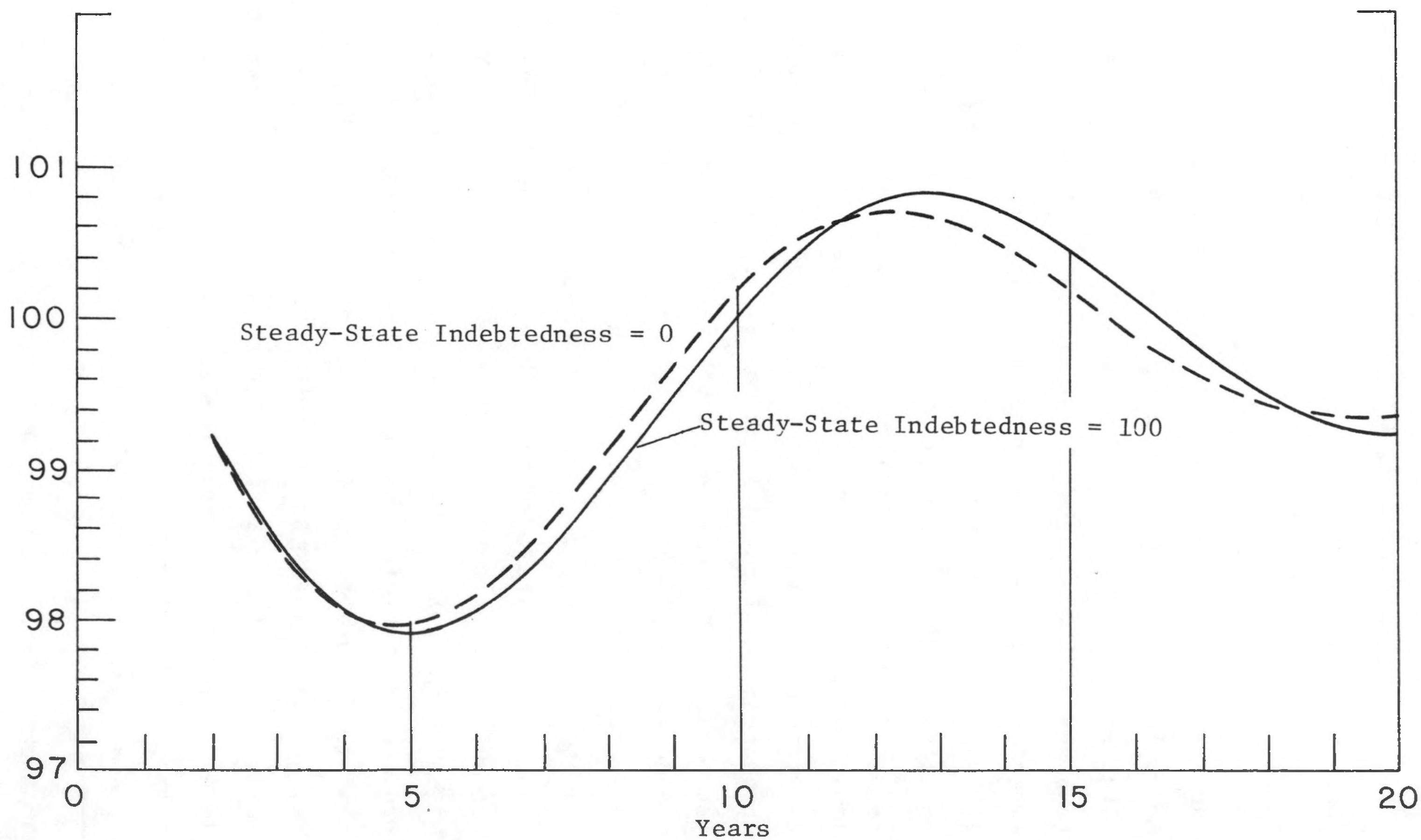


Figure 12

CHOICE OF INSTRUMENTS WITH RESIDUAL BOND FINANCING

Money Is the Intermediate Target

RESPONSE OF REAL OUTPUT (Y)

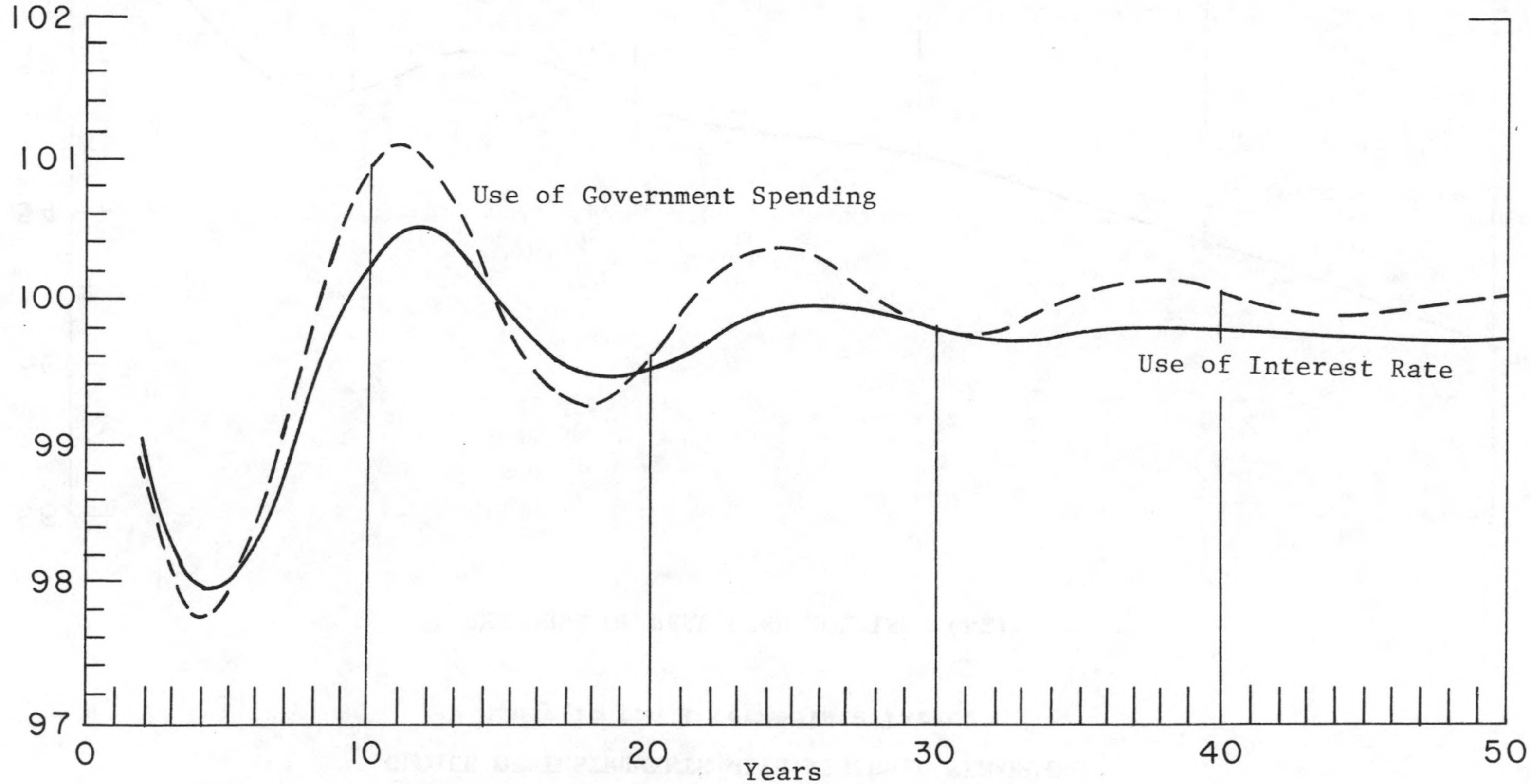
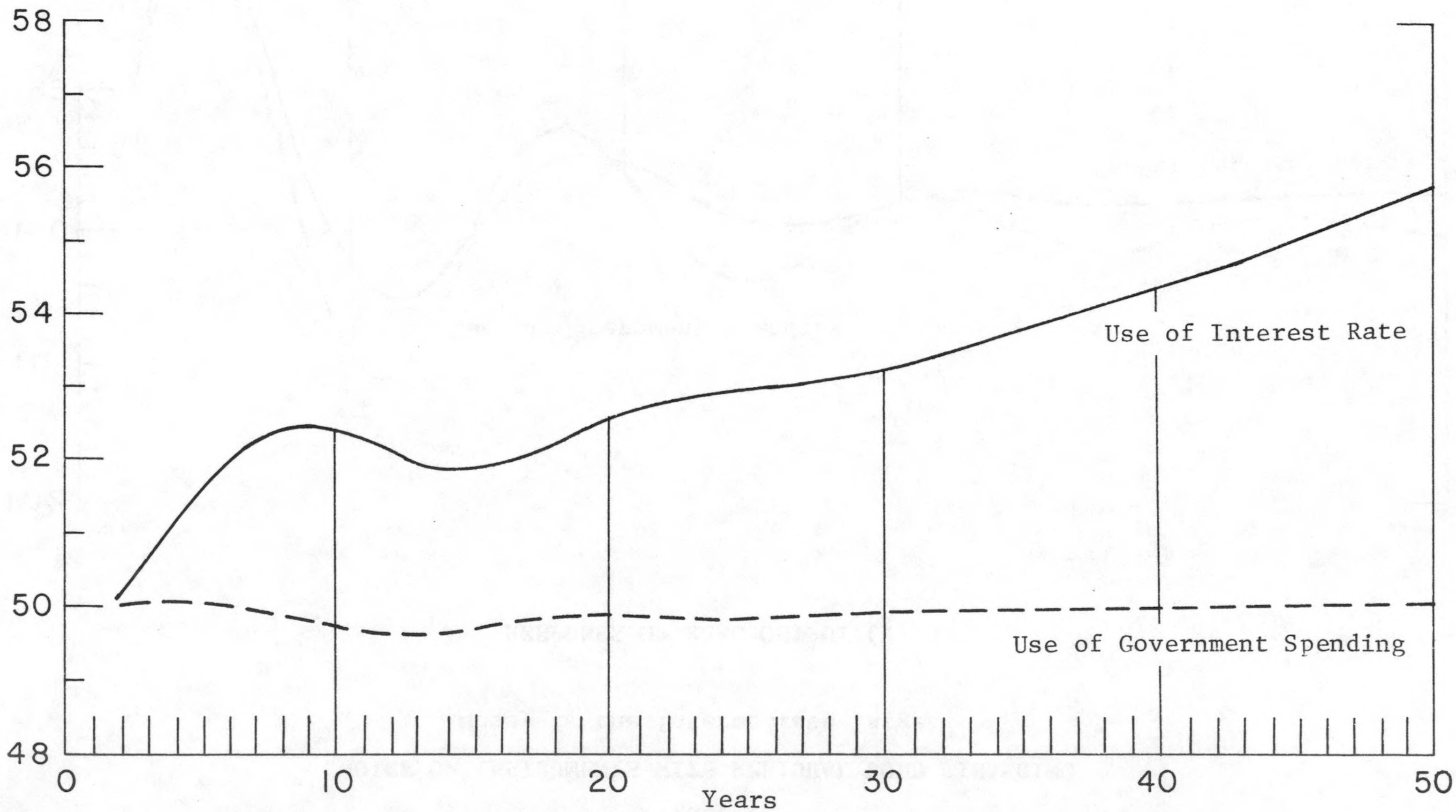


Figure 13

CHOICE OF INSTRUMENTS WITH RESIDUAL FINANCING

Money Is the Intermediate Target

RESPONSE OF REAL BOND HOLDINGS (BR)



continuing increase in real interest rates so that output and money demand are brought towards their steady state values. This spiral would be exacerbated if money demand also depended on wealth. The model therefore never settles down to a new steady state. When real interest rates are exogenous and government spending moves to achieve money targets, real bond holdings do not grow without limit, and a new steady state is eventually reached.

Chapter 7
CONCLUSIONS

It is perhaps useful to end by repeating some of the conclusions that emerge from both the mathematical derivations and the simulation results.

First, it is clear that a gradual deceleration of the money supply or of nominal income will not be accompanied by a smooth deceleration of inflation. What we have considered is a step reduction in the intermediate targets, and these are accompanied by decreases in output relative to potential and an overshoot of inflation (below its steady state level), and possibly subsequent cycles in both variables. The same type of behaviour, probably in more extreme form, would occur if intermediate targets were lowered periodically as part of a program of gradualism.

A second conclusion, arrived at with the simplest model and in the model when it is generalized to account for asset stocks and open economy considerations, is that a policy aimed at nominal income growth will be accompanied by less severe swings in real variables and the inflation rate than a policy oriented towards the money supply, where money demand depends inversely on the level of nominal interest rates. When money is targetted, the interaction of an interest-elastic money demand, sluggish price adjustment and adaptive expectations of inflation may in fact produce an unstable path for the economy. Even if it does not, it will likely involve cycles because of the following mechanism. Although slack in output markets reduces the rate of increase in prices and lower growth in nominal income leads to lower growth in money demand, lower inflation lowers nominal interest rates which increases money demand, requiring a further decrease in output. Money targetting thus involves maintaining restraint longer in downturns and maintaining stimulation longer in upturns, leading to more cyclical behaviour than does targetting nominal income directly. It is worth repeating here the limited scope of the analysis: it is assumed that the authorities have equal knowledge of the relationships explaining aggregate demand and the demand for money, and that there are no lags in the availability of the data for either real or financial variables. If these assumptions do not hold, as they do not in the real world, then the practical advantages of targetting a financial variable may outweigh the considerations brought out here.

A third theme of this paper has been that if one considers using both monetary and fiscal aggregate demand policies, in the sense of real interest rates and tax-financed government spending, as instruments to hit an intermediate target, one may be preferable to the other because of their different effects on the economy. In particular each will have a different effect on the capital stock, on indebtedness to foreigners, and on the stock of government debt.

As concerns the capital stock, the use of real interest rates as instrument will involve instability if induced aggregate supply effects are larger than the aggregate demand effects, the latter deriving from wealth effects on consumption and from the continuing flow of replacement investment. If aggregate demand effects do dominate and the system is stable, use of real interest rates need not produce more cyclical adjustment than government spending, however. In fact, making aggregate supply move procyclically may dampen cycles by preventing the gap between actual and potential output from becoming too large.

As concerns the accumulation of indebtedness vis-à-vis foreigners, which is equal to the current account deficit, use of real interest rates as the control instrument to moderate the growth in spending can be expected to lead to a weaker current account than use of tax-financed government spending reduction, because the former will tend to involve a lower price of foreign exchange. However, we have argued above that, in the light of both simulation results and experience, the magnitudes are not such that this channel will produce a fundamentally different dynamic behaviour. The level of indebtedness to foreigners may itself be important, because if it is large enough it may imply perverse response to aggregate demand policies: however, the critical value here would seem to be well beyond that relevant to Canada.

Our model confirms the dangers inherent in a policy that uses real interest rates to hit an intermediate target--whether money or nominal income--and sets taxes and government expenditures independently of the stock of government debt. In such a context a slowing of aggregate spending leads to an increase in government debt that will feed upon itself, and the result will be an unstable path for the government debt and eventually for the real economy. On the other hand, use of tax-financed government spending as instrument to achieve either intermediate target will produce a stable adjustment, and government debt will not exhibit explosive growth.

Characterizing the dynamics of adjustment of such a complicated model is difficult, but several interesting conclusions emerge. Asset stocks are a possible source of explosive behaviour but not of cycles. Indeed, their presence may be associated with non-cyclical behaviour where cycles existed otherwise, or with dampened cycles. The cyclical process inherent in our model results from the interaction of sticky prices and adaptive expectations of inflation. Targetting money rather than nominal income makes the resulting cycles more extreme, because of cyclical variations in velocity.

APPENDIX A
DERIVATION OF CHARACTERISTIC EQUATIONS IN CHAPTER 5

The table below presents the result of linearizing the model in Chapter 5 around the new steady state; the order of the equations is (14), (17), (18), (16) and (13).

THE SYSTEM OF EQUATIONS FOR THE OPEN-ECONOMY MODEL

$$\begin{vmatrix} (1-c_1)\eta & 0 & -(1-c_1+i) & 0 & a_2 & (c_1 r^*+c_2) \\ 0 & -\gamma D & \rho D+\Phi(1-\beta\gamma) & 1 & h(1-\beta\gamma)D & 0 \\ 0 & 1/a_1 & -(a_0/a_1)g & 0 & 1 & a_0/a_1 \\ 0 & 0 & -\beta\Phi & D & -\beta hD & 0 \\ 0 & 0 & i & 0 & -(gD+a_2) & (D-r^*) \end{vmatrix} \begin{vmatrix} \delta \ln GOV \\ \delta r \\ \delta \ln Y \\ \delta\pi \\ \delta \ln t \\ \delta f/Y \end{vmatrix}$$

$$= \begin{vmatrix} -xa_Y & -c_1 g & 0 & 0 \\ 0 & 0 & \Phi(1-\beta\gamma) & 1 \\ 0 & (1/a_1) & 0 & 0 \\ 0 & 0 & -\beta\Phi & 0 \\ xa_Y & g & 0 & 0 \end{vmatrix} \begin{vmatrix} \delta \ln Y^* \\ \delta r^* \\ \delta \ln Y^C \\ \delta\mu \end{vmatrix}$$

where η = GOV/Y, g = f/Y, a_Y = Y*/Y, all evaluated at steady state values, and $a_2 = x' a_Y - i - i'$.

Note that term a_2 is related to the Marshall-Lerner condition and it must be positive for the trade balance to improve in response to a devaluation, if only relative prices are considered. The term can be written in a more familiar form by noting that if trade is initially balanced,

$$x\ a_y = i,$$

and thus

$$a_2 = xa_y \left(\frac{x'}{x} - \frac{i'}{i} - 1 \right)$$

and a_2 is positive if and only if the sum of the magnitudes of export and import price elasticities exceed unity. It will be assumed that they are.

The characteristic equations for the use of the real interest rate or tax-financed government spending are obtained by evaluating the determinant of the left-hand-side matrix, with either column two or column one deleted. In each case the other instrument is being kept constant at its steady state value. We will call these determinants Δ_i and Δ_{ii}, respectively.

First note that the value of the determinant is unchanged by elementary row or column operations. Let us add -D times row 2 to row 4, and add g times column 6 to column 3. When column 2 is deleted, the result is as follows.

$$\Delta_i = \begin{vmatrix} (1-c_1)\eta & [-(1-c_1+i)+g(c_1r^*+c_2)] & 0 & a_2 & (c_1r^*+c_2) \\ 0 & \rho D+\phi(1-\beta\gamma) & 1 & h(1-\beta\gamma)D & 0 \\ 0 & 0 & 0 & 1 & a_0/a_1 \\ 0 & -[\rho D^2+\phi(1-\beta\gamma)D+\beta\phi] & 0 & -hD[(1-\beta\gamma)D+\beta] & 0 \\ 0 & i+g(D-r^*) & 0 & -(gD+a_2) & (D-r^*) \end{vmatrix}$$

$$= (1-c_1)\eta\,(-1) \begin{vmatrix} 0 & 1 & a_0/a_1 \\ -A(D) & -hD[(1-\beta\gamma)D+\beta] & 0 \\ i+g(D-r^*) & -(gD+a_2) & (D-r^*) \end{vmatrix}$$

$$= -(1-c_1)\eta\{A(D)[(D-r^*)+(a_0/a_1)(gD+a_2)]$$

$$+ hD[(1-\beta\gamma)D+\beta]\,(a_0/a_1)[i+g\,(D-r^*)]\} \qquad (A1)$$

where $A(D) = \rho D^2+\phi(1-\beta\gamma)D+\beta\phi$.

Similarly, when one deletes column 1,

$$\Delta_{ii} = \begin{vmatrix} 0 & [-(1-c_1+i)+g(c_1r^*+c_2)] & 0 & a_2 & (c_1r^*+c_2) \\ -\gamma D & \rho D+\phi(1-\beta\gamma) & 1 & h(1-\beta\gamma)D & 0 \\ 1/a_1 & 0 & 0 & 1 & a_0/a_1 \\ \gamma D^2 & -[\rho D^2+\phi(1-\beta\gamma)D+\beta\phi] & 0 & -hD[(1-\beta\gamma)D+\beta] & 0 \\ 0 & i+g(D-r^*) & 0 & -(gD+a_2) & (D-r^*) \end{vmatrix}$$

$$= (1/a_1)\{A(D)[a_2(D-r^*)+(gD+a_2)(c_1r^*+c_2)]$$

$$+ hD[(1-\beta\gamma)D+\beta][(1-c_1+i)D - r^*(1-c_1)(1+i) + ic_2]\}$$

$$+ \gamma D^2\{(1-c_1+i)[(1+g\ a_0/a_1)D - r^* + a_0a_2/a_1]$$

$$- a_2[(a_0g/a_1)(D-r^*)+ia_0/a_1]-(c_1r^*+c_2)[(a_0g/a_1)(gD+a_2)-i]\} \qquad \text{(A2)}$$

Let us consider various subcases.

If imported goods prices have no direct effect on domestic prices, so that $h = 0$, and money demand is inelastic, so γ that $= 0$, we have for each of the two cases:

$$\Delta_i = -(1-c_1)\eta\{A(D)[(1+ga_0/a_1)D - r^* + a_2a_0/a_1]\},$$

whose roots are the roots to $A(D) = 0$ and

$$D = \frac{r^*-a_2a_0/a_1}{1+ga_0/a_1} \qquad \text{(A3)}$$

and

$$\Delta_{ii} = (1/a_1)\{A(D)[(a_2+g(c_1r^*+c_2))D-a_2(r^*(1-c_1)-c_2)]\}$$

whose roots are the roots to $A(D) = 0$ and

$$D = \frac{a_2(r^*(1-c_1)-c_2}{a_2+g(c_1 r^*+c_2)} \tag{A4}$$

Consider, on the contrary, the situation where the effect of foreign prices on domestic prices is very large, so $b \to 0$ and $h \to \infty$. This may occur because of lack of market power or domestic firms producing undifferentiated goods, combined with indexation of wages to the cost of living. Here, the characteristic equations are

$$[(1-\beta\gamma)D+\beta][(a_0/a_1)[i+g(D-r^*)] = 0 \tag{A5i}$$

and

$$[(1-\beta\gamma)D+\beta][(1-c_1+i)D-r^*(1-c_1)(1+i)+ic_2] \tag{A5ii}$$

Monetary and fiscal policies share a common root, $-\beta/(1-\beta\gamma)$, which is stable provided the Cagan condition holds. The remaining roots are

$$D = r^*-i/g \tag{A6i}$$

for fiscal policy, which is stable only for a net creditor country with $0<g<i/r^*$, and

$$D = \frac{r^*(1-c_1)(1+i)-ic_2}{1-c_1+i} \tag{A6ii}$$

for monetary policy, which is likely to be unstable unless the marginal propensity to consume, c_1, is close to unity, as wealth effects in consumption are typically thought very small. This example illustrates, first, the fact that the net creditor/debtor position may make an important differences to the dynamics, and, second, that there is a fundamental asymmetry in the two policies.

A final example is of interest. Suppose that there is no effect of the level of net claims on foreigners on the exchange rate, so $a_0 = 0$. This is the case where there are no deviations from interest parity, and the country is small in that it can lend indefinitely without affecting the world rate of interest or borrow indefinitely without foreigners demanding a risk premium. Then it can be seen directly from (A1) that fiscal policy is unstable, as one of the roots is r^*. What has happened is that in this case we have reverted to a Blinder-Solow world where there is nothing to prevent borrowing (in this case, borrowing by the private sector from foreigners) to service debt, which, not surprisingly, will cause deviations from equilibrium to feed upon themselves. Thus, allowing for the possibility that endogenous risk premiums cause deviations from interest parity makes an important qualitative difference to open economy models.

APPENDIX B

TABLE 1

THE EQUATIONS OF THE SIMULATION MODEL

(When the real interest rate is used as instrument)

```
MODEL: PMAR

SYMBOL DECLARATIONS

ENDOGENOUS:
    B   DF   DK   DNP   DPI   DRL   F   K   M   P   PFX   PI   PW   R   RL   Y

DEFINITION:
    DNB   DNPFX   DNPW   IM   SBAL   T   TAX   TBAL   X   YC

EXOGENOUS:
    GOV   MU   N   PIW   RW   TBAR   YW

COEFFICIENT:
    ALPHA   A0   A00   A1   BETA   B0   C1   C2   D   DELTA   GAMMA   I0   I1   M0
    PHI   RHO   THETA   X0   X1

PARAMETER:
    SWF   SWK

EQUATIONS

    1:        DNB == PI

    2:        T == PW*PFX/P

    3:        DNPW == PIW

    4:        TAX == GOV+(R+PI)*B/P-DNB*B/P

    5:        DNPFX == DEL(1 : LOG(PFX))

    6:        IM == I0*T**I1*Y

    7:        X == X0*T**X1*YW

    8:        YC == K**ALPHA*N**(1-ALPHA)

    9:        Y = C1*(Y-TAX+RW*F+R*B/P)+C2*(F+B/P+K)+X-T*IM+DK+D*K+GOV
```

TABLE B-1
(Continued)

```
10:        DRL = DELTA*(R-RL)

11:        DK = THETA*(ALPHA*Y/(RL+D)-K)

12:        TBAL == X-T*IM

13:        SBAL == (RW+DEL(1 : LOG(T)))*F

14:        DF = SBAL+TBAL

15:        LOG(PFX) = LOG(P/PW)+1/A1*(RW-R)-A0/A1*(F/Y-A00)+LOG(
           TBAR)

16:        DNP = B0*(PI+PHI*LOG(Y/YC))+(1-B0)*(DNPFX+DNPW)

17:        DPI = BETA*(DNP-PI)

18:        LOG(M) = M0+LOG(P)+RHO*LOG(Y)-GAMMA*(R+PI)

19:        DEL(1 : K) = 0.5*SWK*(DK+DK(-1))

20:        DEL(1 : F) = 0.5*SWF*(DF+DF(-1))

21:        DEL(1 : LOG(P)) = 0.5*(DNP+DNP(-1))

22:        DEL(1 : PI) = 0.5*(DPI+DPI(-1))

23:        DEL(1 : LOG(M)) = 0.5*(MU+MU(-1))

24:        DEL(1 : LOG(B)) = 0.5*(DNB+DNB(-1))

25:        DEL(1 : LOG(PW)) = 0.5*(DNPW+DNPW(-1))

26:        DEL(1 : RL) = 0.5*(DRL+DRL(-1))
```

TABLE B-2

PARAMETERS AND INITIAL VALUES OF VARIABLES APPEARING IN THE SIMULATION MODEL
Parameter values (when targetting money)

PMA -

ALPHA	0.375	A0	0.1
A00	0.	A1	0.25
BETA	0.4	B0	0.8
C1	0.5	C2	0.02
D	0.13	DELTA	0.3
GAMMA	2.	I0	0.2
I1	-1.	M0	-1.14155
PHI	0.5	RHO	0.8
SWF	1.	SWK	1.
THETA	0.2	X0	0.02
X1	1.25		

Initial value of variables

Name	Definition	Value
B	Stock of government debt	50.0
DF	Change in real net claims on foreigners	0.0
DK	Net investment	0.0
DNP	Change in log of prices	0.1
DPI	Change in inflation expectations	0.0
DRL	Change in real long rate	0.0
F	Real net claims on foreigners	0.0
GOV	Real government spending	23.0
K	Real capital stock	250.0
M	Money stock	10.0
MU	Rate of growth (in logs) of money	0.1
N	Labour force	57.708
P	Price level	1.0
PFX	Price of foreign exchange	1.0
PI	Inflation expectations	0.1
PIW	Inflation expectations abroad	0.1
PW	Foreign price level	1.0
R	Short-term real rate of interest	0.02
RL	Long-term real rate of interest	0.02
RW	Foreign short-term real rate of interest	0.02
TBAR	Equilibrium real exchange rate	1.0
Y	Output	100.0
YW	Foreign output	1000.0

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